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Non-Local Theory of Bose-Einstein Condensate and Stopping of Light

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Abstract

The problem of an adequate description of the transport processes in Bose-Einstein condensates (CBE), including space-temporal evolution of CBE in a gravitational field is considered. The full nonlocal system for the CBE evolution is delivered including particular case and analytical solutions. We show (analytically) that a black hole can evolve in the Bose-Einstein condensate (CBE) regime. At the same time, there are modes in which black hole flickering occurs. Quantization of the black holes flickering is discovered. The corresponding nonlocal hydrodynamic equations indicated for fermions gas.

Keywords

Nonlocal Physics, Transport Processes in Bose-Einstein Condensate, Black Hole Flickering, Light Stopping, Nonlocal Hydrodynamic Equations for Fermions Gas

1. Introduction

As is known, the Bose-Einstein distribution describes the distribution of identical particles with zero or integer spin over energy levels, provided that the interaction of particles in the system is weak and can be neglected. In 1924, in the journal Zeitschrift für Physik [1], an article by Shatiendranath Bose on quantum photon statistics was published, in which he derived Planck's quantum law of radiation without any reference to classical physics.

In 1925, based on Bose's work, Einstein [2] theoretically predicted the existence of a Bose-Einstein condensate as a consequence of wave mechanics. Einstein suggested that cooling bosonic atoms to a very low temperature would cause them to fall (or "condense") into the lowest available quantum state. The term "condensation" corresponds to some analogy with the process of condensation of a gas into a liquid, although these phenomena are different—Bose-Einstein condensation occurs in the pulse space, and the distribution of particles in ordinary coordinate space does not change.

At the same time, they often talk about a new form of matter. Exactly:

1) At a very low but finite temperature, a macroscopic number of atoms or molecules fill one energy level.

2) The gas consists of non-interacting particles.

It would seem that the existence of a finite temperature should inevitably lead to thermal chaotic motion of particles. This circumstance caused the rejection of the theory by many major theoretical physicists.

However, subsequent experiments have confirmed the possibility of the existence of such effects at the macroscopic level.

In 1995, Eric Cornell and Carl Wyman from the National Institute of Standards and Technology of the USA, using laser cooling, managed to cool about 2 thousand rubidium-87 atoms to a temperature of 20 nanokelvins and experimentally confirm the existence of Bose-Einstein condensate in gases.

They were awarded the Nobel Prize in Physics in 2001 together with Wolfgang Ketterle, who four months later received Bose-Einstein condensate from sodium atoms using the principle of holding atoms in a magnetic trap.

The Nobel Prize in Physics 2001 was awarded jointly to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman "for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates."

Subsequently, Bose-Einstein condensate was obtained for many substances (see, for example, [3]). In 2010, the Bose-Einstein condensate of photons was obtained for the first time [4].

In 2014, employees of the Cold Atom Laboratory (CAL) NASA and scientists from the California Institute of Technology in Pasadena managed to create a Bose-Einstein condensate in an Earth prototype of an installation designed to work on the International Space Station. A fully functional installation for creating Bose-Einstein condensate in zero gravity was sent to the ISS in the summer of 2018. In 2020, Bose-Einstein condensate was first obtained on board the ISS, [5].

The appearance of a superconducting Bose-Einstein condensate is possible within the framework of the nonlocal theory of superconductivity (see also [6] [7] [8]).

The further development of experimental technology has made it possible to significantly reduce the speed of light in the medium and approach a complete stop of light (see, for example, [9]).

However, there is a problem of an adequate description of the transport processes in Bose-Einstein condensates (CBE), including space-temporal evolution of CBE in a gravitational field. This work is devoted to solving this problem. We intend to show (analytically) that a black hole can evolve in the Bose-Einstein condensate (CBE) regime. At the same time, there are modes in which black hole flickering occurs.

2. The Nonlocal Hydrodynamic Equations

The generalized hydrodynamic equations (GHE) can be obtained from the non-local kinetic equation in the frame of the Enskog procedure, [10] [11] [12] [13] [14]:

Continuity equation for species α

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)\right.\right.\\ \left.\left.+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)+\overleftrightarrow{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=R_\alpha.\end{gathered}\tag{2.1}$$

Continuity equation for mixture

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)\right.\right.\\ \left.\left.+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)+\overleftrightarrow{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=0.\end{gathered}\tag{2.2}$$

