

# Persistence and Extinction of a Non-Autonomous Plant Disease Model with Roguing\*

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## Abstract

On the basis of analyzing the shortages of present studies on plant disease model for autonomous phenomenon, and considering the actual situation, this paper applies the joint factors of environmental change and the infectivity for latent plants into the system; therefore we deal with a non-autonomous plant disease model with roguing. Some sufficient conditions are established for extinction of diseases and permanence of the system in this paper.

## Keywords

Non-Autonomous Plant Disease Model, Roguing, Extinction, Permanence

## 1. Introduction

In the real world, the phenomenon of plant virus diseases is quite widespread; a lot of plant diseases are closely related to people's lives. In the Middle Ages, European barley developed ergot disease. Eating barley mixed with ergot caused people to be psychedelic, turned black for their limbs, and gangrened. After eating sweet potato with black spot disease by livestock, it can induce asthma or even death in severe cases. Papaya (known as "Beneficial Fruit King") virus disease in Guangdong Province can't be cultivated so far. The potato in Ireland stopped production due to potato late blight, which caused 1.1 million people died of starvation, and forced 1.64 million people to flee to North America in 1845-1846 [1]. In 1970, because of the pandemic of corn spot disease in the United States, the yield was reduced by fifteen percent, which was about 16.5 billion kg, and the economic loss was 11 billion US dollars [2].

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Therefore, we know a variety of plant diseases which play important roles in not only threatening plant's lives but also lead to catastrophic economic consequences. With the development of modern technology and the world economy, the problem of plant virus diseases attracts more and more attentions. Millions of plants suffer or die of various virus diseases every year. Plant viruses are important constraints to crop production, and cause serious losses in yield and quality of many crops grown in agriculture, horticulture and forestry [3] [4] [5] [6] [7]. Among various methods to control plant virus diseases in the real world, there are great research efforts on chemical methods, but no chemicals can routinely be used to control viruses in the crop [8]. Therefore, it is necessary to take effective measures to control the spread of plant virus. Methods to prevent virus from reaching the host are well-developed. Cultural control strategy is the most commonly method for controlling plant diseases, and roging is one of the methods for cultural control strategy [9] [10] [11] [12] [13].

Considerable researches have been using mathematical models for plant virus diseases to describe, analyze, and predict epidemics of plant diseases for the ultimate purposes of developing and testing control strategies and tactics for plant protection [12]-[19]. A mathematical model was proposed by Fishman *et al.* for the temporal spread of an epidemic in a closed plant population with periodic of the diseased plants [12]. Here, Chan and Jeger [20] considered a plant virus disease model:

$$\begin{cases} S'(t) = r(K - N(t)) - \mu S(t) - k_1 S(t) \frac{I(t)}{K}, \\ E'(t) = k_1 S(t) \frac{I(t)}{K} - (\mu + k_2) E(t), \\ I'(t) = k_2 E(t) - (\mu + k_3) I(t) \\ R'(t) = k_3 I(t) - (\mu + \alpha) R(t). \end{cases} \quad (1.1)$$

The model is derived from the following assumptions.

- The plant population was divided into susceptible  $S$ , latently infected  $E$ , infectious  $I$  and post-infectious  $R$  categories.
- There was natural mortality  $\mu$ , which was not attributed to disease and was common to each category. The disease led to an additional mortality  $\alpha$  in the post-infectious category owing to the cumulative effect of the disease.
- There was maximum plant population size  $K$ , defined in terms of agronomic considerations. The actual total population size was presented by  $N(N = S + E + I + R)$ . Recruitment to the population was by replanting at a rate proportional  $r$  to the difference between the actual number of plants present  $N$  and maximum population size  $K$ .
- The rate of infection was determined by the product number of susceptible and infected plants, and a constant rate  $k_1$  (plants per infectious plants per unit time) divided by  $K$  to give uniform dimensions for the rate parameters.
- There were the conversion rates of disease progression  $k_2$  and  $k_3$ , which were from latent to infectious and from infectious to post-infectious, respec-

tively.

The authors studied the disease management by roguing infected plants and replanting susceptible plants in model (1.1). And they also analyzed the case that the roguing was imposed at a constant proportional rate  $\eta$  in the latent stage, infectious stage, or post-infectious stage with respect to system (1.1).

In epidemiology, the latent period is the interval between the onset of spore germination and the appearance of the next spore generation [21]. Therefore, we consider that the plant still has a certain infectivity during the latent period. The importance of the latent period in model has been emphasized by Gumpert *et al.* [22]. A model was researched by Nakasuji *et al.* to depict the quantitative changes in pathogen, vector, and host plant populations with a latent period [23]. In addition, the case of the roguing is studied in their work by Chan *et al.* for latent plants [24]. Therefore, it is necessary to degrade both the latent plant and the infected plant at the same time for system (1.1).

However, because biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces in the system with a fluctuating environment. As far as we know, almost references considered plant disease with both seasonality and human-to-human transmission. Occurrence of plant disease is typically seasonal due climatic factors, physical, and biological factor. Therefore, it is interesting to formulate a more realistic mathematical model with the seasonality of the changing environment (see [19] [20] [21] [22] [23] [25]).

