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Performance of Continuous Wavelet Transform over Fourier Transform in Features Resolutions

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Abstract

This study presents a comparative analysis of two image enhancement techniques, Continuous Wavelet Transform (CWT) and Fast Fourier Transform (FFT), in the context of improving the clarity of high-quality 3D seismic data obtained from the Tano Basin in West Africa, Ghana. The research focuses on a comparative analysis of image clarity in seismic attribute analysis to facilitate the identification of reservoir features within the subsurface structures. The findings of the study indicate that CWT has a significant advantage over FFT in terms of image quality and identifying subsurface structures. The results demonstrate the superior performance of CWT in providing a better representation, making it more effective for seismic attribute analysis. The study highlights the importance of choosing the appropriate image enhancement technique based on the specific application needs and the broader context of the study. While CWT provides high-quality images and superior performance in identifying subsurface structures, the selection between these methods should be made judiciously, taking into account the objectives of the study and the characteristics of the signals being analyzed. The research provides valuable insights into the decision-making process for selecting image enhancement techniques in seismic data analysis, helping researchers and practitioners make informed choices that cater to the unique requirements of their studies. Ultimately, this study contributes to the advancement of the field of subsurface imaging and geological feature identification.

Keywords

Continuous Wavelet Transform (CWT), Fast Fourier Transform (FFT), Reservoir Characterization, Tano Basin, Seismic Data, Spectral Decomposition

1. Introduction

Hydrocarbon reservoirs are an integral component of the global energy system, and their exploration and production necessitate accurate characterization. The petroleum industry's ability to identify unique geological and petrophysical properties in subsurface formations, such as channels, faults, fractures, and stratigraphic boundaries, is critical [1].

Reservoir characterization is a critical process in optimizing exploration and production in the oil and gas industry. It involves the integration of data from various sources such as seismic surveys, well logs, and geological investigations [2]. The process is aimed at identifying flow units that exhibit similar geological characteristics and consistent fluid properties, which are essential for efficient production strategies. An in-depth analysis of well log data provides valuable information on reservoir properties, aiding in the identification of hydrocarbon-bearing zones and estimating recoverable volumes [3].

To enhance the accuracy of reservoir characterization, a multidisciplinary approach that combines insights from geological, petrophysical, and geophysical fields is essential. It is worth noting that reservoir characterization is a dynamic process that requires continuous updates as new data becomes available [4].

This paper seeks to address an existing gap in literature by conducting a comparative analysis of Continuous Wavelet Transform (CWT) and Fast Fourier Transform (FFT) to enhance image clarity in seismic attribute analysis. The study aims to advance reservoir feature identification methodologies by revealing the distinct strengths and limitations of FFT and CWT. The research findings will contribute to the ongoing refinement of reservoir characterization techniques, providing valuable insights for optimizing seismic attribute analysis and improving the identification of reservoir features in subsurface exploration [5] [6] [7].

2. Fourier and Wavelet Transform

Seismic data has been established as a valuable source of information for analyzing the composition of rocks and fluids within pores. Hydrocarbon-saturated formations have unique mineralogical compositions and fluid properties that create specific frequencies in seismic data. Spectral decomposition techniques like discrete Fourier transform, S-transform, and time-frequency continuous wavelet transform are used to identify these frequencies [8].

Spectral decomposition dissects seismic signals into constituent frequencies, providing insights into phase and amplitude tuned to specific wavelengths. This technique has various applications, such as determining layer thickness, visualizing stratigraphy, and detecting hydrocarbons. However, spectral decomposition is non-unique, yielding multiple time-frequency analyses from a single seismic trace [9].

There are several methods for conducting spectral decomposition, including discrete Fourier transform (DFT), maximum entropy method (MEM), continuous wavelet transform (CWT), and matching pursuit decomposition (MPD), each with specific advantages. The choice of method depends on the analysis goals, with FFT offering better frequency localization but sacrificing vertical resolution, while CWT enhances vertical resolution [10] [11].

The distinct frequencies in seismic data of hydrocarbon-saturated formations contribute to the success of spectral decomposition. Various methods like Short Time window Fourier Transform (STFT), Wavelet Transform, S-Transform (ST), Matching Pursuit Decomposition (MPD), and Empirical Mode Decomposition are employed for this purpose. Therefore, it is important to consider the strengths and limitations of these methods for effective subsurface geological feature delineation [12] [13] [14].

The FFT is a widely utilized mathematical algorithm in seismic analysis, which efficiently converts signals from the time domain to the frequency domain. Its significance lies in the ability to identify the dominant frequency components associated with subsurface geological features. This makes it particularly valuable for processing large amounts of data and characterizing complex structures [14] [15] [16].

The CWT is a highly effective tool in providing a joint time-frequency representation of seismic signals. Unlike the FFT, the CWT has the ability to capture subtle variations in seismic signals that are associated with complex geological structures and reservoir features. Being adaptable to varying signal frequencies and having the ability to localize features, the CWT holds immense value in detecting seismic anomalies in non-stationary scenarios [17] [18] [19].

The CWT as a mathematical technique analyses signals in both time and frequency domains. Employing scaled and translated wavelets, the CWT captures non-stationary features in signals, making it well-suited for seismic data analysis. Key considerations include selecting a suitable wavelet function, with commonly used options like the Morlet, Mexican hat, and Haar wavelets [20] [21]. The CWT involves convolving the wavelet with the signal at different scales and translations, creating a time-frequency representation known as the scalogram [22] [23].

Mathematically expressed as:

$$\mathrm{CWT}(s,\tau)=\int_{-\infty}^{\infty} x(t)\psi^{*}(st-\tau)\,dt \tag{1}$$

where $x(t)$ is the signal being analysed, ψ is the complex conjugate of the wavelet, s is the scale parameter that controls the width of the wavelet and the level of detail in the analysis. Small values of s lead to narrower and higher frequency wavelet while large values are used to capture lower frequency information. τ is the translation parameter which represent the position of the wavelet along the time axis. Varying τ allows the analysis of different sections of the signal.