Momentum equation for species α

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\ &\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right)\right]\\ &-\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\ &\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\overleftrightarrow{\mathrm{I}}-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right.\right.\right.\\ &\left.+p_\alpha\overleftrightarrow{\mathrm{I}}\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\left(\mathbf{v}_0\mathbf{v}_0\right)\mathbf{v}_0+2\overleftrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot\left(p_\alpha\mathbf{v}_0\right)\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\overleftrightarrow{\mathrm{I}}p_\alpha\mathbf{v}_0\right)\\ &\left.\left.-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\left[\mathbf{v}_0\times\mathbf{B}\right]\right]\right\}\\ &=\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.\end{aligned}\tag{2.3}$$

Momentum equation for mixture

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\ &\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\sum_\alpha\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right)\right]\\ &-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\end{aligned}$$

$$-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0\times\mathbf{B}\Big]\Big\}\times\mathbf{B}+\frac{\partial}{\partial\mathbf{r}}\cdot\Big\{\rho\mathbf{v}_0\mathbf{v}_0+p\ddot{\mathrm{I}}-\sum_\alpha\tau_\alpha\Big[\frac{\partial}{\partial t}\big(\rho_\alpha\mathbf{v}_0\mathbf{v}_0$$

$$+p_\alpha\ddot{\mathrm{I}}\big)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\left(\mathbf{v}_0\mathbf{v}_0\right)\mathbf{v}_0+2\ddot{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot\left(p_\alpha\mathbf{v}_0\right)\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\ddot{\mathrm{I}}p_\alpha\mathbf{v}_0\right) \tag{2.4}$$

$$-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\left[\mathbf{v}_0\times\mathbf{B}\right]\Big]\Big\}=0$$

Energy equation for α species

$$\frac{\partial}{\partial t}\Big\{\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha-\tau_\alpha\Big[\frac{\partial}{\partial t}\Big(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\Big)$$

$$+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\Big]\Big\}$$

$$+\frac{\partial}{\partial\mathbf{r}}\cdot\Big\{\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0-\tau_\alpha\Big[\frac{\partial}{\partial t}\Big(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0$$

$$+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\Big)+\frac{\partial}{\partial\mathbf{r}}\cdot\Big(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\ddot{\mathrm{I}}$$

$$+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\ddot{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\ddot{\mathrm{I}}\Big)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}$$

$$-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]$$

$$-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\Big]\Big\}-\Big\{\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0-\tau_\alpha\Big[\mathbf{F}_\alpha^{(1)}\cdot\Big(\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)$$

$$+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial\mathbf{r}}\cdot p_\alpha\ddot{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\Big)\Big]\Big\}$$

$$=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha. \tag{2.5}$$

Energy equation for mixture

$$\frac{\partial}{\partial t}\Big\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\Big[\frac{\partial}{\partial t}\Big(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\Big)$$

$$+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\Big]\Big\}$$

$$+\frac{\partial}{\partial\mathbf{r}}\cdot\Big\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0+\mathbf{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\Big[\frac{\partial}{\partial t}\Big(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0$$

$$+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\Big)+\frac{\partial}{\partial\mathbf{r}}\cdot\Big(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\ddot{\mathrm{I}}$$

$$+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\ddot{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\ddot{\mathrm{I}}\Big)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}$$

$$-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]$$

$$-\varepsilon_\alpha n_\alpha \frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)}\Bigg]\Bigg\}-\Bigg\{\mathbf{v}_0\cdot\sum_\alpha \rho_\alpha \mathbf{F}_\alpha^{(1)}-\sum_\alpha \tau_\alpha \mathbf{F}_\alpha^{(1)}\cdot\Bigg[\frac{\partial}{\partial t}\left(\rho_\alpha \mathbf{v}_0\right)$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha \overset{\leftrightarrow}{I}-\rho_\alpha \mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\Bigg]=0. \tag{2.6}$$

The force dimension, $\left[F_\alpha^{(1)}\right]=\frac{cm}{s^2}$. Here $\mathbf{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\mathbf{B}$ —magnetic induction, $\overset{\leftrightarrow}{I}$ —unit tensor, q_α —charge of the α-component particle, p_α —static pressure for α-component, ε_α —internal energy for the particles of α-component, $\mathbf{v}_0$ —hydrodynamic velocity for mixture, τ_α —non-local parameter.

The system of Equations (2.1)-(2.6) is a consequence of a nonlocal kinetic equation with respect to a single-particle distribution function

$$\frac{Df_\alpha}{Dt}-\frac{D}{Dt}\tau_\alpha\frac{Df_\alpha}{Dt}=J^B, \tag{2.7}$$

where J^B is a local collision integral. It is important to notice that causes exist:

1) $\tau_\alpha>0$, in the case of approximation of a non-local term that occurs when the meshing equations of the Bogolyubov chain break, against the direction of the arrow of time (approximation by the accomplished past).

2) $\tau_\alpha<0$, in the case of approximation of a non-local term that occurs when the Bogolyubov chain meshing equations break, in the direction of the arrow of time (approximation by hypothetical future).