Based on the above factors, here we propose and analyze a more realistic mathematical model as the following non-autonomous form of system (1.1):

$$\begin{cases} S'(t) = r(t)(K - S(t)) - \mu(t)S(t) - k_1(t)S(t)\frac{I(t)}{K} - k_4(t)S(t)E(t), \\ E'(t) = k_1(t)S(t)\frac{I(t)}{K} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t), \\ I'(t) = k_2(t)E(t) - (\mu(t) + k_3(t) + \eta_2(t))I(t), \\ R'(t) = k_3(t)I(t) - (\mu(t) + \alpha(t))R(t). \end{cases} \quad (1.2)$$

where  $k_4$  denotes the horizontal transmission rate by vectored transmission,  $\eta_1$  and  $\eta_2$  denote the roguing (removing) rates of latently infected and infectious plants respectively. Note that the variable  $R$  does not appear in the first three equations of system (1.2). This allows us to attack (1.2) by studying the subsystem:

$$\begin{cases} S'(t) = r(t)(K(t) - S(t)) - \mu(t)S(t) - k_1(t)S(t)\frac{I(t)}{K(t)} - k_4(t)S(t)E(t), \\ E'(t) = k_1(t)S(t)\frac{I(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t), \\ I'(t) = k_2(t)E(t) - (\mu(t) + k_3(t) + \eta_2(t))I(t), \end{cases} \quad (1.3)$$

with initial values

$$S(0) > 0, E(0) \geq 0, I(0) \geq 0. \tag{1.4}$$

This paper is arranged in five parts. Section 2 will introduce preliminaries setting and propositions, which we use to analyze the long-time behavior of system (1.3) in the following sections. The extinction conditions of the disease of system (1.3) will be given in Section 3. In Section 4, we will discuss the permanence of the infectious plants. Some of the corresponding conclusions in this paper are presented in Section 5.

## 2. Preliminaries

First, we give some assumptions as follows:

(A<sub>1</sub>) Functions  $r(t), K(t), \mu(t), k_1(t), k_2(t), k_4(t), \eta_1(t)$  and  $\eta_2(t)$  are positive, bounded and continuous on  $[0, +\infty)$ .

(A<sub>2</sub>) There exist constants  $\omega_i > 0, (i = 1, 2, 3)$  such that:

$$\begin{aligned} \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_1} \frac{k_1(s)}{k(s)} ds &> 0, \\ \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_2} r(s)k(s) ds &> 0, \\ \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_3} \mu(s) ds &> 0 \end{aligned}$$

Next, we denote by  $x(t)$  and  $y(t)$  the solutions of:

$$\frac{dx(t)}{dt} = r(t)k(t) - \mu(t)x(t), \tag{2.1}$$

and

$$\frac{dy(t)}{dt} = r(t)k(t) - (\mu(t) + r(t) + \eta_1(t) + \eta_2(t) + k_3(t))y(t), \tag{2.2}$$

respectively. From [[26], Lemma 2.1], we have the following results for system (2.1-2.2).

**Lemma 2.1.** 1) There exist constants  $m' > 0$  and  $M > 0$ , such that every positive solution of Equation (2.1),  $x(t)$ , satisfies:

$$0 < m' \leq \liminf_{t \rightarrow +\infty} x(t) \leq \limsup_{t \rightarrow +\infty} x(t) \leq M < +\infty.$$

2) There exist constants  $m > 0$  and  $M_1 > 0$ , such that every positive solution of Equation (2.2),  $y(t)$ , satisfies:

$$0 < m \leq \liminf_{t \rightarrow +\infty} y(t) \leq \limsup_{t \rightarrow +\infty} y(t) \leq M_1 < +\infty.$$

3) The solution  $(S(t), E(t), I(t))$  of system (1.3) with initial value (1.4) exists, uniformly bounded and

$$S(t) > 0, E(t) > 0, I(t) > 0$$

for all  $t > 0$ .

**Theorem 2.1.** Suppose that (A<sub>1</sub>) and (A<sub>2</sub>) hold, then there must be constants  $m > 0$  and  $M > 0$  satisfying

$$0 < m \leq \liminf_{t \rightarrow +\infty} N^*(t) \leq \limsup_{t \rightarrow +\infty} N^*(t) \leq M < +\infty.$$

Proof. Set  $(S(t), E(t), I(t))$  be any positive solution of system (1.3) with initial conditions (1.4). We can see that  $N^*(t) = S(t) + E(t) + I(t)$  means the size of total plants at time  $t$  for system (1.3). From system (1.3), we can get

$$\frac{dN^*(t)}{dt} \leq r(t)k(t) - \mu(t)N^*(t),$$

for all  $t \geq 0$ . By comparison theorem, we have that there exists constant  $T_1 > 0$  such that

$$N^*(t) \leq \bar{N}(t), \quad (2.3)$$

for all  $t \geq T_1$ , where  $\bar{N}(t)$  is the solution of (2.1) with the condition  $x(T_1) = N^*(T_1)$ . On the other hand, from system (1.3), we can get

$$\frac{dN^*(t)}{dt} \geq r(t)k(t) - (\mu(t) + r(t) + \eta_1(t) + \eta_2(t) + k_3(t))N^*(t),$$

for all  $t \geq T_1$ . By comparison theorem, we have that there exists constant  $T_2 > T_1$  such that

$$N^*(t) \geq \underline{N}(t), \quad (2.4)$$

for all  $t \geq T_2$ , where  $\underline{N}(t)$  is the solution of (2.2) with the condition  $y(T_2) = N^*(T_2)$ . From inequalities (2.3) and (2.4), we can easily obtain

$$\underline{N}(t) \leq N^*(t) \leq \bar{N}(t), \quad (2.5)$$

for all  $t \geq T_2$ . From the above conclusions 1) and 2) of Lemma 2.1 and inequality (2.5), we have

$$\begin{aligned} 0 < m \leq \liminf_{t \rightarrow +\infty} \underline{N}(t) &\leq \liminf_{t \rightarrow +\infty} N^*(t) \leq \limsup_{t \rightarrow +\infty} N^*(t) \\ &\leq \limsup_{t \rightarrow +\infty} \bar{N}(t) \leq M < +\infty, \end{aligned} \quad (2.6)$$

that is to say

$$0 < m \leq \liminf_{t \rightarrow +\infty} N^*(t) \leq \limsup_{t \rightarrow +\infty} N^*(t) \leq M < +\infty.$$

The proof is completed.

