

# Tsunami and Hubble Expansion in the Frame of Unified Nonlocal Theory

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## Abstract

Nonlocal physics is applied for investigation of the tsunami wave movement. It is established that tsunami movement and the Hubble effect of the Universe expansion can be considered in the frame of the same mathematical theory. Moreover, it can be said that tsunami is Hubble effect in the Earth conditions. The corresponding results of mathematical modeling are shown.

## **Keywords**

Nonlocal Physics, Tsunami Movement, Hubble Effect, Transport Processes in Nonlocal Hydrodynamics, Soliton Theory

## 1. Introduction: About Tsunami and Hubble Effects

Tsunami is one of the most dangerous events caused by the ocean, for example, tsunami that occurred at 07:58:53 local time on 26 December 2004 (00:58:53 GMT).

Here we remind only some facts we need for the construction of the mathematical model of tsunami events. Tsunami is a single wave convoying by a series of ocean waves (like a wave train). Tsunami waves can be as huge as 100 feet. The first wave of a tsunami is only the harbinger of the tsunami; successive waves get bigger and stronger. The fast-moving water associated with tsunami can reach a maximum vertical height on shore above tens of meters.

The length of a tsunami can be as long as 100 kilometers. Deep in the ocean, tsunamis are only 1 - 3 feet tall. In other words, tsunami can be even less than 30 centimeters in height and can pass off unnoticed. Tsunamis can travel up to the speed of 500 - 800 kilometers per hour.

Let us consider now shortly the destiny of the most famous classic paper in the annals of science. I mean Edwin Hubble's 1929 PNAS article on the observed

relation between distance and recession velocity of galaxies—the Hubble Law [1]. Hubble's remarkable observational relation was obtained using 24 nearby galaxies for which both measured velocities and distances were available. Evaluating his data, Hubble concludes: "For such scanty material, so poorly distributed, the results are fairly definite." Hubble showed that galaxies are receding away from us with a velocity u that is proportional to their distance x from us: more distant galaxies recede faster than nearby galaxies. This relation

 $u = \mathbf{H}x \tag{1.1}$ 

is the well-known Hubble Law. Its graphic representation is the Hubble Diagram indicates a constant expansion of the cosmos ones. The slope of the relation, H, is the Hubble Constant; it represents the constant rate of cosmic expansion. Strictly speaking this rate changes with time throughout the life of the universe. Over the decades since Hubble's discovery, numerous observations of the Hubble Law have been carried out to much greater distances and with much higher precision using a variety of modern standard candles, including Supernovae type Ia (SNIa). The linear relation observed at small distances starts deviating from linearity at large distances due to the specific cosmology of the universe, including the cosmic mass density. You can find the corresponding details in [2]. Then the Hubble Parameter H(t) depends on time. The expansion rate at the present time is about 70 km/s/Mpc. The inverse of the Hubble Constant is the Hubble Time, which reflects the time since a linear cosmic expansion has begun. This estimation leads to the age of the Universe from the Big-Bang to today about  $\sim$ 14 billion years. Hubble's diagram shows that a static universe (that Einstein and others assumed in 1917) does not exist.

Hubble's values for his distances in 1929 were, however, rather wrong. But despite this large difference and its major implications for the expansion rate and age of the universe, Hubble's fundamental discovery of the expanding universe is not affected; the underlying linear relation remains unchanged.

From the first glance, we deal with the absolutely different physical systems which need the different mathematical descriptions. But this is not the case. Really:

1) Both events can be considered as 1D movement of soliton waves in space.

2) The movement of Hubble and tsunami waves is a product of the selforganization of matter in the gravitational field.

3) Both waves are longitudinal waves.

4) The action of the gravitational field is significant only in the longitudinal (*x*) direction. For the tsunami wave motion, the Earth's gravitation is not significant.

#### 2. Generalized Hydrodynamic Equations

The generalized hydrodynamic Equations (GHE) can be obtained from the nonlocal kinetic equation in the frame of the Enskog procedure [3]-[8]. Generally speaking to GHE should be added the system of generalized Maxwell Equations (for example in the form of the generalized Poisson equation for electric potential) and gravitational equations (for example in the form of the generalized Poisson equation for gravitational potential). For example

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{F}^{(1)} = -4\pi\gamma \left[ \rho - \tau \left( \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v}_0 \right) \right].$$
(2.1)

(Continuity equation for species  $\alpha$ )

$$\frac{\partial}{\partial t} \left\{ \rho_{\alpha} - \tau_{\alpha} \left[ \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \rho_{\alpha} \mathbf{v}_{0} \right) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \rho_{\alpha} \mathbf{v}_{0} \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} \right) + \vec{\mathbf{I}} \cdot \frac{\partial \rho_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\} = R_{\alpha}.$$
(2.2)

(Continuity equation for mixture)

$$\frac{\partial}{\partial t} \left\{ \rho - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_{0}) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_{0} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0}) + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0}) + \mathbf{I} \cdot \frac{\partial \rho_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\} = 0.$$

$$(2.3)$$

(Momentum equation for species  $\alpha$ )

$$\frac{\partial}{\partial t} \left\{ \rho_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{\partial p_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\}$$

$$- \mathbf{F}_{\alpha}^{(1)} \left[ \rho_{\alpha} - \tau_{\alpha} \left( \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_{0}) \right) \right] - \frac{q_{\alpha}}{m_{\alpha}} \left\{ \rho_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{\partial p_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\} \times \mathbf{B}$$

$$+ \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + p_{\alpha} \mathbf{\tilde{I}} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + p_{\alpha} \mathbf{\tilde{I}}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} (\mathbf{v}_{0} \mathbf{v}_{0}) \mathbf{v}_{0} \right]$$

$$+ 2 \mathbf{\tilde{I}} \left\{ \frac{\partial}{\partial \mathbf{r}} \cdot (p_{\alpha} \mathbf{v}_{0}) \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{\tilde{I}} p_{\alpha} \mathbf{v}_{0}) - \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} \mathbf{v}_{0} - \rho_{\alpha} \mathbf{v}_{0} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} [\mathbf{v}_{0} \times \mathbf{B}] \mathbf{v}_{0}$$

$$- \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} [\mathbf{v}_{0} \times \mathbf{B}] \right\} = \int m_{\alpha} \mathbf{v}_{\alpha} J_{\alpha}^{st,el} d\mathbf{v}_{\alpha} + \int m_{\alpha} \mathbf{v}_{\alpha} J_{\alpha}^{st,inel} d\mathbf{v}_{\alpha}.$$
(2.4)

