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A Mathematical Model for Spread of COVID-19 in the World

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Abstract

In this article, the validity of a previously developed model for Coronavirus pandemic is verified by testing it against the global infection cases complied by the World Health Organization. The results further support the validity of the model and its assumptions as was previously verified by the United States data.

Keywords

Coronavirus, Pandemic, World, COVID-19

1. Introduction

Epidemic diseases have haunted mankind forever and have wiped out large fractions of the populations throughout the history. The Spanish flu during 1918-1919 infected nearly one-third of the world population and claimed at least 50 million lives [1]. In more recent years, the longest lasting pandemic of HIV/AIDS, which peaked during the period of 2005-2012, has claimed more than 36 million lives worldwide [2] [3].

The currently active COVID-19 pandemic, which outbroke in December of 2019 [4], has so far infected 172 million people worldwide and has claimed 3.69 million lives [5]. Due to its economic and political impacts, COVID-19 pandemic has prompted a number of investigations into the dynamics of its spread and predictions of its future development. For example, Leung *et al.* [6] studied the transmissibility and severity of the first wave of the disease in China. Kissler *et al.* [7] projected the transmission dynamics of the disease through the postpandemic period. Haushofer and Metcalf [8] investigated the effectiveness of various interventions in a pandemic. Sanche *et al.* [9] studied the high contagiousness and rapid spread of the coronavirus. Finally, Kaxiras and Neofotistos

[10] investigated the multiple epidemic wave model of the COVID-19 pandemic.

Recently, we developed a new model for the Coronavirus pandemic. This model contains only two adjustable parameters as compared to as many as 5 parameters in other models [11]. The model was tested against the number of Coronavirus cases in the United States during the period of January 2020 to January 2021, and the results showed good agreement between the model and the data. In this work, we extend the application of the model to the number of cases in the entire world, for which enough data was not available during the previous work.

2. The Model

Recently, we developed a simple model for the spread of Coronavirus pandemic (COVID-19). This model is based on the normal uninhibited population growth,

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN \tag{1}$$

where N is the instantaneous population and k is the growth constant, which is the probability of a member of the population to double per unit time. In a pandemic, however, the rate of disease spread at any time is proportional to the number of the remaining healthy population,

$$\frac{\mathrm{d}n}{\mathrm{d}t} = k(N-n) \tag{2}$$

where N is the total population, which is a constant, and n is the infected population at time t. Furthermore, in this model, instead of treating k as a constant, it is assumed to be proportional to the fraction of the infected population [11]. Thus,

$$k = k_0 \frac{n}{N} \tag{3}$$

where now k_0 is a constant. Therefore, the differential equation for the growth of the infected population becomes

$$\frac{\mathrm{d}n}{\mathrm{d}t} = k(N-n) = k_0 \frac{n}{N}(N-n) \tag{4}$$

which reduces to

$$\frac{\mathrm{d}n}{n(N-n)} = \frac{k_0}{N}\mathrm{d}t \tag{5}$$

Integration of this equation gives

$$\frac{n}{N-n} = A\mathrm{e}^{k_0 t} \tag{6}$$

where A is related to the integration constant. Applying the initial conditions $n(0) = n_0$, gives

$$A = \frac{n_0}{N-n_0} \tag{7}$$

Therefore, Equation (6) reduces to [11]

$$n(t) = \frac{n_0 N}{n_0 + (N - n_0)e^{-k_0 t}} \quad (8)$$

Furthermore, since in a pandemic situation, the initial infected population is much smaller than the total population, $n_0 \ll N$, this equation further reduces to

$$n(t) = \frac{n_0 N}{n_0 + N e^{-k_0 t}} \quad (9)$$

which gives the number of the infected population as a function of time. The S-shape graph of this equation is usually referred to as the logistic growth curve [11] [12].

3. Application of the Model to World COVID-19 Data

As stated in a previous section, we recently tested the model described above against the COVID-19 data in the United States during the period of January 2020 to January 2021. The results showed good agreement between the model and the data even though the model contains only two adjustable parameters, namely n_0 and N.

At the time that the previous work was completed, we could not find reliable comprehensive data for the entire world. However, recently we were able to find enough worldwide data to test this model against them [13]. To do so, we complied the world data in increments of 7-days due to this period correlating to the common measurement of time known as a "week" making the data more comprehensible. Thus, in **Table 1**, Day 1 corresponds to December 30, 2019 and Day 533 corresponds to June 20, 2021.