This complex-valued function visualizes the correlation strength and phase between the wavelet and signal at various time-frequency locations, aiding in the identification of time-localized frequency variations. Despite its computational intensity, the CWT provides a detailed view of signal frequency changes over time, with peaks in the scalogram indicating significant features.

The FFT is mathematically expressed as:

$$F(\tau,\omega)=\int_{-\infty}^{\infty} f(t)\,w(t-\tau)\,\mathrm{e}^{-i\omega t}\,dt \tag{2}$$

where $F(\tau,\omega)$ is the Short-Time Fourier Transform (STFT) of the signal at time τ and frequency ω, $f(t)$ is the input signal, $w(t-\tau)$ is the window function applied to the signal to isolate a segment for analysis. The window function helps in focusing on a specific time interval for analysis. $\mathrm{e}^{-i\omega t}$ is the complex exponential function with $\omega = 2\pi f$ being the angular frequency component of interest [24].

In signal and image processing, FFT plays a crucial role in analyzing frequency components. Employing a decimation in time (DIT) form, the algorithm divides the sequence into even and odd indices, recursively computing FFT for sub-sequences and combining them. The core "butterfly" operation involves complex multiplications and additions, and twiddle factors account for rotation and scaling. FFT's applications span signal, image, audio, and communication processing, offering an efficient means of frequency content analysis, particularly for large datasets [25] [26].

The FFT and CWT are both important techniques in signal processing and analysis. The FFT provides a representation of a signal in the frequency domain, while the CWT offers a simultaneous representation in both the time and frequency domains [27] [28]. The FFT has a fixed frequency resolution, while the CWT offers variable resolution in both domains. Windowing is crucial in signal processing, and while the FFT requires explicit windowing, the CWT adapts to local signal characteristics [29]. The FFT is well-suited for analyzing stationary signals, while the CWT is more suitable for signals with time-varying characteristics [30]. The choice between FFT and CWT depends on the specific characteristics of the signal being analyzed and the analysis requirements [8] [31] [32] [33] [34] [35].

The FFT stands as a widely applied algorithm in signal processing, offering distinct strengths and limitations [36]. Notably, FFT excels in computational efficiency, significantly reducing the time complexity of Fourier Transform calculations, making it particularly advantageous for real-time applications and large datasets [15]. Furthermore, FFT provides a clear frequency representation of a signal, facilitating tasks such as frequency analysis, spectral analysis, and filtering. Its ability to efficiently decompose a signal into its frequency components aids in the identification of dominant frequencies and harmonic relationships.

However, FFT does come with certain limitations. One notable weakness is its assumption of signal stationarity, meaning that it assumes a constant frequency over the entire signal duration. This assumption may limit its effectiveness when dealing with non-stationary signals or those with rapidly changing frequencies [15] [37].

Moreover, the discrete nature of FFT may lead to spectral leakage issues, especially when analyzing signals with non-integer multiples of the sampling frequency. This phenomenon can lead to inaccuracies in frequency representation and complicate the interpretation of results [38] [39].

Despite these weaknesses, FFT remains a cornerstone in signal processing due to its efficiency and effectiveness. Its widespread use in applications such as audio processing, telecommunications, and image analysis underscores its importance in various fields.

The CWT offers a robust analytical approach, showcasing several strengths and weaknesses in its application to signal analysis [40]. On the positive side, CWT excels in providing simultaneous time-frequency localization, making it well-suited for the examination of non-stationary signals where frequencies change over time. Its versatility becomes apparent when dealing with signals exhibiting variable frequencies, as CWT's adaptability allows for the selection of wavelets tailored to specific signal characteristics.

A key advantage lies in CWT's multi-resolution analysis capability, enabling the examination of a signal at different scales [41]. This proves valuable for detecting features of various sizes within the signal, making it a versatile tool in fields such as signal processing, image analysis, and pattern recognition, particularly for identifying transient events [42].

However, the application of CWT comes with certain drawbacks. The computational complexity of the transform, especially for large datasets or high-resolution wavelets, may limit its practicality in real-time processing or resource-constrained systems [43] [44]. Additionally, the subjective nature of scale selection and the challenge of interpreting results demand expertise in signal processing and wavelet theory [44].

Furthermore, CWT is sensitive to boundary effects, which can impact the accuracy of the analysis, particularly at the edges of the signal [31]. Additionally, its inherent continuous nature may pose challenges when applied directly to discrete signals, necessitating discretization methods that may result in information loss [45] [46].

The CWT stands as a powerful tool for signal analysis, providing a comprehensive view of both time and frequency domains. While its strengths make it invaluable in various applications, users must be mindful of its computational demands, the intricate procedure of scale selection, and the possible difficulties in understanding the outcomes.

3. Application of FFT and CWT in Feature Detection and Enhancement

Enhancing image clarity in seismic data is essential for the accurate interpretation of subsurface structures and improved seismic analysis, given that seismic data reflects sound wave reflections in the Earth's subsurface, offering valuable insights into subsurface geology and potential hydrocarbon reservoirs. Geoscientists and geophysicists can benefit from applying image clarity enhancement techniques to extract more precise information from seismic data [47] [48].

The FFT technique is useful for analyzing the frequency content of seismic signals and identifying distinct subsurface features. FFT-based filtering techniques can be used to get rid of noise and unwanted frequencies, which will make seismic reflections clearer and improv the signal-to-noise ratio. Additionally, FFT is instrumental in migration algorithms, contributing to improved imaging of subsurface structures by mitigating artifacts and enhancing the resolution of seismic images [23] [49].