(Momentum equation for mixture)

$$\frac{\partial}{\partial t} \left\{ \rho \mathbf{v}_{0} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{\partial p_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\}$$

$$- \sum_{\alpha} \mathbf{F}_{\alpha}^{(1)} \left[ \rho_{\alpha} - \tau_{\alpha} \left( \frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_{\alpha} \mathbf{v}_{0}) \right) \right] - \sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \left\{ \rho_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{\partial p_{\alpha}}{\partial \mathbf{r}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \times \mathbf{B} \right] \right\} \times \mathbf{B}$$

$$+ \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_{0} \mathbf{v}_{0} + p \mathbf{\tilde{I}} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + p_{\alpha} \mathbf{\tilde{I}} \right] + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} (\mathbf{v}_{0} \mathbf{v}_{0}) \mathbf{v}_{0}$$

$$+ 2 \mathbf{\tilde{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p_{\alpha} \mathbf{v}_{0}) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{\tilde{I}} p_{\alpha} \mathbf{v}_{0}) - \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} \mathbf{v}_{0} - \rho_{\alpha} \mathbf{v}_{0} \mathbf{F}_{\alpha}^{(1)}$$

$$- \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \left[ \mathbf{v}_{0} \times \mathbf{B} \right] \mathbf{v}_{0} - \frac{q_{\alpha}}{m_{\alpha}} \rho_{\alpha} \mathbf{v}_{0} \left[ \mathbf{v}_{0} \times \mathbf{B} \right] \right\} = 0.$$
(2.5)

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(Energy equation for  $\alpha$  species)

$$\frac{\partial}{\partial t} \left\{ \frac{\rho_{\alpha} v_{0}^{2}}{2} + \frac{3}{2} p_{\alpha} + \varepsilon_{\alpha} n_{\alpha} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{\rho_{\alpha} v_{0}^{2}}{2} + \frac{3}{2} p_{\alpha} + \varepsilon_{\alpha} n_{\alpha} \right) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} + \frac{5}{2} p_{\alpha} \mathbf{v}_{0} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} \right) - \mathbf{F}_{\alpha}^{(1)} \cdot \rho_{\alpha} \mathbf{v}_{0} \right] \right\} \\
+ \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} + \frac{5}{2} p_{\alpha} \mathbf{v}_{0} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} \right) \right] \right\} \\
+ \frac{\delta}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} + \frac{5}{2} p_{\alpha} \mathbf{v}_{0} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} \right) \right] \right\} \\
+ \frac{\delta}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} + \frac{5}{2} p_{\alpha} \mathbf{v}_{0} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} - \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0} \right) \right] \right\} \\
+ \frac{\delta}{2} \rho_{\alpha} \mathbf{v}_{0} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} \right\} \\
+ \frac{\delta}{2} \frac{\rho_{\alpha}}{\rho_{\alpha}} \mathbf{I} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_{0} \mathbf{v}_{0} + \varepsilon_{\alpha} \frac{p_{\alpha}}{m_{\alpha}} \mathbf{I} \right\} \\
- \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_{0} \mathbf{v}_{0} + \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{v}_{0}^{2} \mathbf{I} \\
- \frac{1}{2} \rho_{\alpha} v_{0}^{2} \mathbf{F}_{\alpha}^{(1)} - \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} - \frac{\rho_{\alpha} v_{0}^{2}}{2} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{v}_{0} \mathbf{v}_{0} \\
- \varepsilon_{\alpha} n_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} \mathbf{v}_{0} \mathbf{v}_{0} + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \mathbf{I} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right] \\
= \int \left( \frac{m_{\alpha} v_{\alpha}^{2}}{2} + \varepsilon_{\alpha} \right) J_{\alpha}^{st,el} \mathbf{d} \mathbf{v}_{\alpha} + \int \left( \frac{m_{\alpha} v_{\alpha}^{2}}{2} + \varepsilon_{\alpha} \right) J_{\alpha}^{st,inel} \mathbf{d} \mathbf{v}_{\alpha}.$$

$$(2.6)$$

(Energy equation for mixture)