Next, we consider the following non-autonomous linear equation:

$$\frac{dZ(t)}{dt} = r(t)K(t) - \left[ r(t) + \mu(t) + \left( \frac{k_1(t)}{K(t)} + k_4(t) \right) \bar{N}(t) \right] Z(t). \quad (2.7)$$

**Lemma 2.2.** Suppose that  $(A_i)$  holds and there exist constants  $\omega_i > 0, (i = 4, 5)$  such that

$$\begin{aligned} \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_4} r(s)k(s) ds &> 0, \\ \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_5} \left[ r(s) + \mu(s) + \left( \frac{k_1(s)}{K(s)} + k_4(s) \right) \bar{N}(s) \right] ds &> 0, \end{aligned}$$

then there exists  $T_3 > T_2$ , such that  $S(t) \geq \underline{S}(t)$  for all  $t \geq T_3$ , where  $\underline{S}(t)$  is the solution of system (2.7) with the condition  $Z(T_3) = S(T_3)$ .

For  $p > 0$  and  $t > 0$ , we define:

$$G(p, t) = \left( \frac{pk_1(t)}{K(t)} + k_4(t) \right) \bar{N}(t) - \left( 1 + \frac{1}{p} \right) k_2(t) - \eta_1(t) + k_3(t) + \eta_2(t),$$

and

$$W(p, t) = pE(t) - I(t), \tag{2.8}$$

where  $(S(t), E(t), I(t))$  is any solution of system (1.3). We use the following lemmas in order to investigate the long-time behavior of system (1.3).

**Lemma 2.3.** If there exist positive constants  $p > 0$  and  $T_1' \geq T_3$  such that  $G(p, t) < 0$  for all  $t \geq T_1'$ , then there exists  $T_2' \geq T_1'$  such that either  $W(p, t) > 0$  for all  $t \geq T_2'$  or  $W(p, t) \leq 0$  for all  $t \geq T_2'$ .

**Proof.** Suppose that there does not exist  $T_2' \geq T_1'$  such that either  $W(p, t) > 0$  for all  $t \geq T_2'$  or  $W(p, t) \leq 0$  for all  $t \geq T_2'$  hold. Then there necessarily exists  $s \geq T_1'$  such that

$$W(p, s) = 0 \text{ and } \frac{dW(p, s)}{ds} > 0. \tag{2.9}$$

So we have

$$pE(s) = I(s), \tag{2.10}$$

and

$$\begin{aligned} & p \left\{ k_1(s)S(s) \frac{I(s)}{K(s)} + k_4(s)S(s)E(s) - (\mu(s) + k_2(s) + \eta_1(s))E(s) \right\} \\ & - \left\{ k_2(s)E(s) - (\mu(s) + k_3(s) + \eta_2(s))I(s) \right\} \\ & = I(s) \left\{ \frac{pk_1(s)}{K(s)}S(s) + (\mu(s) + k_3(s) + \eta_2(s)) \right\} \\ & + pE(s) \left\{ k_4(s)S(s) - (\mu(s) + k_2(s) + \eta_1(s)) - \frac{k_2(s)}{p} \right\} > 0 \end{aligned} \tag{2.11}$$

Substituting (2.10) into (2.11) we can get

$$\begin{aligned} 0 &< pE(s) \left\{ \frac{pk_1(s)}{K(s)} + k_4(s)S(s) - \left( 1 + \frac{1}{p} \right) k_2(s) - \eta_1(s) + k_3(s) + \eta_2(s) \right\} \\ &\leq pE(s) \left\{ \frac{pk_1(s)}{K(s)} + k_4(s)N^*(s) - \left( 1 + \frac{1}{p} \right) k_2(s) - \eta_1(s) + k_3(s) + \eta_2(s) \right\} \\ &\leq pE(s)G(p, s) \end{aligned}$$

From 3) of Lemma 2.1, we have  $G(p, s) > 0$ , which is a contradiction.  $\square$

### 3. Extinction of Infectious Plants

In this section, we obtain conditions for the extinction of infectious plants of system (1.3).

**Theorem 3.1.** If there exist positive constants  $\lambda > 0, p > 0$  and  $T_1' \geq T_3$  such that

$$R_1(\lambda, p) := \limsup_{t \rightarrow +\infty} \int_t^{t+\lambda} \left\{ \left( \frac{pk_1(s)}{K(s)} + k_4(s) \right) \bar{N}(s) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds < 0, \tag{3.1}$$

$$R_1^*(\lambda, p) := \limsup_{t \rightarrow +\infty} \int_t^{t+\lambda} \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds < 0, \quad (3.2)$$

and  $G(p, t) < 0$  for all  $t \geq T_1'$ , then the infectious plants of system (1.3) is extinct.