On the other hand, the CWT offers a multiscale analysis of seismic data, enabling the identification of features at different scales in the subsurface. CWT's unique ability to provide both frequency and temporal localization proves beneficial in precisely localizing seismic events, such as fault lines or stratigraphic features. Furthermore, CWT can be applied for anomaly detection in seismic data, highlighting irregularities or subtle variations indicative of geological structures [50] [51] [52].

A combined approach utilizing both FFT and CWT techniques presents a comprehensive strategy for feature extraction in seismic data. FFT can be employed for initial frequency domain analysis, while CWT can refine the analysis by capturing localized variations, resulting in enhanced seismic imaging and clearer identification of subsurface structures [47] [53].

To assess the quality of enhanced seismic images, metrics such as entropy and FSIM (Feature Similarity Index) are valuable tools. Lower entropy values signify clearer images with more defined structures, while FSIM evaluates structural similarity between original and enhanced images, providing an assessment of image quality. Application-specific metrics, such as fault detection rates or accuracy in identifying stratigraphic features, offer targeted evaluations [53] [54] [55] [56].

The incorporation of FFT and CWT techniques into seismic data analysis holds the potential to improve resolution, reduce noise, and enhance overall clarity in subsurface images. This, in turn, facilitates more accurate geological interpretations and informed decision-making in the oil and gas exploration industry. Combining these metrics is advised when presenting results in order to provide a thorough assessment that takes into account the particular needs of the application in question.

When synthetic data is subjected to FFT and CWT, the resulting output is as follows (**Figures 1-3**). The data's sampling rate is 1000 Hz with a duration of 2 seconds.

4. Application to Real Data

The subject of this study is the Tano Basin, which is identified as a pull-apart basin modified by wrenching during the Cretaceous period. Positioned as the eastern extension of the Cote D'Ivoire-Ghana Basin, it originated due to trans-tensional movement during the separation of Africa and South America, leading to the opening of the Atlantic in the Albian epoch. The dynamic geological processes

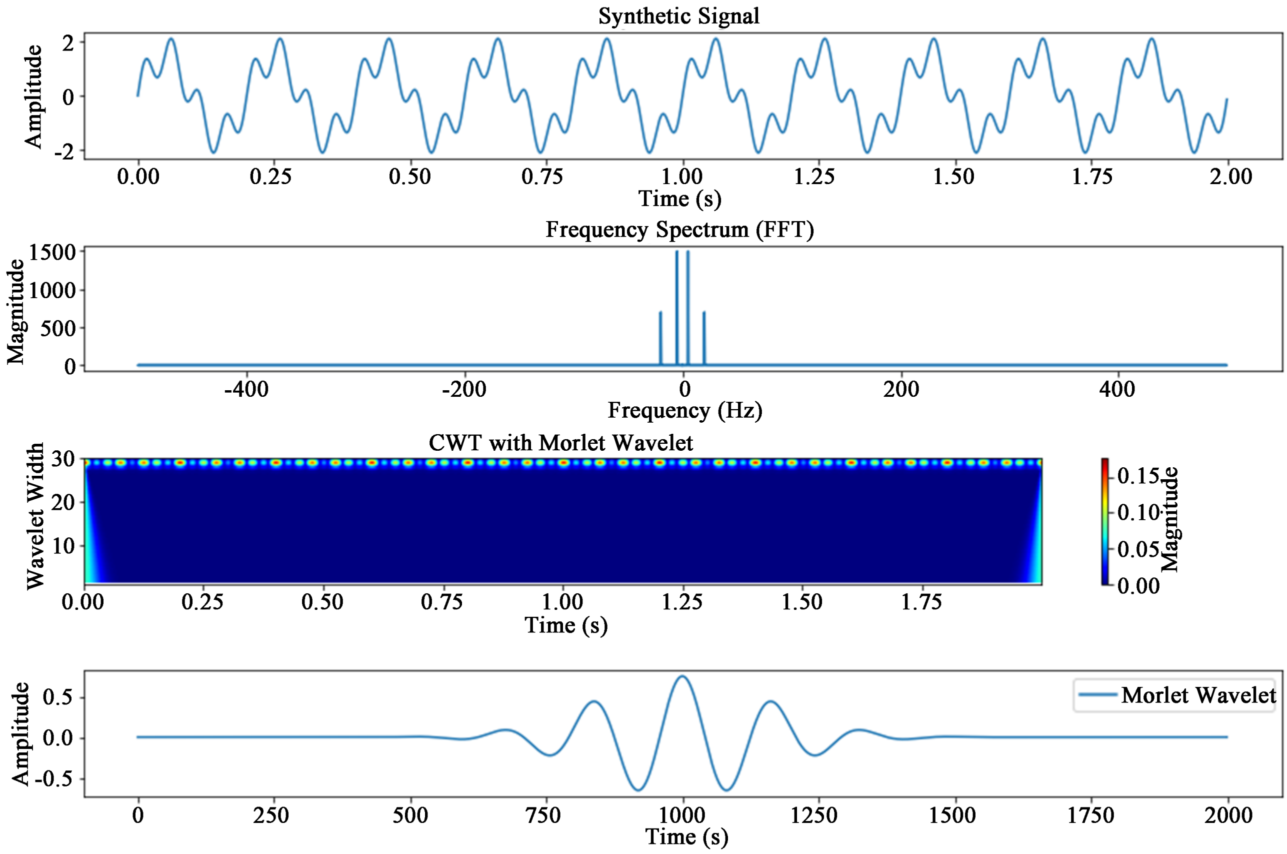


Figure 1. Signal processing with Morlet wavelet for FFT and CWT.