3) $\tau_\alpha=0$, the local case (Boltzmann theory).

3. The Basic of Nonlocal Equations for Non-Interacting Bosons, $p_\alpha=0$

We remember that a finite temperature does not lead to thermal chaotic motion of boson particles. Then $p_\alpha=0$ and the system of Equations (2.1)-(2.6) can be simplified, namely:

Continuity equation for species α

$$\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\Bigg\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\Bigg[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\Bigg]\Bigg\}=R_\alpha. \tag{3.1}$$

Continuity equation for mixture

$$\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\Bigg\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\Bigg[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\Bigg]\Bigg\}=0. \tag{3.2}$$

Momentum equation for species α

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho_\alpha \mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0-\rho_\alpha \mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0\times\mathbf{B}\right]\right\}\\
&-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0)\right)\right]-\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0)\right.\right.\\
&\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0-\rho_\alpha \mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho_\alpha \mathbf{v}_0\mathbf{v}_0\right.\\
&-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0-\mathbf{F}_\alpha^{(1)}\rho_\alpha \mathbf{v}_0-\rho_\alpha \mathbf{v}_0\mathbf{F}_\alpha^{(1)}\right.\\
&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}\\
&=\int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.
\end{aligned}\tag{3.3}$$

Momentum equation for mixture

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho \mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0-\rho_\alpha \mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0\times\mathbf{B}\right]\right\}\\
&-\sum_\alpha\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha \mathbf{v}_0)\right)\right]-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0)\right.\right.\\
&\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0-\rho_\alpha \mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho \mathbf{v}_0\mathbf{v}_0\right.\\
&-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0-\mathbf{F}_\alpha^{(1)}\rho_\alpha \mathbf{v}_0-\rho_\alpha \mathbf{v}_0\mathbf{F}_\alpha^{(1)}\right.\\
&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha \mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}=0.
\end{aligned}\tag{3.4}$$

Energy equation for α species

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2}+\varepsilon_\alpha n_\alpha-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\varepsilon_\alpha n_\alpha \mathbf{v}_0\right)\right.\right.\\
&\left.\left.-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha \mathbf{v}_0\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\varepsilon_\alpha n_\alpha \mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\varepsilon_\alpha n_\alpha \mathbf{v}_0\right)\right.\right.\\
&+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha n_\alpha \mathbf{v}_0\mathbf{v}_0\right)-\rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}\\
&\left.\left.-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)}\right]\right\}-\left\{\rho_\alpha \mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\right.\\
&\left.-\tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha \mathbf{v}_0\mathbf{v}_0-\rho_\alpha \mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\}\\
&=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.
\end{aligned}\tag{3.5}$$

Energy equation for mixture

$$\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)\right.\right.$$
$$\left.\left.-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\mathbf{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\right.\right.\right.$$
$$\left.+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)} \tag{3.6}$$
$$\left.\left.-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\mathbf{v}_0\times\mathbf{B}\right]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}-\left\{\mathbf{v}_0\cdot\sum_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.$$
$$\left.-\sum_\alpha\tau_\alpha\mathbf{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\right]\right\}=0.$$

I remind that the force dimension, $\left[F_\alpha^{(1)}\right]=\frac{cm}{s^2}$. Here $\mathbf{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\mathbf{B}$—magnetic induction, $\overset{\leftrightarrow}{I}$—unit tensor, q_α—charge of the α-component particle, p_α—static pressure for α-component, ε_α—internal energy for the particles of α-component, $\mathbf{v}_0$—hydrodynamic velocity for mixture, τ_α—non-local parameter.

4. System of Non-Local Equations for the Case $p_\alpha = 0$, $\mathbf{v}_0 = 0$

In the following we intend to consider the particular case of the basic nonlocal equations taking into account the important applications.

Assume that there is no directional motion of boson particles with hydrodynamic velocity, $\mathbf{v}_0 = 0$.

We find:

Continuity equation for species α

$$\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\frac{\partial\rho_\alpha}{\partial t}\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}\right)=R_\alpha. \tag{4.1}$$

Continuity equation for mixture

$$\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\frac{\partial\rho_\alpha}{\partial t}\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\sum_\alpha\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}=0. \tag{4.2}$$

Momentum equation for species α

$$\frac{\partial}{\partial t}\left(\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}\right)-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\frac{\partial\rho_\alpha}{\partial t}\right]-\frac{q_\alpha}{m_\alpha}\left(\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}\times\mathbf{B}\right)=0. \tag{4.3}$$