$$\begin{aligned} \frac{\partial}{\partial t} \left\{ \frac{\rho v_0^2}{2} + \frac{3}{2} p + \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{\rho_{\alpha} v_0^2}{2} + \frac{3}{2} p_{\alpha} + \varepsilon_{\alpha} n_{\alpha} \right) \right) \right. \\ \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 + \frac{5}{2} p_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right) - \mathbf{F}_{\alpha}^{(1)} \cdot \rho_{\alpha} \mathbf{v}_0 \right] \right\} \\ \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho v_0^2 \mathbf{v}_0 + \frac{5}{2} p \mathbf{v}_0 + \mathbf{v}_0 \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \right) \right] \right\} \\ \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho v_0^2 \mathbf{v}_0 + \frac{5}{2} p \mathbf{v}_0 + \mathbf{v}_0 \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \right) \right] \right\} \\ \left. + \frac{5}{2} \rho_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right] + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \mathbf{v}_0 + \frac{7}{2} p_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p_{\alpha} v_0^2 \mathbf{v}_0^2 \mathbf{I} \right] \\ \left. + \frac{5}{2} \frac{p_{\alpha}^2}{\rho_{\alpha}} \mathbf{I} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_{\alpha} \frac{p_{\alpha}}{m_{\alpha}} \mathbf{I} \right] - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 - p_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{I} \right] \\ \left. - \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{F}_{\alpha}^{(1)} - \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} \rho_{\alpha} - \frac{\rho_{\alpha} v_0^2}{2} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] \\ \left. - \varepsilon_{\alpha} n_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \varepsilon_{\alpha} n_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right] \right\} - \left\{ \mathbf{v}_0 \cdot \sum_{\alpha} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \sum_{\alpha} \tau_{\alpha} \left[ \mathbf{F}_{\alpha}^{(1)} \cdot \left( \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0 \right) \right] \\ \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \mathbf{I} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - q_{\alpha} n_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right\} \right\} = 0.$$

Here  $\mathbf{F}_{\alpha}^{(1)}$  are the forces of the non-magnetic origin, **B**—magnetic induction,  $\mathbf{\ddot{I}}$ —unit tensor,  $q_{\alpha}$ —charge of the  $\alpha$ -component particle,  $p_{\alpha}$ —static pressure for  $\alpha$ —component,  $\varepsilon_{\alpha}$ —internal energy for the particles of  $\alpha$ —

component,  $\mathbf{v}_0$ —hydrodynamic velocity for mixture,  $\tau_{\alpha}$ —non-local parameter.

The non-locality parameter  $\tau$  plays the same role as the transport coefficients in local hydrodynamics. The different models can be introduced for the  $\tau$  definition, but the corresponding results are not much different like in local kinetic theory for different models of the interaction of particles.

Generally speaking, parameter  $\tau$  is the mean time of the information transfer in adjacent physical volumes. Obviously, the time  $\tau$  should tend to diminish with increasing of the velocities *u* of particles invading in the nearest neighboring physically infinitely small volume:

$$\tau = \frac{H_{\tau}}{u^n}.$$
 (2.8)

But the value  $\tau$  cannot depend on the velocity direction and naturally to tie  $\tau$  with the particle kinetic energy, then

$$\tau = \frac{H_{\tau}}{mu^2},\tag{2.9}$$

where  $H_{\tau}$  is a coefficient of proportionality, which reflects the state of physical system. In the simplest case  $H_{\tau}$  is equal to Plank constant  $\hbar$  and relation (2.9) became compatible with the Heisenberg relation.

GHE are extremely important for astrophysics special cases when density  $\rho \rightarrow 0$  (the initial stage of evolution of the Universe, the Big Bang; transport processes in physical vacuum) and when density  $\rho \rightarrow \infty$  (evolution of the black hole). Both limiting cases have no physical or mathematical meaning in "classical" hydrodynamics. Thus, we have a unified statistical theory of dissipative structures, which has a hydrodynamic shape defined by the genesis of GHE.

## 3. The 1D Non-Stationary System of Equations for Description of Hubble and Tsunami Waves

Let us derive the system 1D non-stationary nonlocal equations. For the Poisson nonlocal equation we have from (2.1)

$$\frac{\partial^2 \Psi}{\partial x^2} = 4\pi \gamma \left[ \rho - \tau \left( \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) \right) \right], \tag{3.1}$$

where *u* is velocity along the *x* axis;  $\Psi$  is gravitational potential (its dimension is cm<sup>2</sup>/s<sup>2</sup>) and  $\gamma = 6.67430 (15) \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$  or

$$\gamma \approx 6.6 \times 10^{-8} \frac{\mathrm{cm}^3}{\mathrm{g} \cdot \mathrm{s}^2}$$
(3.2)

is gravitational constant.

Let us introduce the moving coordinate system and coordinate  $\mathcal{E} = x - Ct$ .

$$= x - Ct . \tag{3.3}$$

For the scales  $\rho_0, u_0, x_0 = u_0 t_0, \Psi_0 = u_0^2$ , and conditions  $\tilde{C} = C/u_0 = 1$ , the equations take the form

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{\xi}^2} = 4\pi \gamma \rho_0 \frac{x_0^2}{u_0^2} \left[ \tilde{\rho} - \tilde{\tau} \left( -\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho} \tilde{u}) \right) \right]$$
(3.4)

Introducing the scale for  $\gamma$  as

$$\gamma_0 = \frac{u_0^2}{\rho_0 x_0^2},$$
(3.5)

we reach

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{\xi}^2} = 4\pi \tilde{\gamma} \left[ \tilde{\rho} - \tilde{\tau} \left( -\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho} \tilde{u}) \right) \right].$$
(3.6)

The dimensionless parameter  $\tilde{\gamma}$  is of the principal significance for the numerical investigation. Let us introduce the Hubble parameter H as

$$H = u_0 / x_0$$
 (3.7)

For the cosmic situation H  $\approx 2.2 \times 10^{-18} \text{ s}^{-1}$ . Then in the general case, we find

$$\gamma_0 = \frac{\mathrm{H}^2}{\rho_0} \,. \tag{3.8}$$

For water if  $\rho_0 = 1 \text{ g/cm}^3$  we find  $\gamma_{0,w} = H_w^2 \frac{\text{cm}^3}{\text{g} \cdot \text{s}^2}$ ,

For the length L of the tsunami wave equal to 100 km and velocity u equal to 600 km per hour we have

$$\gamma_0 = \frac{6 \times 10^7 \times 6 \times 10^7}{3600 \times 3600 \times 10^{14}} = \frac{36}{3600 \times 3600} = \frac{1}{3.6 \times 10^5} \frac{\text{cm}^3}{\text{g} \cdot \text{s}^2}$$
(3.9)

and

$$\tilde{\gamma} = \frac{\gamma}{\gamma_0} = 6.67 \times 10^{-8} \times 3.6 \times 10^5 = 2.4 \times 10^{-2}.$$
(3.10)

Results of the Hubble expansion essentially depend on the space density chosen for calculations.