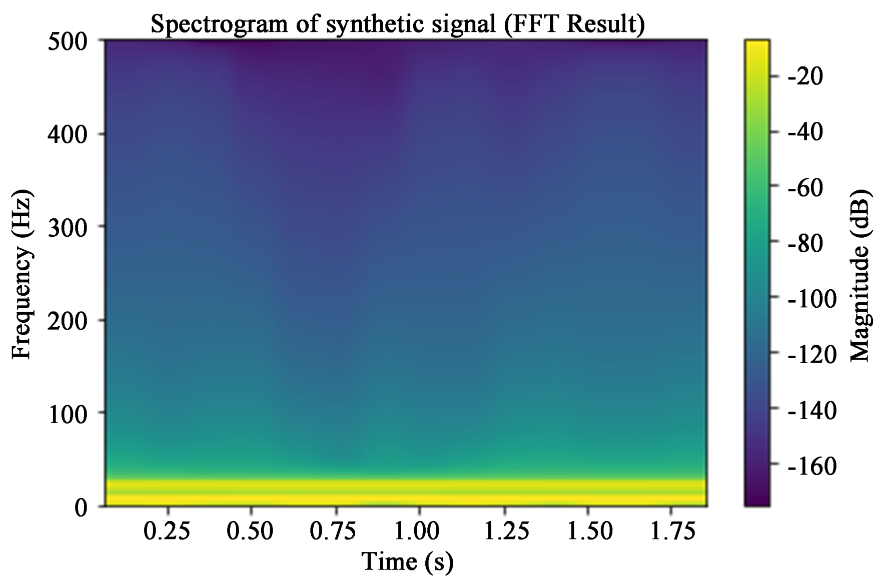


Figure 2. Signal processing with FFT (Spectrogram).

during this period, marked by active rifting and subsidence, gave rise to the development of a deep basin.

The Tano Basin is a multifaceted geological arrangement that encompasses a rift section that features shallow marine to continental deposits, as well as a

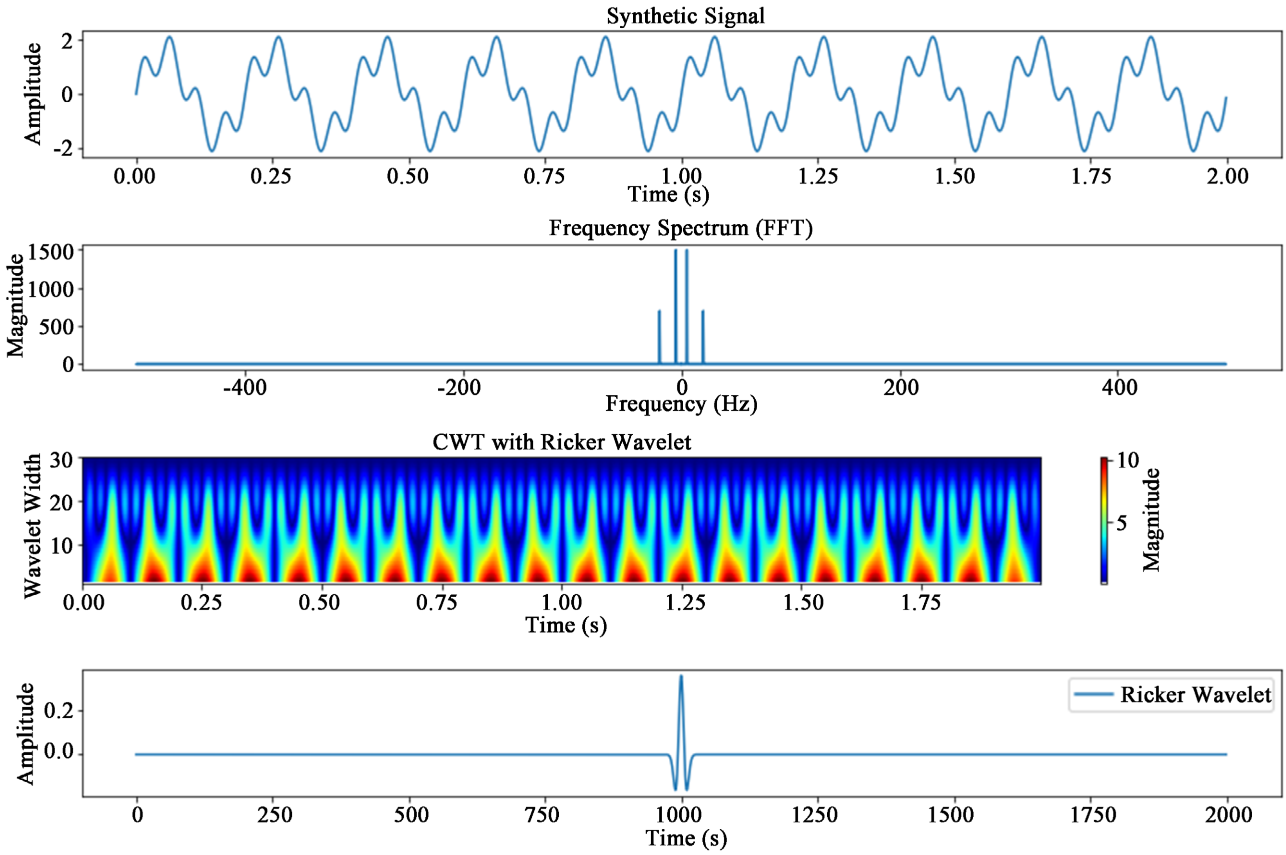


Figure 3. Signal processing with Ricker wavelet for FFT and CWT.

significant Upper Cretaceous drift section highlighted by basin floor fans, channel systems, and stratigraphic traps. The primary play type being investigated is the Cretaceous Play, which involves Cenomanian-Turonian and Albian shales as source rocks, with Turonian slope fan turbidite sandstones and Albian sandstones in tilted fault blocks serving as reservoirs. The trapping mechanisms are both stratigraphic and structural in nature.