In the absence of an external magnetic field we have

$$\frac{\partial}{\partial t}\left(\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}\right)-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\frac{\partial\rho_\alpha}{\partial t}\right]=0. \tag{4.4}$$

Momentum equation for mixture

$$\frac{\partial}{\partial t}\sum_\alpha\tau_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}-\sum_\alpha\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\frac{\partial\rho_\alpha}{\partial t}\right]-\sum_\alpha\frac{q_\alpha}{m_\alpha}\tau_\alpha\left[\rho_\alpha\mathbf{F}_\alpha^{(1)}\times\mathbf{B}\right]=0. \tag{4.5}$$

In the absence of an external magnetic field we have

$$\frac{\partial}{\partial t}\sum_{\alpha}\tau_{\alpha}\rho_{\alpha}\mathbf{F}_{\alpha}^{(1)}-\sum_{\alpha}\mathbf{F}_{\alpha}^{(1)}\left[\rho_{\alpha}-\tau_{\alpha}\frac{\partial\rho_{\alpha}}{\partial t}\right]=0. \tag{4.6}$$

Energy equation for α species

$$\begin{aligned}&\frac{\partial}{\partial t}\left[\varepsilon_{\alpha}n_{\alpha}-\tau_{\alpha}\frac{\partial}{\partial t}\left(\varepsilon_{\alpha}n_{\alpha}\right)\right]+\frac{\partial}{\partial\mathbf{r}}\cdot\left[\tau_{\alpha}\varepsilon_{\alpha}n_{\alpha}\mathbf{F}_{\alpha}^{(1)}\right]-\tau_{\alpha}\rho_{\alpha}\mathbf{F}_{\alpha}^{(1)}\cdot\mathbf{F}_{\alpha}^{(1)}\\&=\int\left(\frac{m_{\alpha}v_{\alpha}^{2}}{2}+\varepsilon_{\alpha}\right)J_{\alpha}^{st,el}\mathrm{d}\mathbf{v}_{\alpha}+\int\left(\frac{m_{\alpha}v_{\alpha}^{2}}{2}+\varepsilon_{\alpha}\right)J_{\alpha}^{st,inel}\mathrm{d}\mathbf{v}_{\alpha}.\end{aligned} \tag{4.7}$$

But there is no dependence on velocity, then

$$\frac{\partial}{\partial t}\left[\varepsilon_{\alpha}n_{\alpha}-\tau_{\alpha}\frac{\partial}{\partial t}\left(\varepsilon_{\alpha}n_{\alpha}\right)\right]+\frac{\partial}{\partial\mathbf{r}}\cdot\left[\tau_{\alpha}\varepsilon_{\alpha}n_{\alpha}\mathbf{F}_{\alpha}^{(1)}\right]-\tau_{\alpha}\rho_{\alpha}\mathbf{F}_{\alpha}^{(1)}\cdot\mathbf{F}_{\alpha}^{(1)}=0. \tag{4.8}$$

Energy equation for mixture

$$\frac{\partial}{\partial t}\left[\sum_{\alpha}\varepsilon_{\alpha}n_{\alpha}-\sum_{\alpha}\tau_{\alpha}\frac{\partial}{\partial t}\left(\varepsilon_{\alpha}n_{\alpha}\right)\right]+\frac{\partial}{\partial\mathbf{r}}\cdot\sum_{\alpha}\tau_{\alpha}\varepsilon_{\alpha}n_{\alpha}\mathbf{F}_{\alpha}^{(1)}-\sum_{\alpha}\tau_{\alpha}\rho_{\alpha}\mathbf{F}_{\alpha}^{(1)}\cdot\mathbf{F}_{\alpha}^{(1)}=0. \tag{4.9}$$

If the forces $\mathbf{F}_{\alpha}^{(1)}$ are absent continuity equation for species α takes the form for non-interaction particles

$$\frac{\partial}{\partial t}\left\{\rho_{\alpha}-\tau_{\alpha}\frac{\partial\rho_{\alpha}}{\partial t}\right\}=0 \tag{4.10}$$

or

$$\rho_{\alpha}-\tau_{\alpha}\frac{\partial\rho_{\alpha}}{\partial t}=const=C_{1\alpha}. \tag{4.11}$$

For the particles of the same masses m but different energies we reach

$$n_{\alpha}-\tau_{\alpha}\frac{\partial n_{\alpha}}{\partial t}=\frac{C_{1\alpha}}{m}. \tag{4.12}$$