**Proof.** From Lemma 2.3, we only have to consider the following two cases.

- 1)  $pE(t) > I(t)$  for all  $t \geq T_2'$ .
- 2)  $pE(t) \leq I(t)$  for all  $t \geq T_2'$ .

First we consider the case 1). From the second equation of system (1.3), we can get

$$\begin{aligned} \frac{dE(t)}{dt} &= k_1(t)S(t)\frac{I(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ &\leq \left[ \frac{k_1(t)}{K(t)}I(t) + k_4(t)E(t) \right] N^*(t) - [\mu(t) + k_2(t) + \eta_1(t)]E(t) \\ &\leq E(t) \left\{ \left[ \frac{pk_1(t)}{K(t)} + k_4(t) \right] \bar{N}(t) - [\mu(t) + k_2(t) + \eta_1(t)] \right\}. \end{aligned}$$

Hence, we have

$$E(t) < E(T_2') \exp \left( \int_{T_2'}^t \left\{ \left[ \frac{pk_1(s)}{K(s)} + k_4(s) \right] \bar{N}(s) - [\mu(s) + k_2(s) + \eta_1(s)] \right\} ds \right) \quad (3.3)$$

for all  $t \geq T_2'$ . From (3.1) we see that there exist constants  $\delta_1 > 0$  and  $T_3' > T_2'$  such that

$$\int_t^{t+\lambda} \left\{ \left[ \frac{pk_1(s)}{K(s)} + k_4(s) \right] \bar{N}(s) - [\mu(s) + k_2(s) + \eta_1(s)] \right\} ds < -\delta_1, \quad (3.4)$$

for all  $t \geq T_3'$ . From (3.3) and (3.4), we have  $\lim_{t \rightarrow +\infty} E(t) = 0$ . Then it follows from  $pE(t) > I(t)$  for all  $t \geq T_2'$  that  $\lim_{t \rightarrow +\infty} I(t) = 0$ .

Next we consider the case 2). Since we have  $E(t) \leq \frac{1}{p}I(t)$  for all  $t \geq T_2'$ , it follows From the third equation of system (1.3) that

$$\begin{aligned} \frac{dI(t)}{dt} &= k_2(t)E(t) - (\mu(t) + k_3(t) + \eta_2(t))I(t) \\ &\leq \frac{1}{p}k_2(t)I(t) - (\mu(t) + k_3(t) + \eta_2(t))I(t) \\ &= I(t) \left\{ \frac{k_2(t)}{p} - (\mu(t) + k_3(t) + \eta_2(t)) \right\} \end{aligned}$$

Hence, we can easily get

$$I(t) \leq I(T_2') \exp \left( \int_{T_2'}^t \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds \right) \quad (3.5)$$

for all  $t \geq T_2'$ . Now it follows from (3.2) that there exist constants  $\delta_2 > 0$  and  $T_4' > T_2'$  such that

$$\int_t^{t+\lambda} \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds < -\delta_2 \quad (3.6)$$

for all  $t \geq T'_4$ . From (3.5) and (3.6), we have  $\lim_{t \rightarrow +\infty} I(t) = 0$ .  $\square$

### 4. Permanence of Infectious Plants

In this section, we obtain sufficient conditions for the permanence of system (1.3).

**Theorem 4.1.** If there exist constants  $\lambda > 0, p > 0$  and  $T'_1 \geq T_3$  such that

$$R_2(\lambda, p) := \liminf_{t \rightarrow +\infty} \int_t^{t+\lambda} \left\{ \frac{pk_1(s)}{K(s)} + k_4(s)N(s) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds > 0, \tag{4.1}$$

$$R_2^*(\lambda, p) := \liminf_{t \rightarrow +\infty} \int_t^{t+\lambda} \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds > 0, \tag{4.2}$$

and  $G(p, t) < 0$  for all  $t \geq T'_1$ , then the infectious plants of system (1.3) is permanent.

Before we give the proof of Theorem 4.1, we introduce the following lemma.

**Lemma 4.1.** If there exist constants  $\lambda > 0, p > 0$  and  $T'_1 \geq T_3$  such that (4.1), (4.2) and  $G(p, t) < 0$  hold for all  $t \geq T'_1$ , then  $W(p, t) \leq 0$  for all  $t \geq T'_2 \geq T'_1$ , where  $T'_2$  is given as in lemma 2.3.

**Proof.** From Lemma 2.3 we have only two cases,  $W(p, t) > 0$  for all  $t \geq T'_2$  or  $W(p, t) \leq 0$  for all  $t \geq T'_2$ . Suppose that  $W(p, t) > 0$  for all  $t \geq T'_2$ . Then we have  $E(t) > I(t)/p$  for all  $t \geq T'_2$ . It follows from the third equation of system (1.3) that

$$\frac{dI(t)}{dt} > I(t) \left\{ \frac{k_2(t)}{p} - (\mu(t) + k_3(t) + \eta_2(t)) \right\}$$

for all  $t \geq T'_2$ . Hence, we have

$$I(t) > I(T'_2) \exp \left( \int_{T'_2}^t \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds \right), \tag{4.3}$$

for all  $t \geq T'_2$ . From the equality (4.2), we see that there exist constants  $\eta' > 0$  and  $T > 0$  such that

$$\int_t^{t+\lambda} \left\{ \frac{k_2(s)}{p} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds > \eta', \tag{4.4}$$

for all  $t \geq T$ . Since the inequality (4.3) holds for all  $t \geq \max(T'_2, T)$ , it follows from (4.4) that  $\lim_{t \rightarrow +\infty} I(t) = +\infty$ . This contradicts with the boundedness of  $I$ , stated in 3) of Lemma 2.1.