The Ghanaian segment of the basin has been recognized for its hydrocarbon potential since the 1890s, primarily based on onshore oil seeps. Figure 4 provides an overview of the study area, highlighting its geological features and significance [57] [58] [59] [60].

The study methodology involves obtaining high-quality 3D seismic data and conducting a preprocessing step to enhance its quality for in-depth subsurface analysis. This includes generating images of subsurface structures to facilitate meaningful attribute analysis. The analytical phase employs CWT with the Morlet wavelet and FFT to extract valuable information and refine the dataset. The seismic data has a sampling rate of 4 ms. The final stage assesses the reliability and robustness of attribute analysis results.

The method enhances reservoir study with subtle insights into subsurface structures. Results of CWT and FFT applied to seismic data from Tano Basin are shown below (Figures 5-10).

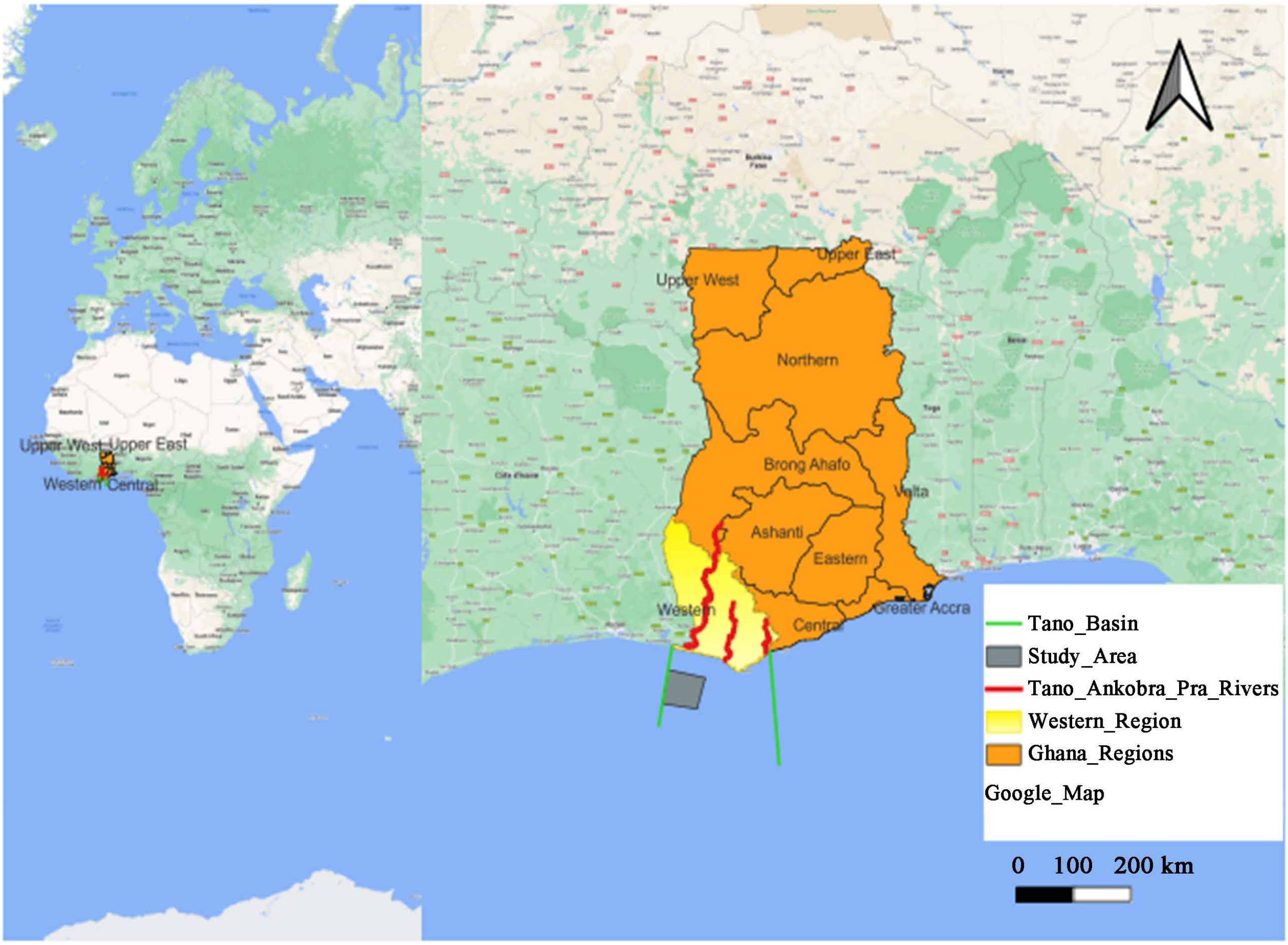
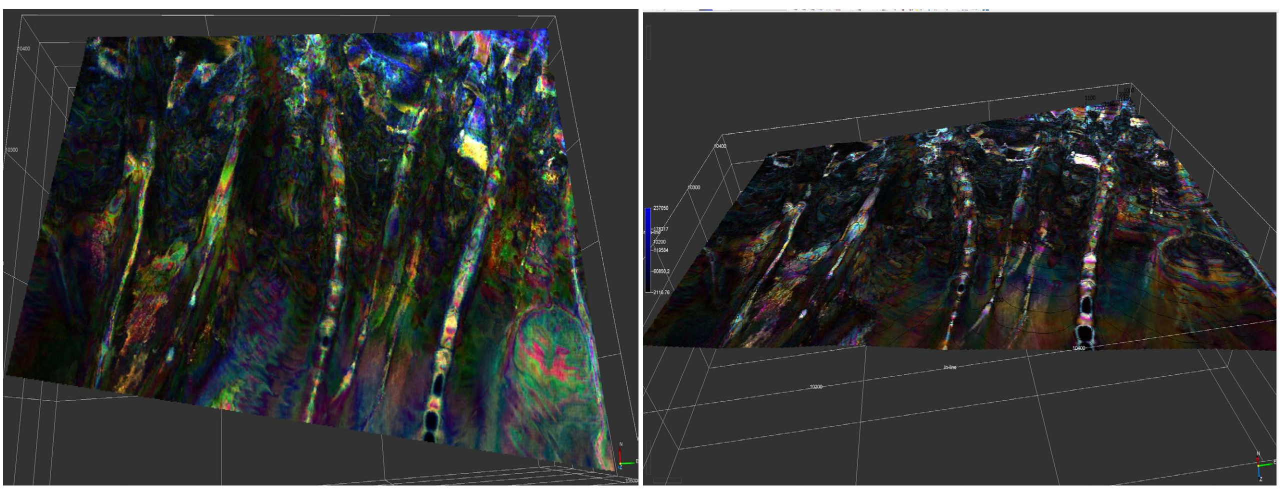
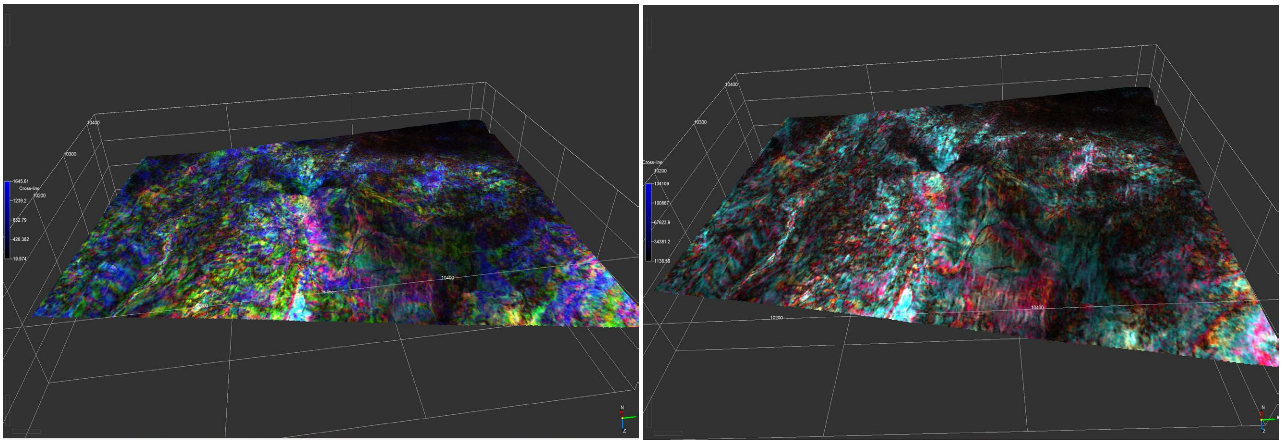