For the average number n_{α} of particles in a given state ε_{α} according to Bose- Einstein statistics, we get

$$n_{\alpha}-n_{\alpha}\mathrm{e}^{\frac{\varepsilon_{\alpha}}{k_BT}}\mathrm{e}^{-\frac{\mu}{k_BT}}=-g_{\alpha}n_{0}, \tag{4.13}$$

with $\varepsilon_{\alpha}>\mu$ and where g_{α} is the degeneracy of energy level α, ε_{α} is the energy of the α-th state, μ is the chemical potential, k_B is the Boltzmann constant, and T is absolute temperature. Compare (4.12) and (4.13) we find

$$n_{\alpha}\mathrm{e}^{\frac{\varepsilon_{\alpha}}{kT}}\mathrm{e}^{-\frac{\mu}{kT}}=\tau_{\alpha}\frac{\partial n_{\alpha}}{\partial t} \tag{4.14}$$

or

$$\tau_{\alpha}=\mathrm{e}^{\frac{\varepsilon_{\alpha}-\mu}{kT}}\left[\frac{\partial\ln n_{\alpha}}{\partial t}\right]^{-1}. \tag{4.15}$$

The value $\left[\frac{\partial\ln n_{\alpha}}{\partial t}\right]^{-1}$ is slowly changing value, then

$$\left[\frac{\partial \ln n_\alpha}{\partial t}\right]^{-1} \sim const = \tau_{\alpha 0} \tag{4.16}$$

and

$$\tau_\alpha = \tau_{\alpha 0} \mathrm{e}^{\frac{\varepsilon_\alpha - \mu}{kT}}. \tag{4.17}$$

Intermediate conclusion:

1) Bose-Einstein statistics is a deep particular case of nonlocal physics.

2) It is known that appearance of the sign "minus" in the left hand side of Equation (4.12) corresponds to approximation against to the arrow in the Bogolubov chain, [11].

3) It is known that appearance of the sign "plus" in the left hand side of Equation (4.12) corresponds to approximation along time arrow in the Bogolyubov chain, [12] and leads to Fermi-Dirac statistics.

4) The special nonlocal hydrodynamic system for fermions corresponds to the same system of Equations (2.1)-(2.6) but with changing $\tau \Rightarrow -\tau$. In other words, approximations in the Bogolyubov chain can be made

a) Using either a deterministic past (Bose-Einstein statistics)

or

b) Using a hypothetical future (Fermi-Dirac statistics).

It is interesting to note that the model (using approximations in the Bogolyubov chain by a hypothetical future) can lead to non-physical consequences. That is why (implicitly) restrictions are introduced into the theory of fermions, for example, the Pauli principle. By the way the second approximation leads to the oscillating Universe with attenuation (model belonging to St. John).

5. System of Equations for Light in the Case $\rho_\alpha = 0$, $\mathbf{v}_0 = const$

In this case continuity equation and momentum equations disappear in the system

(3.1)-(3.6). We have only energy equation

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\varepsilon_\alpha n_\alpha - \tau_\alpha\left[\frac{\partial}{\partial t}(\varepsilon_\alpha n_\alpha) + \frac{\partial}{\partial \mathbf{r}}\cdot(\varepsilon_\alpha n_\alpha \mathbf{v}_0)\right]\right\} \\ + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{\varepsilon_\alpha n_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(\varepsilon_\alpha n_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot(\varepsilon_\alpha n_\alpha \mathbf{v}_0 \mathbf{v}_0) - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)}\right]\right\} = 0.\end{gathered} \tag{5.1}$$

Let us introduce photon energy for the volume unit

$$E_\alpha = \varepsilon_\alpha n_\alpha \tag{5.2}$$

and rewrite (5.1)

$$\begin{gathered}\frac{\partial}{\partial t}\left\{E_\alpha - \tau_\alpha\left[\frac{\partial E_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot(E_\alpha \mathbf{v}_0)\right]\right\} \\ + \frac{\partial}{\partial \mathbf{r}}\cdot\left\{E_\alpha \mathbf{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}(E_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}}\cdot(E_\alpha \mathbf{v}_0 \mathbf{v}_0) - E_\alpha \mathbf{F}_\alpha^{(1)}\right]\right\} = 0.\end{gathered} \tag{5.3}$$

For the 1D case we find from (5.3)

$$\frac{\partial}{\partial t}\left\{E_\alpha-\tau_\alpha\left[\frac{\partial E_\alpha}{\partial t}+v_0\frac{\partial E_\alpha}{\partial x}\right]\right\} \\ +\frac{\partial}{\partial x}\left\{E_\alpha v_0-\tau_\alpha\left[v_0\frac{\partial E_\alpha}{\partial t}+v_0^2\frac{\partial E_\alpha}{\partial x}-E_\alpha F_\alpha^{(1)}\right]\right\}=0. \tag{5.4}$$