Voyager-2 has detected an increase in the density of space beyond the Solar System. Voyager-1 sent the same message.

As it is known the Voyager-2 spacecraft has detected an increase in the density of space beyond the Solar System. In November 2018, 41 years after launch, the Voyager-2 spacecraft finally crossed the border of the heliosphere—the region of space in which the Solar System is located. And as the device moves away from the heliopause—the boundary of the heliosphere—the density of space increases.

Voyager-1, which entered interstellar space in 2012, found a similar density gradient elsewhere. New data from the second Voyager shows that the increase in density may be a large-scale feature of space beyond the Solar System.

When the Voyager probes crossed the heliopause, their plasma wave instruments measured the electron density of the plasma. Voyager-1 at that moment recorded a density of 0.055 electrons per cubic centimeter, and Voyager-2— 0.039. After passing another 20 astronomical units (2.9 billion km) in space, Voyager-1 reported an increase in plasma density by half: up to 0.13 electrons per cubic centimeter. The electron rest mass is about  $0.9109 \times 10^{-27} \text{ g} \approx 10^{-27} \text{ g}$ . In this case

$$\gamma_0 = \frac{2.2 \times 10^{-18} \times 2.2 \times 10^{-18}}{0.13 \times 0.91 \times 10^{-27}} = 4.09 \times 10^{-8} \frac{\text{cm}^3}{\text{g} \cdot \text{s}^2}$$
(3.11)

and

$$\tilde{\gamma} = \frac{\gamma}{\gamma_0} = \frac{6.67 \times 10^{-8}}{4.09 \times 10^{-8}} = 1.63.$$
(3.12)

Let us transform the nonlocal continuity equation.

$$\frac{\partial}{\partial t} \left\{ \rho - \tau \left[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) \right] \right\} + \frac{\partial}{\partial x} \left\{ \rho u - \tau \left[ \frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} (\rho u^2) + \frac{\partial \rho}{\partial x} + \rho \frac{\partial \Psi}{\partial x} \right] \right\} = 0.$$
(3.13)

Introducing the wave  $\xi$  variable one obtains

$$\frac{\partial}{\partial\xi} \left\{ \rho u - \tau \left[ \frac{\partial}{\partial\xi} \left( \rho u^2 \right) + \frac{\partial p}{\partial\xi} + \rho \frac{\partial \Psi}{\partial\xi} \right] \right\} = 0$$
(3.14)

or using the scales  $p_0 = \rho_0 u_0^2$ ,  $\rho_0, u_0, x_0 = u_0 t_0, \Psi_0 = u_0^2$  we fined the dimensionless form of equation

$$\frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u} - \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^2 \right) + \frac{\partial \tilde{p}}{\partial \tilde{\xi}} + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0$$
(3.15)

or

$$\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} - \frac{\partial \tilde{\rho}\tilde{u}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{p} + \tilde{\rho}\tilde{u}^2 + \tilde{\rho} - 2\tilde{\rho}\tilde{u} \right] + \tilde{\rho}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0$$
(3.16)

We turn to the transformation of the motion equation

$$\frac{\partial}{\partial t} \left\{ \rho u - \tau \left[ \frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} (\rho u^2) + \frac{\partial p}{\partial x} + \rho \frac{\partial \Psi}{\partial x} \right] \right\} + \frac{\partial \Psi}{\partial x} \left[ \rho - \tau \left( \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) \right) \right] \\ + \frac{\partial}{\partial x} \left\{ \rho u^2 + p - \tau \left[ \frac{\partial}{\partial t} (\rho u^2 + p) + \frac{\partial}{\partial x} (\rho u^3 + 3pu) + 2\rho u \frac{\partial \Psi}{\partial x} \right] \right\} = 0$$
(3.17)

In the dimensionless form Equation (3.17) is written as follows

$$\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left[ \tilde{\rho} - \tilde{\tau} \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u} \right) \right] + \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u}^2 + \tilde{p} - \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^3 + 3 \tilde{p} \tilde{u} \right) + 2 \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0 \ (3.18)$$

or

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{2} + \tilde{p} - \tilde{\rho} \tilde{u} \right) + \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left( 2\tilde{\rho} \tilde{u}^{2} - \tilde{\rho} \tilde{u} + 2\tilde{p} - \tilde{\rho} \tilde{u}^{3} - 3\tilde{p} \tilde{u} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} \\
+ \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left\{ \tilde{\rho} - \tilde{\tau} \left[ -\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u} \right) \right] \right\} - 2 \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} = 0$$
(3.19)