Figure 4. Tano basin within West Africa.



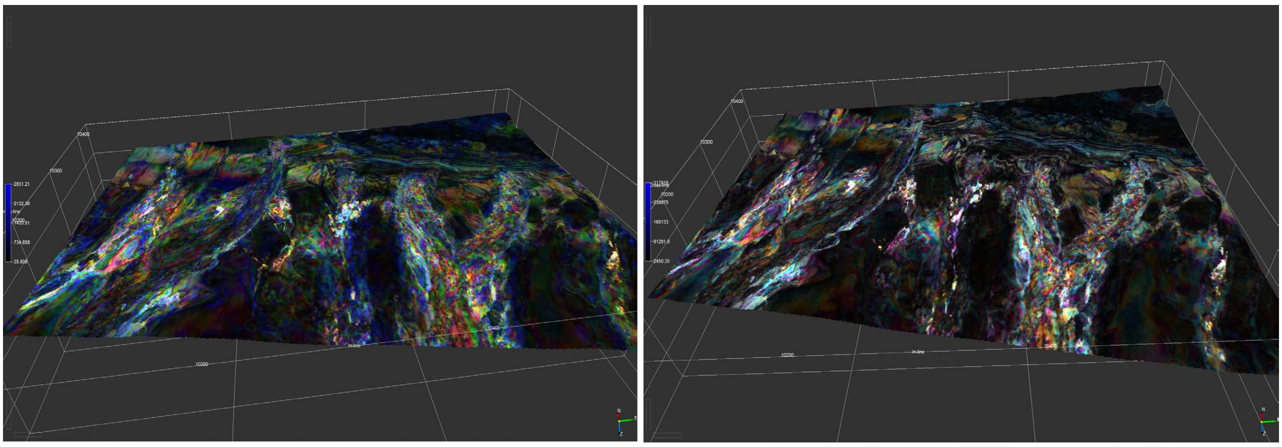
(a) (b)

Figure 5. (a) Displays a seismic image featuring a braided channel characterized by sediment deposition between the channels. Positioned in the lower right corner is a potential diapir composed of either salt or shale. The clarity of this image is notably enhanced in the CWT representation; (b) Presents the same image processed using FFT. However, the features appear less distinct, with a reduced colour contrast affecting the clarity of the potential salt or shale diapir, particularly in the lower right corner.