or

$$\frac{\partial}{\partial t}\left\{E_\alpha-\tau_\alpha\left[\frac{\partial E_\alpha}{\partial t}+v_0\frac{\partial E_\alpha}{\partial x}\right]\right\} \\ +\frac{\partial}{\partial x}\left\{E_\alpha v_0-\tau_\alpha\left(v_0\frac{\partial E_\alpha}{\partial t}+v_0^2\frac{\partial E_\alpha}{\partial x}\right)\right\}+\frac{\partial}{\partial x}\left\{\tau_\alpha E_\alpha F_\alpha^{(1)}\right\}=0. \tag{5.5}$$

or

$$\frac{\partial}{\partial t}\left\{E_\alpha-\tau_\alpha\left[\frac{\partial E_\alpha}{\partial t}+v_0\frac{\partial E_\alpha}{\partial x}\right]\right\} \\ +v_0\frac{\partial}{\partial x}\left\{E_\alpha-\tau_\alpha\left(\frac{\partial E_\alpha}{\partial t}+v_0\frac{\partial E_\alpha}{\partial x}\right)\right\}+\frac{\partial}{\partial x}\left\{\tau_\alpha E_\alpha F_\alpha^{(1)}\right\}=0. \tag{5.6}$$

Let us introduce the substation derivative

$$\frac{DE_\alpha}{Dt}=\frac{\partial E_\alpha}{\partial t}+v_0\frac{\partial E_\alpha}{\partial x} \tag{5.7}$$

We find

$$\frac{\partial}{\partial t}\left[E_\alpha-\tau_\alpha\frac{DE_\alpha}{Dt}\right]+v_0\frac{\partial}{\partial x}\left\{E_\alpha-\tau_\alpha\frac{DE_\alpha}{Dt}\right\}+\frac{\partial}{\partial x}\left\{\tau_\alpha E_\alpha F_\alpha^{(1)}\right\}=0. \tag{5.8}$$

or

$$\frac{DE_\alpha}{Dt}-\frac{\partial}{\partial t}\left[\tau_\alpha\frac{DE_\alpha}{Dt}\right]-v_0\frac{\partial}{\partial x}\left\{\tau_\alpha\frac{DE_\alpha}{Dt}\right\}+\frac{\partial}{\partial x}\left\{\tau_\alpha E_\alpha F_\alpha^{(1)}\right\}=0. \tag{5.9}$$

Let be $\tau_\alpha = const$, then

$$\frac{DE_\alpha}{Dt}-\tau_\alpha\frac{\partial}{\partial t}\left[\frac{DE_\alpha}{Dt}\right]-v_0\tau_\alpha\frac{\partial}{\partial x}\left[\frac{DE_\alpha}{Dt}\right]+\tau_\alpha\frac{\partial}{\partial x}\left[E_\alpha F_\alpha^{(1)}\right]=0. \tag{5.10}$$

or

$$\frac{DE_\alpha}{Dt}-\tau_\alpha\left[\frac{\partial}{\partial t}\frac{DE_\alpha}{Dt}+v_0\frac{\partial}{\partial x}\frac{DE_\alpha}{Dt}\right]+\tau_\alpha\frac{\partial}{\partial x}\left[E_\alpha F_\alpha^{(1)}\right]=0. \tag{5.11}$$

or

$$\frac{DE_\alpha}{Dt}-\tau_\alpha\frac{D}{Dt}\frac{DE_\alpha}{Dt}=-\tau_\alpha\frac{\partial}{\partial x}\left[E_\alpha F_\alpha^{(1)}\right]. \tag{5.12}$$

This is an analogue of the nonlocal kinetic equation with approximation against time arrow! [10].

The particular cases:

A) If $\tau_\alpha = 0$ we reach trivial local energy conservation along the motion trajectory

$$\frac{DE_{\alpha}}{Dt}=0\,. \tag{5.13}$$

B) If $F_{\alpha}^{(1)}=0$ we have

$$\frac{DE_{\alpha}}{Dt}=\tau_{\alpha}\frac{D}{Dt}\frac{DE_{\alpha}}{Dt}. \tag{5.14}$$

or

$$\tau_{\alpha}\frac{D}{Dt}\ln\frac{DE_{\alpha}}{Dt}=1. \tag{5.15}$$

6. System of Nonlocal Transport Equations for the Stopped Light $v_0=0$ (Photon Condensate, $\rho_{\alpha}=0$, $\mathbf{v}_0=0$)