Using a non-linear least-squares analysis, we fitted Equation (9) to the data in **Table 1** via the adjustable parameters n_0 and k as described in reference [11]. For the total population N we used the current population of the world, 7.7×10^9 (7.7 billion). This resulted in the following values of the parameters:

$$n_0 = (7.01 \pm 0.63) \times 10^6 \quad \text{and} \quad k = (6.37 \pm 0.19) \times 10^{-3}\ \text{day}^{-1} \quad (10)$$

Figure 1 shows the graph of Equation (9) with these parameters together with the actual world data from **Table 1**.

4. Conclusions

As can be seen from **Figure 1**, the overall agreement between our model and the World Coronavirus data is reasonably good. We do see, however, some variations in the data resulting in the model overestimating in the actual data in some regions and underestimating in some other regions. This is attributed to changes in the dynamics of the disease spread due to modifications in the parameters of the pandemic such as mask mandates, social distancing, and vaccination efforts. The willingness of population to adhere to parameter modifications as well as the revisions made by the CDC and WHO regarding mandates also influenced

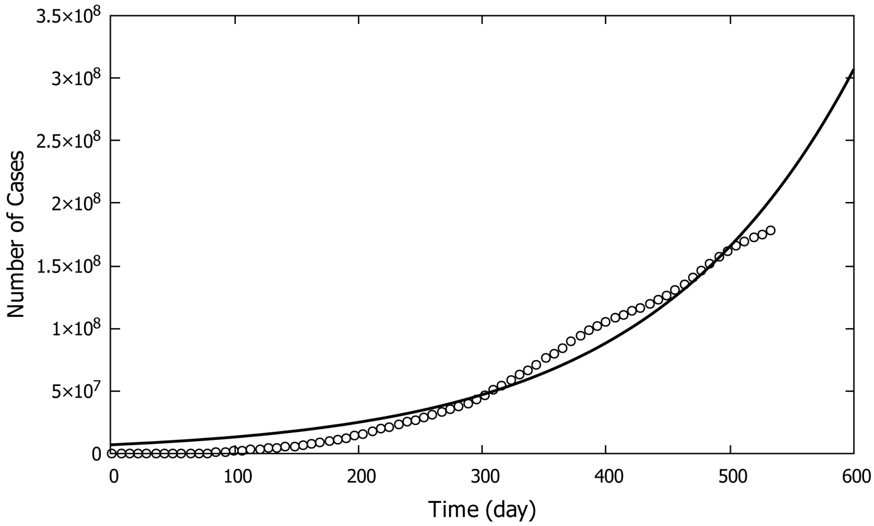

Figure 1. Worldwide COVID cases as a function of time.

Table 1. Number of the world coronavirus cases from December 30, 2019 to June 20, 2021 [13]. The data obtained from WHO is continuously revised as more data come in meaning the data which was collected originally may be slightly altered if viewed at a further date.

Day	Cases	Day	Cases	Day	Cases	Day	Cases
1	1	141	5,225,157	281	37,710,539	421	113,561,976
8	45	148	5,947,720	288	40,268,140	428	116,312,221
15	137	155	6,786,661	295	43,290,917	435	119,279,244
22	2061	162	7,675,625	302	46,759,504	442	122,600,095
29	14,644	169	8,694,929	309	50,556,433	449	126,432,603
36	37,657	176	9,833,792	316	54,636,216	456	130,535,153
43	69,375	183	11,137,846	323	58,781,647	463	135,122,255
50	79,005	190	12,566,581	330	62,849,434	470	140,398,755
57	87,426	197	14,133,450	337	67,027,757	477	146,142,306
64	107,466	204	15,877,378	344	71,394,221	484	151,882,470
71	161,243	211	17,724,078	351	76,044,276	491	157,375,571
78	314,646	218	19,568,304	358	80,182,403	498	162,207,358
85	675,304	225	21,459,711	365	84,325,552	505	166,083,578
92	1,174,742	232	23,272,553	372	89,371,212	512	169,647,532
99	1,729,560	239	25,128,039	379	94,230,121	519	172,678,347
106	2,264,263	246	27,091,286	386	98,498,696	526	175,346,072
113	2,819,049	253	29,028,553	393	102,292,594	533	177,875,936
120	3,367,817	260	31,121,103	400	105,520,024		
127	3,953,175	267	33,223,301	407	108,392,203		
134	4,551,838	274	35,342,832	414	110,878,947		

fluctuations in data. These fluctuations, however, average out resulting in the effective parameters n_0 and N given by Equation (10).

The agreement between Equation (9) and the world data on COVID-19 cases is indicative of validity of the model and the corresponding underlying assumptions, which is further evidenced by agreement of the model with the United States data alone. Furthermore, since the model does not depend on the specifics of COVID-19, it can be applied to any pandemic in general.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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