Finally we reach the energy equation

$$\frac{\partial}{\partial t} \left\{ \rho u^{2} + 3p - \tau \left[ \frac{\partial}{\partial t} \left( \rho u^{2} + 3p \right) + \frac{\partial}{\partial x} \left( \rho u^{3} + 5pu \right) + 2\rho u \frac{\partial \Psi}{\partial x} \right] \right\}$$

$$+ \frac{\partial}{\partial x} \left\{ \rho u^{3} + 5pu - \tau \left[ \frac{\partial}{\partial t} \left( \rho u^{3} + 5pu \right) + \frac{\partial}{\partial x} \left( \rho u^{4} + 8pu^{2} + 5\frac{p^{2}}{\rho} \right) \right]$$

$$+ \frac{\partial\Psi}{\partial x} \left\{ 3\rho u^{2} + 5p \right\} + 2\frac{\partial\Psi}{\partial x} \left\{ \rho u - \tau \left[ \frac{\partial}{\partial t} \left( \rho u \right) + \frac{\partial}{\partial x} \left( \rho u^{2} + p \right) + \rho \frac{\partial\Psi}{\partial x} \right] \right\} = 0$$

$$(3.20)$$

We search the wave solutions using the  $\xi$ -variable. In the dimensionless form energy equation is written as

$$\frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u}^{3} + 5 \tilde{p} \tilde{u} - \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{4} + 8 \tilde{p} \tilde{u}^{2} + 5 \frac{\tilde{p}^{2}}{\tilde{\rho}} \right) + \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left( 3 \tilde{\rho} \tilde{u}^{2} + 5 \tilde{p} \right) \right] \right\} + 2 \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u} - \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{2} + \tilde{p} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0$$
(3.21)

or

$$-\frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u}^{2} + 3\tilde{p} - \tilde{\tau} \left[ -\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{2} + 3\tilde{p} \right) + \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{3} + 5\tilde{p} \tilde{u} \right) + 2\tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} \\ + \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u}^{3} + 5\tilde{p} \tilde{u} - \tilde{\tau} \left[ -\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{3} + 5\tilde{p} \tilde{u} \right) + \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{4} + 8\tilde{p} \tilde{u}^{2} + 5\frac{\tilde{p}^{2}}{\tilde{\rho}} \right) \right] \right\} \\ + \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left( 3\tilde{\rho} \tilde{u}^{2} + 5\tilde{p} \right) \right] + 2\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left\{ \tilde{\rho} \tilde{u} - \tilde{\tau} \left[ -\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u} \right) + \frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{2} + \tilde{p} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0$$

$$(3.22)$$

or

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^{2} + 3\tilde{p} - \tilde{\rho} \tilde{u}^{3} - 5\tilde{p} \tilde{u} \right) 
- \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \frac{\partial}{\partial \tilde{\xi}} \left( 2\tilde{\rho} \tilde{u}^{3} + 10\tilde{p}\tilde{u} - \tilde{\rho} \tilde{u}^{2} - 3\tilde{p} - \tilde{\rho} \tilde{u}^{4} - 8\tilde{p} \tilde{u}^{2} - 5\frac{\tilde{p}^{2}}{\tilde{\rho}} \right) \right\} 
+ \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \left( 3\tilde{\rho} \tilde{u}^{2} + 5\tilde{p} \right) \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} - 2\tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} - 2\frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} 
+ 2\tilde{\tau} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \left[ -\frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho} \tilde{u}) + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho} \tilde{u}^{2} + \tilde{p}) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] = 0$$
(3.23)

Let us concentrate the scales:

$$\gamma = 6.6 \times 10^{-8} \text{ cm}^3 / (\text{g} \cdot \text{s}^2)$$
 ant its scale  $\gamma_0 = \frac{u_0^2}{\rho_0 x_0^2}$ ; the pressure scale

 $p_0 = \rho_0 u_0^2$ ; the gravitation potential scale  $\Psi_0 = u_0^2$  and other scales  $\rho_0, u_0, x_0 = u_0 t_0$ . We should choose the time scale  $t_0$ . With this aim we intend to apply the relation (2.9) in the form

$$\tilde{\tau} = \frac{1}{\tilde{u}^2} \tag{3.24}$$

or in the dimension form

$$\tau = \frac{1}{u^2} u_0^2 t_0 = \frac{1}{u^2} u_0 x_0 = \frac{\nu_0^{\kappa in}}{u^2},$$
(3.25)

where the kinematic viscosity is introduced as a scale  $\upsilon_0^{\kappa in} = u_0 x_0$ . Then we obtain the physically transparent result: the nonlocal parameter is proportional to the kinematic viscosity and inversely proportional to the velocity squared. These scales correlate with the Heisenberg principle of the uncertainty principle.

Let us consider another variant of the  $\tilde{\gamma}$  estimation. Effects of gravitational self-catching should be typical for Universe. The existence of "Hubble boxes" is typical blocks of the nearby Universe. Gravitational self-catching takes place for Big Bang having given birth to the global expansion of Universe, but also for Little Bang in so called Local Group of galaxies. Then the evolution of the Local Group (the typical Hubble box) is a really fruitful field for testing of different theoretical constructions (see Figure 1). The data were obtained by Karachentsev and his collaborators in 2002-2007 in observation with the Hubble Space Telescope [9]. Each point corresponds to a galaxy with measured values of distance and line-of-site velocity in the reference frame related to the center of the Local Group. The diagram shows two distinct structures, the Local Group and the local flow of galaxies. The galaxies of the Local Group occupy a volume with the radius up to ~1.1 - 1.2 Mpc, but there are no galaxies in the volume whose radius is less than 0.25 Mpc. These galaxies move both away from the center (positive velocities) and toward the center (negative velocities). These galaxies form a gravitationally bound quasi-stationary system. Their average radial velocity is equal to zero. The galaxies of the local flow are located outside the group and all of them are moving from the center (positive velocities) beginning their



Figure 1. Velocity-distance diagram for galaxies at distances of up to 3 Mpc for local group of galaxies.

motion near  $R \approx 1$  Mpc with the velocity  $v \sim 50$  km/s. By the way the measured by Karachentsev the average Hubble parameter for the Local Group is 72 ±  $6 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ .

A unit of measurement of distances in astronomy, approximately equal to the average distance from the Earth to the Sun. Currently, it is assumed to be exactly equal to 149,597,870,700 meters. The astronomical unit is used mainly to measure distances between objects of the Solar system, exoplanetary systems, as well as between components of binary stars.

Let be  $x_0$  is astronomical unit (*au*);  $x_0^2 = 2.25 \times 10^{26} \text{ cm}^2$ ,  $u_0 = 100 \text{ km/s} = 10^7 \text{ cm/s}$ . Then

$$\frac{\gamma x_0^2}{u_0^2} = \frac{6.6 \times 10^{-8} \times 2.25 \times 10^{26}}{10^{14}} = 14.85 \times 10^4 \frac{\text{cm}^3}{\text{g} \cdot \text{s}^2} \cdot \text{s}^2 \cong 1.5 \times 10^5 \frac{\text{cm}^3}{\text{g} \cdot \text{s}^2} \cdot \text{s}^2 \quad (3.26)$$

Sun mass  $M_{sun}$  is equal approximately to  $2 \times 10^{33}$  g. The mean density in the nearby cosmic space can be estimated as

$$\rho_0 = \frac{M_c}{\frac{4}{3}\pi R_{au}^3} = \frac{2 \times 10^{33}}{\frac{4}{3}\pi \times \left(1.5 \times 10^{13}\right)^3} = \frac{10^{33}}{\pi \times 2.25 \times 10^{39}} = 0.14 \times 10^{-6} \frac{g}{\text{cm}^3} \quad (3.27)$$

and

$$\tilde{\gamma} = \frac{\gamma}{\gamma_0} = \frac{\gamma \rho_0 x_0^2}{u_0^2} = 1.5 \times 0.14 \times 10^{-1} = 0.021.$$
(3.28)

Let us go now to the mathematical modeling. As a result the non-local system of hydrodynamic equations is written as

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{\xi}^2} = 4\pi \tilde{\gamma} \Biggl[ \tilde{\rho} - \tilde{\tau} \Biggl( -\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho}\tilde{u}) \Biggr) \Biggr],$$
(3.29)

$$\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} - \frac{\partial \tilde{\rho}\tilde{u}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} \left\{ \tilde{\tau} \left[ \frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{p} + \tilde{\rho}\tilde{u}^2 + \tilde{\rho} - 2\tilde{\rho}\tilde{u} \right] + \tilde{\rho}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0,$$
(3.30)

$$\begin{aligned} \frac{\partial}{\partial \xi} \left( \tilde{\rho} \tilde{u}^{2} + \tilde{p} - \tilde{\rho} \tilde{u} \right) + \frac{\partial}{\partial \xi} \left\{ \tilde{\tau} \left[ \frac{\partial}{\partial \xi} \left( 2 \tilde{\rho} \tilde{u}^{2} - \tilde{\rho} \tilde{u} + 2 \tilde{p} - \tilde{\rho} \tilde{u}^{3} - 3 \tilde{p} \tilde{u} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \xi} \right] \right\} \\ + \frac{\partial \tilde{\Psi}}{\partial \xi} \left\{ \tilde{\rho} - \tilde{\tau} \left[ -\frac{\partial \tilde{\rho}}{\partial \xi} + \frac{\partial}{\partial \xi} \left( \tilde{\rho} \tilde{u} \right) \right] \right\} - 2 \frac{\partial}{\partial \xi} \left\{ \tilde{\tau} \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \xi} \right\} = 0, \\ \frac{\partial}{\partial \xi} \left\{ \tilde{\rho} \tilde{u}^{2} + 3 \tilde{p} - \tilde{\rho} \tilde{u}^{3} - 5 \tilde{p} \tilde{u} \right\} \\ - \frac{\partial}{\partial \xi} \left\{ \tilde{\tau} \frac{\partial}{\partial \xi} \left\{ 2 \tilde{\rho} \tilde{u}^{3} + 10 \tilde{p} \tilde{u} - \tilde{\rho} \tilde{u}^{2} - 3 \tilde{p} - \tilde{\rho} \tilde{u}^{4} - 8 \tilde{p} \tilde{u}^{2} - 5 \frac{\tilde{p}^{2}}{\tilde{\rho}} \right\} \right\} \\ + \frac{\partial}{\partial \xi} \left\{ \tilde{\tau} \left( 3 \tilde{\rho} \tilde{u}^{2} + 5 \tilde{p} \right) \frac{\partial \tilde{\Psi}}{\partial \xi} \right\} - 2 \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \xi} - 2 \frac{\partial}{\partial \xi} \left\{ \tilde{\tau} \tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \xi} \right\} \\ + 2 \tilde{\tau} \frac{\partial \tilde{\Psi}}{\partial \xi} \left[ - \frac{\partial}{\partial \xi} \left( \tilde{\rho} \tilde{u} \right) + \frac{\partial}{\partial \xi} \left( \tilde{\rho} \tilde{u}^{2} + \tilde{p} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \xi} \right] = 0. \end{aligned}$$

$$(3.32)$$

Interesting to notice that in the frame of local hydrodynamics we have the

following system of equations

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{\xi}^2} = 4\pi \tilde{\gamma}_N \tilde{\rho}, \qquad (3.33)$$

$$\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} - \frac{\partial \tilde{\rho} \tilde{u}}{\partial \tilde{\xi}} = 0, \qquad (3.34)$$

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^2 + \tilde{p} - \tilde{\rho} \tilde{u} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0, \qquad (3.35)$$

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} \tilde{u}^2 + 3\tilde{p} - \tilde{\rho} \tilde{u}^3 - 5\tilde{p} \tilde{u} \right) - 2\tilde{\rho} \tilde{u} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0.$$
(3.36)