Here, we use Lemmas 4.1 in order to prove Theorem 4.1.

**Proof.** (Proof of Theorem 4.1). For simplicity, let  $m_\epsilon := m - \epsilon$  and  $M_\epsilon := M + \epsilon$ , where  $\epsilon > 0$  is a constant. From the inequality (2.6) of Theorem 2.1, we can see that for any  $\epsilon > 0$ , there exists  $T \geq T_3$  such that

$$m_\epsilon < \underline{N}(t) \leq N^*(t) \leq \bar{N}(t) < M_\epsilon, \tag{4.5}$$

for all  $t \geq T$ . The inequality (4.1) implies that for sufficiently small  $\eta' > 0$ , there exists  $T'_1 \geq T$  such that



$$\int_t^{t+\lambda} \left\{ \left( \frac{pk_1(s)}{K(s)} + k_4(s) \right) \underline{N}(s) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds > \eta', \quad (4.6)$$

for all  $t \geq T'_1$ . We define

$$\begin{aligned} k_1^+ &:= \sup_{t \geq 0} k_1(t), \quad k_4^+ := \sup_{t \geq 0} k_4(t), \quad \mu^+ := \sup_{t \geq 0} \mu(t), \\ k_3^+ &:= \sup_{t \geq 0} k_3(t), \quad \eta_2^+ := \sup_{t \geq 0} \eta_2(t), \quad K^- := \inf_{t \geq 0} K(t). \end{aligned}$$

From (4.5) and (4.6), we see that for positive constants  $\tilde{\eta} < \eta'$  and  $T'_2 \geq T'_1$  there exist small  $\epsilon_i > 0 (i = 1, 2)$ , such that

$$\int_t^{t+\lambda} \left\{ \left( \frac{pk_1(s)}{K(s)} + k_4(s) \right) (\underline{N}(s) - \epsilon_1 - \tilde{k}\epsilon_2) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds > \tilde{\eta}, \quad (4.7)$$

and

$$\underline{N}(t) - \epsilon_1 - \tilde{k}\epsilon_2 > m_\epsilon, \quad (4.8)$$

hold for all  $t \geq T'_2$ , where  $\tilde{k} := 1 + M_\epsilon \omega_3 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right)$ . From (A<sub>2</sub>),  $\epsilon_2$  can be chosen sufficiently small such that

$$\int_t^{t+\omega_3} \left\{ M_\epsilon \epsilon_2 \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) - \epsilon_1 (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds < -\tilde{\eta}. \quad (4.9)$$

hold for all  $t \geq T'_2$ .

First, we claim that

$$\limsup_{t \rightarrow +\infty} I(t) > \epsilon_2.$$

In fact, if it is not true, then there exists  $T'_3 \geq T'_2$  such that

$$I(t) \leq \epsilon_2, \quad (4.10)$$

for all  $t \geq T'_3$ . Now, we only have to consider two cases as follows:

- 1)  $E(t) \geq \epsilon_1$  for all  $t \geq T'_3$ .
- 2) There exists an  $s_1 \geq T'_3$  such that  $E(s_1) < \epsilon_1$ .

First we consider the case 1). From (4.5) and (4.10) and Lemma 4.1, we have

$$\begin{aligned} E(t) &= E(T'_3) + \int_{T'_3}^t \left\{ k_1(s) S(s) \frac{I(s)}{K(s)} + k_4(s) S(s) E(s) \right. \\ &\quad \left. - (\mu(s) + k_2(s) + \eta_1(s)) E(s) \right\} ds \\ &\leq E(T'_3) + \int_{T'_3}^t \left\{ I(s) \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) (N^*(s) - E(s) - I(s)) \right. \\ &\quad \left. - \epsilon_1 (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds \\ &\leq E(T'_3) + \int_{T'_3}^t \left\{ \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) M_\epsilon \epsilon_2 - \epsilon_1 (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds \end{aligned}$$

for all  $t \geq T'_3$ . Thus, from (4.9), we have  $\lim_{t \rightarrow +\infty} E(t) = -\infty$ , which contradicts with 3) of Lemma 2.1.

Next we consider the case 2). Suppose that there exists an  $s_2 > s_1$  such that  $E(s_2) > \epsilon_1 + M_\epsilon \omega_3 \epsilon_2 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right)$ . Then, we see that there necessarily exists an  $s_3 \in (s_1, s_2)$  such that  $E(s_3) = \epsilon_1$  and  $E(t) > \epsilon_1$  for all  $t \in (s_3, s_2]$ . Let  $n$  be an integer such that  $s_2 \in [s_3 + n\omega_3, s_3 + (n+1)\omega_3)$ . Then from (4.9), we obtain