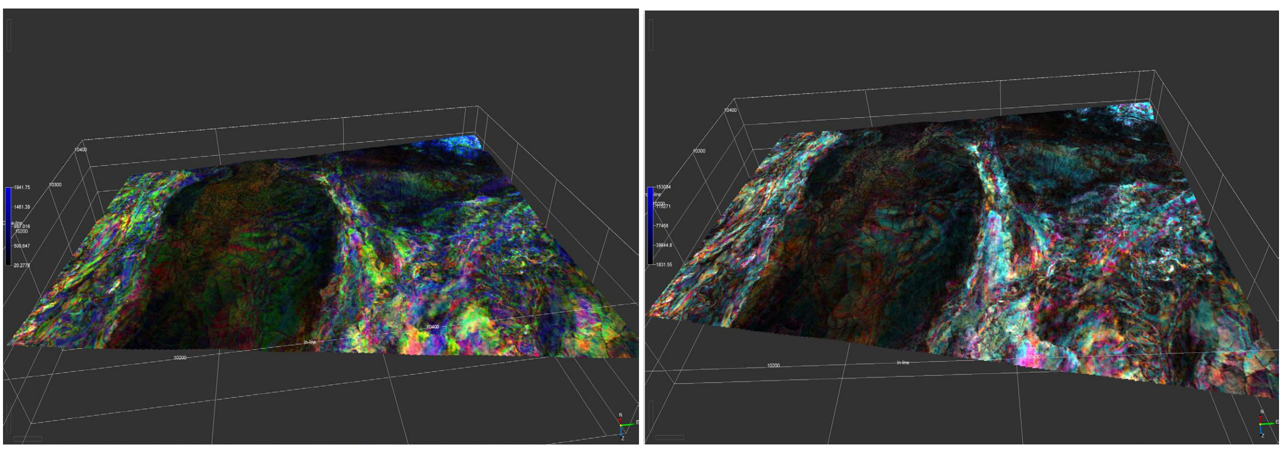
(a) (b)

Figure 6. (a) Illustrates a channel lobe deposit derived from CWT, showcasing improved colour contrast; (b) Depicts a channel lobe deposit obtained through FFT, featuring diminished colour contrast.



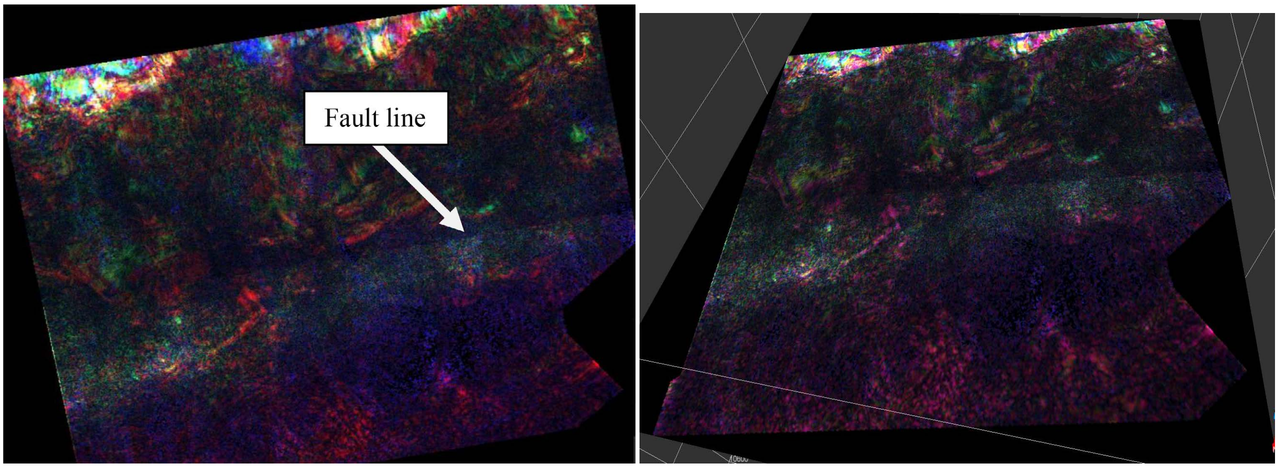
(a) (b)

Figure 7. (a) Portrays a braided channel using CWT, highlighting enhanced colour contrast; (b) Showcases a braided channel derived from FFT, displaying reduced colour contrast.



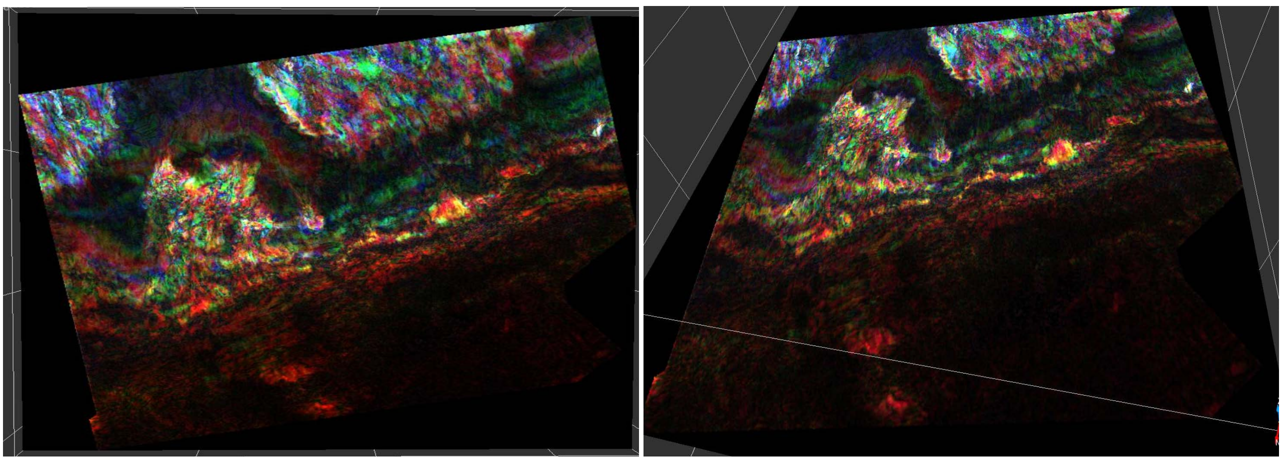
(a) (b)

Figure 8. (a) illustrates a channel featuring channel levee and overbank deposits obtained through CWT; (b) displays a channel with channel levee and overbank deposits obtained through FFT.



(a) (b)

Figure 9. (a) depicts basement rock with a fault, as revealed by CWT; (b) portrays the identical image of basement rock with a fault, now processed through FFT.