In this case continuity equations and momentum equations disappear in the system

(3.1)-(3.6). We have only energy equation

$$\frac{\partial}{\partial t}\left[\varepsilon_{\alpha}n_{\alpha}-\tau_{\alpha}\frac{\partial}{\partial t}(\varepsilon_{\alpha}n_{\alpha})\right]+\frac{\partial}{\partial \mathbf{r}}\cdot\left[\tau_{\alpha}\varepsilon_{\alpha}n_{\alpha}\mathbf{F}_{\alpha}^{(1)}\right]=0\,. \tag{6.1}$$

or for the 1D case

$$\frac{\partial}{\partial t}\left[E_{\alpha}-\tau_{\alpha}\frac{\partial E_{\alpha}}{\partial t}\right]+\tau_{\alpha}\frac{\partial}{\partial x}\left[E_{\alpha}F_{\alpha}^{(1)}\right]=0 \tag{6.2}$$

For the single component species we find

$$\frac{D}{Dt}\left[E-\tau\frac{DE}{Dt}\right]+\tau\frac{\partial}{\partial x}\left(EF^{(1)}\right)=0 \tag{6.3}$$

or

$$\tau\frac{\partial^2 E}{\partial t^2}-\frac{\partial E}{\partial t}-\tau\frac{\partial}{\partial x}\left[EF^{(1)}\right]=0 \tag{6.4}$$

or

$$\tau\frac{\partial^2 E}{\partial t^2}-\frac{\partial E}{\partial t}-\tau E\frac{\partial F^{(1)}}{\partial x}-\tau F^{(1)}\frac{\partial E}{\partial x}=0 \tag{6.5}$$

I remind that $\left[F^{(1)}\right]=\frac{cm}{s^2}$ is acceleration, for example the gravitational acceleration.

Conclusion: *A gradient gravitational stop of light is possible*!

Let us introduce the dimensionless form using scales

$$E=E_0\tilde{E}\,,\ \ \tilde{t}=\frac{t}{\tau}\,,\ \ F^{(1)}=F_0^{(1)}\tilde{F}^{(1)}\,,\ \ x=\tilde{x}F_0^{(1)}\tau^2\,. \tag{6.6}$$

We find the corresponding dimensionless form for the Equation (6.5)

$$\frac{\partial^2\tilde{E}}{\partial\tilde{t}^2}-\frac{\partial\tilde{E}}{\partial\tilde{t}}-\frac{\partial}{\partial\tilde{x}}\left(\tilde{E}\tilde{F}^{(1)}\right)=0 \tag{6.7}$$

or

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2}-\frac{\partial \tilde{E}}{\partial \tilde{t}}-\tilde{F}^{(1)}\frac{\partial \tilde{E}}{\partial \tilde{x}}-\tilde{E}\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}=0, \tag{6.8}$$

I remind that equation of the type

$$y''+ay'+by=0 \tag{6.9}$$

has solutions in the following forms if

1) $\lambda^2=a^2-4b>0$, (6.10)

$$y=C_1\exp\left(\frac{\lambda-a}{2}x\right)+C_2\exp\left(\frac{-\lambda-a}{2}x\right). \tag{6.11}$$

if

2) $\lambda^2=4b-a^2>0$, (6.12)

$$y=\exp\left(-\frac{ax}{2}\right)\left[C_1\sin\left(\frac{\lambda x}{2}\right)+C_2\cos\left(\frac{\lambda x}{2}\right)\right]. \tag{6.13}$$

if

3) $a^2=4b$, (6.14)

$$y=\exp\left(-\frac{ax}{2}\right)\left(C_1x+C_2\right). \tag{6.15}$$

In this particular case we should consider gradient parameters $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}$ and $\frac{\partial \tilde{E}}{\partial \tilde{x}}$ as known values. For the better understanding of situation we suppose

$$\left|\tilde{F}^{(1)}\frac{\partial \tilde{E}}{\partial \tilde{x}}\right| \ll \left\|\tilde{E}\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right\|. \tag{6.16}$$

Therefore we consider equation

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2}-\frac{\partial \tilde{E}}{\partial \tilde{t}}-\tilde{E}\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}=0. \tag{6.17}$$

We have (see (6.9)-(6.11))

$$a=-1,\; b=-\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} \tag{6.18}$$

and

$$\lambda^2=a^2-4b=1+4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}. \tag{6.19}$$

Then if $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}>0$ we find $\lambda^2=a^2-4b=1+4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}>0$ and we obtain the solution

$$\tilde{E}=C_1\exp\left(\frac{\sqrt{1+4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}}+1}{2}\tilde{t}\right)+C_2\exp\left(\frac{-\sqrt{1+4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}}+1}{2}\tilde{t}\right), \tag{6.20}$$

If $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} \ll \frac{1}{4}$ we find

$$\tilde{E} = C_1 \exp\left[\left(1 + \frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right)\tilde{t}\right] + C_2 \exp\left(-\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\tilde{t}\right). \tag{6.21}$$

Remind that in this case $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} > 0$

Let us consider now the case 2. $\lambda^2 = 4b - a^2 > 0$.