$$\begin{aligned} & \epsilon_1 + M_\epsilon \omega_3 \epsilon_2 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right) < E(s_2) \\ & = E(s_3) + \int_{s_3}^{s_2} \left\{ k_1(s)S(s) \frac{I(s)}{K(s)} + k_4(s)S(s)E(s) \right. \\ & \quad \left. - (\mu(s) + k_2(s) + \eta_1(s))E(s) \right\} ds \\ & < \epsilon_1 + \left\{ \int_{s_3}^{s_3+n\omega_3} + \int_{s_3+n\omega_3}^{s_2} \right\} \left\{ \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) \epsilon_2 M_\epsilon - (\mu(s) + k_2(s) + \eta_1(s)) \epsilon_1 \right\} ds \\ & < \epsilon_1 + \int_{s_3+n\omega_3}^{s_2} \left\{ \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) \epsilon_2 M_\epsilon - (\mu(s) + k_2(s) + \eta_1(s)) \epsilon_1 \right\} ds \\ & < \epsilon_1 + \int_{s_3+n\omega_3}^{s_2} M_\epsilon \epsilon_2 \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) ds < \epsilon_1 + M_\epsilon \epsilon_2 \omega_3 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right) \end{aligned}$$

which is a contradiction. Therefore, we see that

$$E(t) \leq \epsilon_1 + M_\epsilon \epsilon_2 \omega_3 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right) \tag{4.11}$$

for all  $t \geq s_1$ . Now, from Lemma 4.1, there exists  $T_4 \geq s_1$  such that  $W(p, t) = pE(t) - I(t) \leq 0$  for all  $t \geq T_4$ . So we have

$$\begin{aligned} \frac{dE(t)}{dt} & = k_1(t)S(t) \frac{I(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ & \geq pk_1(t)S(t) \frac{E(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ & \geq E(t) \left\{ \left( \frac{pk_1(t)}{K(t)} + k_4(t) \right) (N(t) - E(t) - I(t)) - (\mu(t) + k_2(t) + \eta_1(t)) \right\} \\ & \geq E(t) \left\{ \left( \frac{pk_1(t)}{K(t)} + k_4(t) \right) (N(t) - \epsilon_1 - \tilde{k}\epsilon_2) - (\mu(t) + k_2(t) + \eta_1(t)) \right\}. \end{aligned}$$

It follows from (4.10) and (4.11), we have  $E(t) + I(t) \leq \epsilon_1 + \tilde{k}\epsilon_2$  for all  $t \geq T_4$ . Hence, we can get

$$\begin{aligned} E(t) & \geq E(T_4) \exp \left( \int_{T_4}^t \left\{ \left( \frac{pk_1(s)}{K(s)} + k_4(s) \right) (N(s) - \epsilon_1 - \tilde{k}\epsilon_2) \right. \right. \\ & \quad \left. \left. - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds \right). \end{aligned}$$

It follows from (4.7) that  $\lim_{t \rightarrow +\infty} E(t) = +\infty$  and this contradicts with the boundedness of  $E(t)$ , stated in 3) of Lemma 2.1.

Thus, we see that our claim  $\limsup_{t \rightarrow +\infty} I(t) > \epsilon_2$  is true.

Next, we prove

$$\liminf_{t \rightarrow +\infty} I(t) \geq I_1,$$

where  $I_1 = \epsilon_2 e^{-(\mu^+ k_3^+ + \eta_2^+)(C+2\lambda_2)} > 0$  is a constant given in the following. For convenience, we let  $\varphi$  be the least common multiple of  $\omega_3$  and  $\lambda$ . From inequalities (4.7)-(4.9) and assumption (A<sub>2</sub>), we see that there exist constants  $\bar{T}_3 (\geq T'_2)$  and  $\lambda_2 \geq \omega_1$ , which is an integral multiple of  $\varphi$ , and  $\eta^* > 0$  such that

$$\int_t^{t+\lambda_3} \left\{ \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) \epsilon_2 M_\epsilon - (\mu(s) + k_2(s) + \eta_1(s)) \epsilon_1 \right\} ds < -M_\epsilon, \tag{4.12}$$

$$\int_t^{t+\lambda_3} \left\{ \left( \frac{pk_1(s)}{K(s)} + k_4(s) \right) (\underline{N}(s) - \epsilon_1 - \tilde{k}\epsilon_2) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds > \eta^*, \tag{4.13}$$

$$\int_t^{t+\lambda_3} \frac{k_1(s)}{K(s)} ds > \eta^*, \tag{4.14}$$

for all  $t \geq \bar{T}_3$  and  $\lambda_3 \geq \lambda_2$  and  $\lambda_3$  is an integral multiple of  $\varphi$ . Let  $C > 0$  be an integer multiple of  $\lambda_2$  satisfying

$$e^{-\lambda_2(\mu^+ + k_2^+ + \eta_1^+)} \eta^* m_\epsilon \nu_2 e^{\frac{C}{\lambda_2} \eta^*} > \epsilon_1 + M_\epsilon \omega_3 \epsilon_2 \left( \frac{k_1^+}{K^-} + \frac{1}{p} k_4^+ \right) \tag{4.15}$$

where  $\nu_2 := \epsilon_2 e^{-2\lambda_2(\mu^+ + k_3^+ + \eta_2^+)}$ . It follows from  $\limsup_{t \rightarrow +\infty} I(t) > \epsilon_2$  that there are only two possibilities as follows:

- 1)  $I(t) \geq \epsilon_2$  for all  $t \geq \exists \bar{T}_4 \geq \bar{T}_3$ .
- 2)  $I(t)$  oscillates about  $\epsilon_2$  for large  $t \geq \bar{T}_3$ .