From Equation (3.34) follows that

$$\tilde{u} = 1 \tag{3.37}$$

for all  $\tilde{\xi}$  without the soliton formation. Equations (3.35) and (3.36) can be transformed

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} + \tilde{p} - \tilde{\rho} \right) + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0,$$
(3.38)

$$\frac{\partial}{\partial \tilde{\xi}} \left( \tilde{\rho} + 3\tilde{p} - \tilde{\rho} - 5\tilde{p} \right) - 2\tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0, \qquad (3.39)$$

or

$$\frac{\partial \tilde{p}}{\partial \tilde{\xi}} + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0, \qquad (3.40)$$

$$-2\frac{\partial\tilde{p}}{\partial\tilde{\xi}} - 2\tilde{\rho}\frac{\partial\tilde{\Psi}}{\partial\tilde{\xi}} = 0.$$
(3.41)

In the case of local description the motion equation coincide with the energy equation and we find the system of equations

$$\frac{\partial^2 \Psi}{\partial \tilde{\xi}^2} = 4\pi \tilde{\gamma} \tilde{\rho}, \qquad (3.42)$$

$$\frac{\partial \tilde{p}}{\partial \tilde{\xi}} + \tilde{\rho} \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} = 0, \qquad (3.43)$$

$$\tilde{u} = 1 \tag{3.44}$$

As we see "classic" hydrodynamics leads to the unclosed system of equations with the absolutely unsatisfactory results.

Let us show the working Maple program for the case  $\tilde{\tau} = \frac{1}{\tilde{u}^2}$ 

#### dsolve[interactive]({

$$\begin{split} & \text{diff}(r(t)^*(1-u(t)),t) + \text{diff}((1/u(t)^2)^*(\text{diff}(p(t)+r(t)+r(t)^*u(t)^2-2^*r(t)^*u(t),t)),t) + \text{diff}((1/u(t)^2)^*r(t)^*\text{diff}(v(t),t),t) = 0, \end{split}$$

$$\begin{split} & \operatorname{diff}(r(t)^*u(t)^2+p(t)-r(t)^*u(t),t)+\operatorname{diff}((1/u(t)^2)^*\operatorname{diff}(2^*r(t)^*u(t)^2+2^*p(t)^2)^*(t)^*u(t)^2+2^*p(t)^2)^*(t)^*u(t)^2+2^*p(t)^$$

$$\begin{split} & \operatorname{diff}(r(t)^*u(t)^{2}+3^*p(t)-r(t)^*u(t)^{3}-5^*p(t)^*u(t),t)-\operatorname{diff}((1/u(t)^{2})^*\operatorname{diff}(2^*r(t)^*u(t)^{3}+10^*p(t)^*u(t)-r(t)^*u(t)^{2}-3^*p(t)-r(t)^*u(t)^{4}-8^*p(t)^*u(t)^{2}-5^*p(t)^{2}/r(t),t),t)+\operatorname{diff}((1/u(t)^{2})^*\operatorname{diff}(v(t),t)^*(3^*r(t)^*u(t)^{2}+5^*p(t)),t)-2^*r(t)^*\operatorname{diff}(v(t),t)^*(1/u(t)^{2})^*r(t)^*u(t)^*\operatorname{diff}(v(t),t),t)+2^*(1/u(t)^{2})^*\operatorname{diff}(v(t),t),t)+\operatorname{diff}(v(t),t)+\operatorname{diff}(v(t),t)+\operatorname{diff}(v(t),t)^{2}-r(t)^*u(t)^{2}-r(t)^*u(t),t))=0, \end{split}$$

 $diff(v(t),t\$2)=4*Pi*G*(r(t)-(1/u(t)^2)*diff(r(t)*u(t)-r(t),t)),u(0)=1,p(0)=1,r(0)=1,D(u)(0)=0,D(p)(0)=0,$ 

D(r)(0)=0,D(v)(0)=0,v(0)=1}.

The system of generalized quantum hydrodynamic Equations (3.29) - (3.32) has the great possibilities of mathematical modeling as result of changing of eight Cauchy conditions and the dimensionless gravitation  $\tilde{\gamma}$  parameter describing the character features of initial perturbations which lead to the soliton formation.

On this step of investigation, we intend to demonstrate the influence of difference conditions on the soliton formation. The following figures reflect some results of calculations realized according to the system of Equations (3.29) - (3.32) with the help of Maple. The following notations on figures are used: r-density  $\tilde{\rho}$ , u: velocity  $\tilde{u}$ , p: pressure  $\tilde{p}$ , v: self consistent potential  $\tilde{\Psi}$  and G is  $\tilde{\gamma}$ . Explanations placed under all following figures, Maple program contains Maple's notations—for example, the expression D(u)(0) = 0 means in usual notations  $\frac{\partial \tilde{u}}{\partial \tilde{\xi}}(0) = 0$ , independent variable t responds to  $\tilde{\xi}$ .

Regimes can lead to the non-symmetric soliton which occupies a non-symmetric area indicated as lim1 and lim2.

**Figures 2-21** reflect the results of calculations. **Figures 2-5** reflect the results of calculations for the obtained before  $\tilde{\gamma}$ -parameter for moving Hubble and tsunami waves.



Figure 2. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.024$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.



Figure 3. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.024$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{p})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ .



Figure 4. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1.63$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.  $\lim 1 = -3.1535$ ,  $\lim 2 = 4.5267$ .



Figure 5. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1.63$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{p})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ .  $\lim 1 = -3.1535$ ,  $\lim 2 = 4.5267$ .



Figure 6. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.0000001$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.  $\lim 1 = -47.8949$ .



Figure 7. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.0000001$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{p})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ .  $\lim 1 = -47.8949$ .



**Figure 8.** Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.0001$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.  $\lim 1 = -28.8163$ .