But $a = -1, b = -\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}$, then $\lambda^2 = -4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} - 1 > 0$. We should suppose that

$$\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} = -\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| \tag{6.22}$$

or

$$\lambda^2 = 4\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| - 1 > 0 \tag{6.23}$$

or

$$\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| > \frac{1}{4}. \tag{6.24}$$

then

$$\tilde{E} = \exp\left(\frac{\tilde{t}}{2}\right)\left[C_1 \sin\left(\frac{\sqrt{4\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| - 1}}{2}\tilde{t}\right) + C_2 \cos\frac{\sqrt{4\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| - 1}}{2}\tilde{t}\right]. \tag{6.25}$$

If $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} = -\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right|$ and $\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| \gg 1$ we find

$$\tilde{E} = \exp\left(\frac{\tilde{t}}{2}\right)\left[C_1 \sin\left(\sqrt{\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right|}\tilde{t}\right) + C_2 \cos\left(\sqrt{\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right|}\tilde{t}\right)\right]. \tag{6.26}$$

This regime corresponds to shimmering black holes.

$$E = E_0 \exp\left(\frac{t}{2\tau}\right)\left[C_1 \sin\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) + C_2 \cos\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right)\right]. \tag{6.27}$$

Interesting to note that $E(t) = 0$ if

$$C_1 \sin\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) + C_2 \cos\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) = 0 \tag{6.28}$$

or

$$\tan\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) = -\frac{C_2}{C_1} \tag{6.29}$$

Obviously relation (6.29) leads *to quantization of processes*. For example, if $\frac{C_2}{C_1} = -1$ we find

$$\tan\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) = 1 \tag{6.30}$$

and

$$\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t = \frac{\pi}{4} + 2\pi n, \quad n = 0, 1, 2, \cdots \tag{6.31}$$

Finally we consider regime 3 if $a^2 = 4b$. In this case taking into account that $a = -1, b = -\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}$ we obtain

$$\tilde{E} = \exp\left(\frac{\tilde{t}}{2}\right)\left(C_1\tilde{t} + C_2\right). \tag{6.32}$$

Summarizing the last result we write

1) $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} > 0$.

$$\tilde{E} = C_1 \exp\left(\frac{\sqrt{1 + 4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}} + 1}{2}\tilde{t}\right) + C_2 \exp\left(\frac{-\sqrt{1 + 4\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}} + 1}{2}\tilde{t}\right).$$

2) $\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right| > \frac{1}{4}$.

$$\tilde{E} = \exp\left(\frac{\tilde{t}}{2}\right)\left[C_1 \sin\left(\sqrt{\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right|}\tilde{t}\right) + C_2 \cos\left(\sqrt{\left|\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}}\right|}\tilde{t}\right)\right].$$

$$E = E_0 \exp\left(\frac{t}{2\tau}\right)\left[C_1 \sin\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right) + C_2 \cos\left(\sqrt{\left|\frac{\partial F^{(1)}}{\partial x}\right|}t\right)\right].$$

3) $\frac{\partial \tilde{F}^{(1)}}{\partial \tilde{x}} = -\frac{1}{4}$.

$$\tilde{E} = \exp\left(\frac{\tilde{t}}{2}\right)\left(C_1\tilde{t} + C_2\right).$$

In the central part of the Milky Way, at a distance of about 26 thousand light-years from the Sun, there is a compact radio source Sagittarius A*, which is most likely a super-massive black hole with a mass of 4.2 million solar masses. Astronomers have registered quasi-periodic flickering of Sagittarius A*, a super-massive black hole in the center of the Milky Way. Researchers are registering flashes emanating from Sagittarius A* in the radio, near infrared and X-ray ranges. Yuhei Iwata from Keio University together with colleagues observed Sagittarius A* in the millimeter range of electromagnetic waves using the Atacama Large Millimeter Array telescope complex. For 10 days, 70 minutes a day, astronomers recorded how the density of the radiation flux emanating from a source in the center of our galaxy changed. On the resulting light curves, scientists noticed two phenomena: quasi-periodic oscillations that occur about once every half hour and slower, hourly variations. The wobbling shadow of the M87* black hole investigated in [15] [16].

7. Conclusion

We investigated the transport processes in Bose-Einstein condensates (CBE), including space-temporal evolution of CBE in a gravitational field. The full nonlocal system for the CBE evolution is delivered including particular case and analytical solutions. Moreover, the corresponding nonlocal hydrodynamic equations indicated for fermions gas. We show (analytically) that a black hole can evolve in the Bose-Einstein condensate (CBE) regime. At the same time, there are modes in which black hole flickering occurs.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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