In case 1), we have  $\liminf_{t \rightarrow +\infty} I(t) \geq \epsilon_2 \geq I_1$ . In case 2), there necessarily exist two constants  $t_1, t_2 \geq \bar{T}_3 (t_2 \geq t_1)$  such that

$$\begin{cases} I(t_1) = I(t_2) = \epsilon_2, \\ I(t) < \epsilon_2, \text{ for all } t \in (t_1, t_2). \end{cases}$$

a) Suppose that  $t_2 - t_1 \leq C + 2\lambda_2$ . Then, from (1.3) we have

$$\begin{aligned} \frac{dI(t)}{dt} &= k_2(t)E(t) - (\mu(t) + k_3(t) + \eta_2(t))I(t) \\ &\geq -(\mu(t) + k_3(t) + \eta_2(t))I(t) \end{aligned} \tag{4.16}$$

Hence, we obtain

$$I(t) \geq I(t_1) \exp\left(\int_{t_1}^t -(\mu(s) + k_3(s) + \eta_2(s)) ds\right) \geq \epsilon_2 e^{-(\mu^+ + k_3^+ + \eta_2^+)(C+2\lambda_2)} := I_1, \tag{4.17}$$

for all  $t \in (t_1, t_2)$ .

b) Suppose that  $t_2 - t_1 > C + 2\lambda_2$ . Then, from (4.16), we have

$$I(t) \geq \epsilon_2 e^{-(\mu^+ + k_3^+ + \eta_2^+)(C+2\lambda_2)} := I_1$$

for all  $t \in (t_1, t_1 + C + 2\lambda_2)$ . Now, we are in a position to show that  $I(t) \geq I_1$  for all  $t \in [t_1 + C + 2\lambda_2, t_2)$ . Suppose that  $E(t) \geq \epsilon_1$  for all  $t \in [t_1, t_1 + \lambda_2]$ . Then, from (4.12), we have

$$\begin{aligned} & E(t_1 + \lambda_2) \\ & \leq E(t_1) + \int_{t_1}^{t_1 + \lambda_2} \left\{ \left( \frac{k_1(s)}{K(s)} + \frac{k_4(s)}{p} \right) \epsilon_2 M_\epsilon - (\mu(s) + k_2(s) + \eta_1(s)) \epsilon_1 \right\} ds \\ & < M_\epsilon - M_\epsilon = 0 \end{aligned}$$

which is a contradiction. Therefore, there exists an  $s_4 \in [t_1, t_1 + \lambda_2]$  such that  $E(s_4) < \epsilon_1$ . Then, as is in the proof of  $\limsup_{t \rightarrow +\infty} I(t) > \epsilon_2$ , we can show that

$$E(t) \leq \epsilon_1 + M_\epsilon \epsilon_2 \omega_3 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right) \text{ for all } t \geq s_4. \text{ Thus we have}$$

$$E(t) \leq \epsilon_1 + M_\epsilon \epsilon_2 \omega_3 \left( \frac{k_1^+}{K^-} + \frac{k_4^+}{p} \right) \tag{4.18}$$

for all  $t \geq t_1 + \lambda_2 \geq s_4$ . From (4.16), we have

$$I(t) \geq v_2 = \epsilon_2 e^{-2\lambda_2(\mu^+ + k_2^+ + \eta_1^+)}, \tag{4.19}$$

for all  $t \in [t_1, t_1 + 2\lambda_2]$ . Thus, from (4.8), (4.18), (4.19), we have

$$\begin{aligned} \frac{dE(t)}{dt} &= k_1(t)S(t)\frac{I(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ &\geq k_1(t)S(t)\frac{I(t)}{K(t)} - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ &\geq \frac{k_1(t)}{K(t)}(N(t) - E(t) - I(t))I(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ &\geq \frac{k_1(t)}{K(t)}(N(t) - \epsilon_1 - \tilde{k}\epsilon_2)v_2 - (\mu^+ + k_2^+ + \eta_1^+)E(t) \\ &\geq \frac{k_1(t)}{K(t)}m_\epsilon v_2 - (\mu^+ + k_2^+ + \eta_1^+)E(t) \end{aligned}$$

for all  $t \in [t_1 + \lambda_2, t_1 + 2\lambda_2]$ . Hence, from (4.14), we have

$$\begin{aligned} E(t_1 + 2\lambda_2) &\geq e^{-(\mu^+ + k_2^+ + \eta_1^+)(t_1 + 2\lambda_2)} \left\{ E(t_1 + \lambda_2) e^{(\mu^+ + k_2^+ + \eta_1^+)(t_1 + \lambda_2)} \right. \\ &\quad \left. + \int_{t_1 + \lambda_2}^{t_1 + 2\lambda_2} \frac{k_1(s)}{K(s)} m_\epsilon v_2 e^{(\mu^+ + k_2^+ + \eta_1^+)s} ds \right\} \\ &\geq e^{-(\mu^+ + k_2^+ + \eta_1^+)(t_1 + 2\lambda_2)} \int_{t_1 + \lambda_2}^{t_1 + 2\lambda_2} \frac{k_1(s)}{K(s)} m_\epsilon v_2 e^{(\mu^+ + k_2^+ + \eta_1^+)s} ds \\ &\geq e^{-\lambda_2(\mu^+ + k_2^+ + \eta_1^+)} m_\epsilon v_2 \eta^* \end{aligned} \tag{4.20}$$