(a) (b)

Figure 10. (a) showcases a channel featuring channel levee and channel lobe deposits through CWT, demonstrating improved image clarity; (b) displays a channel with channel levee and channel lobe deposits processed through FFT.

5. Discussion

The comparative analysis between the CWT and the FFT in the context of image quality and subsurface structure identification has yielded insightful findings.

The results highlight a notable advantage of CWT in terms of image quality. The CWT's ability to provide simultaneous time-frequency localization allows for a more detailed and accurate representation of image features. Unlike the FFT, which assumes stationarity, the CWT's adaptability to non-stationary signals proves advantageous in capturing subtle variations and intricate patterns within the images. This enhanced image quality could have significant implications in fields where precise feature identification is paramount such as medical imaging or geological exploration.

The superior performance of CWT in the identification of subsurface structures can be observed from the above images. The multi-resolution analysis provided by CWT allows for a comprehensive examination of the signal at different scales, facilitating the detection of subsurface features of varying sizes. In contrast, the FFT, while proficient in frequency domain representation, may struggle with non-stationary signals and the intricate structures present in subsurface imaging scenarios.

It is important to note that these findings do not diminish the significance of FFT, which remains a valuable tool in various signal processing applications. FFT's computational efficiency and clear frequency representation make it well-suited for certain contexts, particularly in cases where stationarity assumptions hold, and a global frequency analysis suffices.

The application of CWT and FFT for image clarity enhancement in reservoir management has several implications for future reservoir studies. These implications can shape the direction of research and industry practices in the exploration and extraction of hydrocarbons

Implication for Reservoir Studies and Management

The FFT has become integral in reservoir studies, playing a key role in analyzing seismic data frequency content. Its versatility significantly enhances understanding across various applications, such as stratigraphic interpretation, fault detection, fluid identification, natural fracture analysis, and time-lapse reservoir monitoring [27] [29] [31] [49] [61] [62] [63]. FFT is used in stratigraphic interpretation to identify frequency content, which helps with stratigraphic layer identification and interpretation [31] [49]. FFT is also valuable in fault detection, mapping, and fluid identification by analyzing seismic signal frequency responses [64] [65]. For natural fractures, FFT analyzes anisotropic frequency responses, influencing reservoir modelling [66]. Beyond static analyses, FFT contributes to time-lapse reservoir monitoring by comparing frequency content changes over time [3] [67]. Its continued application reflects ongoing advancements in seismic analysis, enhancing subsurface reservoir characterization [30] [33] [35].

CWT is a versatile tool in reservoir studies with applications in thin bed detection, fracture characterization, reservoir heterogeneity quantification, resource identification, and time-lapse reservoir monitoring [8] [32] [50] [51] [68] [71]. CWT enhances seismic data resolution in thin bed detection, aiding in the identification of subtle changes associated with thin stratigraphic layers [68]. It is essential to fracture analysis and improves our understanding of reservoir structure [50]. CWT quantifies reservoir heterogeneity by analyzing variations in seismic attributes across different scales [69]. In resource identification, CWT is applied to identify gas hydrate-bearing sediments, effectively analyzing their distribution and concentration within reservoirs [70] [71]. For time-lapse reservoir monitoring, CWT aids in identifying changes in reservoir properties over time, providing crucial information for informed reservoir management [50]. Its simultaneous consideration of time and frequency information enhances our understanding of subsurface structures and refines reservoir characterization [8]

[28] [32] [34].

6. Conclusions

This paper discusses the role of seismic data analysis, specifically focusing on the application of CWT and FFT in reservoir studies within the oil and gas industry. The analysis emphasizes the significance of these techniques in enhancing image clarity for accurate subsurface structure interpretation.

The text details the principles and advantages of both CWT and FFT in spectral decomposition, highlighting their strengths and limitations. FFT is recognized for computational efficiency and frequency domain analysis, while CWT excels in time-frequency localization and adaptability to non-stationary signals. The study proposes a hybrid strategy for feature extraction from seismic data that makes use of both methodologies.

Furthermore, the comparative analysis between CWT and FFT reveals that CWT offers superior image quality, especially in identifying subsurface structures with its multi-resolution analysis. Despite acknowledging the continued value of FFT in specific contexts, the study underlines the potential benefits of incorporating CWT and FFT in reservoir management for tasks like seismic interpretation, fault detection, and well placement optimization.

For reservoir research in the future, developments in computational methods, imaging technologies, and interdisciplinary approaches are essential. The study recommends evolving industry practices to include real-time monitoring, adaptive management, and the integration of machine learning applications. Establishing best practices is believed to require cooperation and standardisation, and problems like computational complexity are viewed as opportunities for further research and innovation.

According to the study's conclusion:

1) A new era of accuracy and productivity in hydrocarbon extraction is ushered in by the improvements in reservoir management brought about by CWT and FFT.

2) Using these technologies is thought to be vital for navigating subsurface conditions and for gaining new insights that will have a big impact on how energy exploration and production are carried out in the future.

3) The future of the energy sector is anticipated to be greatly influenced by the developments in CWT and FFT, which will improve the precision of subsurface structure interpretation and facilitate accurate reservoir management decision-making.

4) The study highlights how important it is to make a context-specific decision between CWT and FFT, emphasising that the decision should be made in accordance with the particular goals and characteristics of the signals that are being investigated in reservoir studies.

5) Establishing best practices is considered to need collaboration and standardisation, while problems like computational complexity are considered as

potential for future study and innovation.

6) The study recommends that industry practices evolve to incorporate enhanced imaging techniques, real-time monitoring, adaptive management, and the integration of machine learning applications.

7) Reservoir studies will increasingly rely on advances in imaging technology, computational techniques, and interdisciplinary approaches. This highlights the necessity of constant innovation in the field.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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