Figure 9. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.0001$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ .  $\lim 1 = -28.8163$ .



Figure 10. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.001$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.  $\lim 1 = -26.0720$ .



Figure 11. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.001$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ .  $\lim 1 = -26.0720$ .



**Figure 12.** Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.1$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.



Figure 13. Soliton formation for the case, density  $r \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $v \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 0.1$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.





**Figure 14.** Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0. lim1 = -3.8605, lim2 = 5.1711.



Figure 15. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0)$  $= 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0. \lim 1 = -3.8605, \lim 2 = 5.1711.$ 



G = 10, p-solid line, u-dashed line, 1im1 = -1.3363, 1im2 = 2.1607

Figure 16. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $\mathbf{u} \leftrightarrow \tilde{u}\left(\tilde{\xi}\right)$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 10$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0)$  $= 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0. \lim 1 = -1.3363, \lim 2 = 2.1607.$ 



Figure 17. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 10$ ;  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{p})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ . lim $\mathbf{1} = -1.3363$ , lim $\mathbf{2} = 2.1607$ .

G = 100, p-solid line, u-dashed line, 1im1 = -0.4480, 1im2 = 0.8462



**Figure 18.** Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 100$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0. lim1 = -0.4480, lim2 = 0.8462.



Figure 19. Soliton formation for the case, density  $\mathbf{r} \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $\mathbf{v} \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $\mathbf{t} \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 100$ ;  $\mathbf{D}(\mathbf{v}) \leftrightarrow \frac{d\tilde{\Psi}}{d\tilde{\xi}}$ ,  $\mathbf{u}(0) = 1$ ,  $\mathbf{p}(0) = 1$ ,  $\mathbf{r}(0) = 1$ ,  $\mathbf{v}(0) = 1$ ,  $\mathbf{D}(\mathbf{u})(0) = 0$ ,  $\mathbf{D}(\mathbf{p})(0) = 0$ ,  $\mathbf{D}(\mathbf{r})(0) = 0$ ,  $\mathbf{D}(\mathbf{v})(0) = 0$ .  $\lim 1 = -0.4480$ ,  $\lim 2 = 0.8642$ .



Figure 20. Soliton formation for the case, pressure  $p \leftrightarrow \tilde{p}(\tilde{\xi})$  (solid line), velocity  $u \leftrightarrow \tilde{u}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1000$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.



Figure 21. Soliton formation for the case, density  $r \leftrightarrow \tilde{\rho}(\tilde{\xi})$  (solid line), potential  $v \leftrightarrow \tilde{\Psi}(\tilde{\xi})$  (dashed line),  $t \leftrightarrow \tilde{\xi}$ ,  $\tilde{\gamma} = 1000$ ; u(0) = 1, p(0) = 1, r(0) = 1, v(0) = 1, D(u)(0) = 0, D(p)(0) = 0, D(r)(0) = 0, D(v)(0) = 0.

As I wrote before the  $\tilde{\gamma}$ -parameter changing corresponds to the very vast diapason. The following are the results of calculations when the gravitational parameter  $\tilde{\gamma}$  changes by ten orders of magnitude.

#### 4. Conclusions

1) The problem of principle significance—is it possible after a perturbation (defined *by Cauchy conditions*) to obtain *the object of the soliton's kind* as a result of the self-organization of gravitation matter? As we see we obtained the positive answer for this question.

2) The main origin of Hubble and tsunami effects is self-catching of expanding matter by the self-consistent gravitational field.

3) Tsunami movement and Hubble expansion have the same physical origin and can be described in the frame of unified nonlocal theory. It means that Hubble expansion can be investigated using tsunami observation and vice versa.

4) The developed theory works in the tremendous diapason of the gravitational  $\tilde{\gamma}$  -parameter changing.

5) Regimes exist of the soliton's evolution when the non-symmetric soliton occupies a limited area of space (indicated as lim1 and lim2) on the left or right, or only a limited area of space.

#### **Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

#### References

- Hubble, E. (1929) A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae. *Proceedings of the National Academy of Sciences of the United States of America*, 15, 168-173.
- [2] Alexeev, B.V. (2017) Nonlocal Astrophysics. Dark Matter, Dark Energy, Physical Vacuum. Elsevier Amsterdam, Netherlands, 454 p.
- [3] Alexeev, B.V. (2004) Generalized Boltzmann Physical Kinetics. Elsevier Amsterdam, Netherlands, 368 p.
- [4] Alexeev, B.V. (2015) Unified Non-Local Theory of Transport Processes. Elsevier Amsterdam, Netherlands, 644 p.
- [5] Alexeev, B.V. (2016) Unified Non-Local Relativistic Theory of Transport Processes. Elsevier Amsterdam, Netherlands, 455 p.
- [6] Alexeev, B.V. (2017) Main Problems of the Theoretical Physics and Artifacts of Local Physics. *Journal of Modern Physics*, 12, 1048-1108. https://doi.org/10.4236/jmp.2021.127066
- [7] Alexeev, B.V. (1994) The Generalized Boltzmann Equation, Generalized Hydrodynamic Equations and Their Applications. *Philosophical Transactions of the Royal Society A*, 349, 417-443. <u>https://doi.org/10.1098/rsta.1994.0140</u>
- [8] Alexeev, B.V. (1995) The Generalized Boltzmann Equation. *Physica A: Statistical Mechanics and Its Applications*, 216, 459-468. https://doi.org/10.1016/0378-4371(95)00044-8
- [9] Karachentsev, I.D. and Kashibadze, O.G. (2006) Masses of the Local Group and of the M81 Group Estimated from Distortions in the Local Velocity Field. *Astrophysics*, 49, 3-18. <u>https://doi.org/10.1007/s10511-006-0002-6</u>