Now we suppose that there exists a  $t_0 > 0$  such that  $t_0 \in (t_1 + 2\lambda_2 + C, t_2)$ ,  $I(t_0) = I_1$  and  $I(t) \geq I_1$  for all  $t \in [t_1, t_0]$ . Note that from Lemma 4.1. without loss of generality, we can assume that  $t_1$  is so large that

$$W(p, t) = pE(t) - I(t) \leq 0 \text{ for all } t \geq t_1 + 2\lambda_2. \text{ Then, from (4.18), we have}$$

$$\begin{aligned} \frac{dE(t)}{dt} &= k_1(t)S(t)\frac{I(t)}{K(t)} + k_4(t)S(t)E(t) - (\mu(t) + k_2(t) + \eta_1(t))E(t) \\ &\geq E(t)\left\{\left(\frac{pk_1(t)}{K(t)} + k_4(t)\right)(N(t) - E(t) - I(t)) - (\mu(t) + k_2(t) + \eta_1(t))\right\} \\ &\geq E(t)\left\{\left(\frac{pk_1(t)}{K(t)} + k_4(t)\right)(N(t) - \epsilon_1 - \tilde{k}\epsilon_2) - (\mu(t) + k_2(t) + \eta_1(t))\right\} \end{aligned}$$

for all  $t \in (t_1 + 2\lambda_2, t_2)$ . Thus, from (4.13) and (4.20), we have

$$\begin{aligned} E(t_0) &\geq E(t_1 + 2\lambda_2) \exp\left(\int_{t_1 + 2\lambda_2}^{t_0} \left\{\left(\frac{pk_1(s)}{K(s)} + k_4(s)\right)(N(s) - \epsilon_1 - \tilde{k}\epsilon_2) \right. \right. \\ &\quad \left. \left. - (\mu(s) + k_2(s) + \eta_1(s))\right\} ds\right) \\ &\geq e^{-\lambda_2(\mu^+ + k_2^+ \eta_1^+)} \eta^* m_\epsilon v_2 e^{\frac{C}{\lambda_2} \eta^*} \end{aligned}$$

Thus, from (4.18), we have

$$e^{-\lambda_2(\mu^+ + k_2^+ \eta_1^+)} \eta^* m_\epsilon v_2 e^{\frac{C}{\lambda_2} \eta^*} \leq \epsilon_1 + M_\epsilon \omega_3 \epsilon_2 \left(\frac{k_1^+}{K^-} + \frac{1}{p} k_4^+\right)$$

which contradicts with (4.15). Therefore,  $I(t) \geq I_1$  for all  $t \in [t_1 + 2\lambda_2 + C, t_2)$ , which implies  $\lim_{t \rightarrow +\infty} I(t) \geq I_1$ .

Since  $\limsup_{t \rightarrow +\infty} I(t) \leq \limsup_{t \rightarrow +\infty} N^*(t) \leq \limsup_{t \rightarrow +\infty} \bar{N}(t) \leq M < +\infty$ , the infectious plants of system (1.3) is permanent.

From limiting system of system (1.3), we can easily see if the infectious plants persist, then system (1.3) will be lasting.  $\square$

## 5. Conclusions

A more objective and meaningful plant virus model with roguing is proposed and analyzed. The model shows rich and complex dynamics. The weaker integral form conditions for permanence and extinction of the model are investigated by constructing auxiliary functions.

Note: If the conclusion of Lemma 2.3 does not hold, we still have another situation for (2.8); then there is  $s' \geq T_1'$  such that

$$W(p, s') = 0 \quad \text{and} \quad \frac{dW(p, s')}{ds} < 0.$$

Accordingly, we can construct an auxiliary function

$$\bar{G}(q, t) = \left(\frac{qk_1(t)}{K(t)} + k_4(t)\right) \underline{S}(t) - \left(1 + \frac{1}{q}\right) k_2(t) - \eta_1(t) + k_3(t) + \eta_2(t),$$

where  $q > 0$  and  $t > 0$ .

Therefore, we have the following similar conclusions for the persistence and extinction of diseases in this section:

**Conclusion 5.1.** If there are positive constants  $\bar{\lambda} > 0, q > 0$  and  $T_1'' \geq T_3$  such

that

$$R_1(\bar{\lambda}, q) := \limsup_{t \rightarrow +\infty} \int_t^{t+\bar{\lambda}} \left\{ \frac{qk_1(s)}{K(s)} + k_4(s)\bar{N}(s) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds < 0, \quad (5.1)$$

$$R_1^*(\bar{\lambda}, q) := \limsup_{t \rightarrow +\infty} \int_t^{t+\bar{\lambda}} \left\{ \frac{k_2(s)}{q} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds < 0, \quad (5.2)$$

and  $\bar{G}(q, t) > 0$  for all  $t \geq T_1^n$ ; then the infectious plants of system (1.3) is extinct.

**Conclusion 5.2.** If there are constants  $\bar{\lambda} > 0, q > 0$  and  $T_1^n \geq T_3$  such that

$$R_2(\bar{\lambda}, q) := \liminf_{t \rightarrow +\infty} \int_t^{t+\bar{\lambda}} \left\{ \frac{qk_1(s)}{K(s)} + k_4(s)\underline{N}(s) - (\mu(s) + k_2(s) + \eta_1(s)) \right\} ds > 0, \quad (5.3)$$

$$R_2^*(\bar{\lambda}, q) := \liminf_{t \rightarrow +\infty} \int_t^{t+\bar{\lambda}} \left\{ \frac{k_2(s)}{q} - (\mu(s) + k_3(s) + \eta_2(s)) \right\} ds > 0, \quad (5.4)$$

and  $\bar{G}(q, t) > 0$  for all  $t \geq T_1^n$ ; then the infectious plants of system (1.3) is permanent.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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