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Slow Strain Waves: Models and Observations, a Review

Victor G. Bykov

Kosygin Institute of Tectonics and Geophysics, Far East Branch, Russian Academy of Sciences, Khabarovsk, Russia
Email: bykov@itig.as.khb.ru

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Abstract

The theoretical discovery of slow strain (tectonic) waves, the so-called strain waves in the Earth, served as a motivation to develop physical backgrounds of the mathematical theory of propagation of these waves and to search for methods of their experimental detection. For fifty years, scientists from different countries in different regions of the Earth, using direct and indirect methods, discovered the migration of crustal deformation and revealed its wave nature, and, therefore, proved the reality of the existence of strain waves of the Earth. This overview briefly describes the history of the development of the concept of strain waves on the Earth, the observation methods and properties of strain waves, and the main types of geological structures generating these waves. The most prominent results of the theoretical, laboratory, and *in-situ* observations of slow strain migration, including slow earthquakes and periodic Episodic Tremor and Slow (ETS) slip effects, are presented. In the near future, studies of slow strain waves may lead to a fundamental revision of the current concepts about the physics of the seismic process.

Keywords

Earthquake Migration, Strain Waves, Solitary Waves, Stick-Slip

1. Introduction

Slow strain (tectonic) waves, the so-called strain waves in the Earth, are one of the most striking discoveries in theoretical geophysics in the last third of the 20th century. The advances in theoretical studies and, primarily, extensive investigations of earthquake migration have aroused a great interest in searching for the possibilities of experimentally detecting the effects of the propagation of these waves. The progress in the formulation of the concept of strain waves in the Earth started just from making attempts to explain the cause of targeted migration of earthquakes in the North Anatolian Fault, Türkiye, detected by Richter in 1958 [1], thus solving one of the seismological problems posed.

Migration of earthquakes is related to the propagation of tectonic stresses that cause an additional load and initiate successive earthquake occurrences in fault segments with high-stress concentration. Migration of the earthquake epicenters at the surface is an external manifestation of strain transfer in the Earth's interiors. Elucidating physical mechanisms responsible for the propagation and redistribution of strain energy, and tectonic stress transfer and relaxation at the boundaries between the blocks and lithospheric plates is the most important problem in recent geodynamics.

All stages in the development of the concept of strain waves on the Earth and their search for, as well, are difficult to describe in the framework of one paper, therefore, we will briefly consider the most important different approaches applied for modeling stress transfer in the Earth's interiors and the associated effects, and the main results of *in-situ* observations and laboratory experiments that may be involved as evidence of really existing waves.

Slow strain waves are mostly excited by natural processes that occur within the crust and the lithosphere and are manifested in variations in seismicity and geophysical fields. The blocky structure of the crust and the lithosphere influences significantly the strain, seismic, filtration, and other processes. It is the blocky structure of the geological medium that leads to the generation of waves of different types including slow strain waves [2], which, in turn, affect different-size geoblocks thus causing their relative motions also capable of generating strain waves, but possessing different wave characteristics (direction, velocity, frequency, and length). Thus, an entire strain wave spectrum appears with a broadly ranging wave velocity (from 1 to 100 km/yr) and length (from 30 to 200 km), to which the faults may respond. Revealing the relationships between the movements of tectonic structures and slow strain wave processes is of prime importance.

The problem posed has been argued for more than 50 years, starting from the publication by Elsasser [3]. Direct instrumental measurements of slow strain waves are difficult to perform due to their superlow velocities and ultralow frequencies. The challenge in direct detection of ultra-long-period strain waves can be explained by the lack of special-type strain sensors or their location schemes efficiently providing a reliable recording of these waves.

In many regions of the world, deformographic, geodetic and hydrologic measurements have thus far detected strain migration at a velocity of about 10 - 100 km/yr and 1 - 10 km/day [4]-[10]. Migration of the earthquake epicenters is consistent in the velocity (10 - 100 km/yr) and direction with the propagation of crustal strain [4] [11] and hydrologic effects [12].

The accumulated facts indicate strain wave processes occurred within the crust at different velocities [13]. The observational results of the targeted earthquake migration, direct and indirect *in-situ* strain wave measurements, or their indications were mostly reported in [4] [11] [14]-[19]. These data are a powerful background for the physical understanding of most geodynamic and seismological problems.

The principal goals of the review are as follows: 1) to give a brief history of the development of the concept of strain waves in the Earth; 2) to analyze the most prominent results of the theoretical, laboratory, and *in-situ* observations of slow strain migration; 3) to present the observation methods and properties of strain waves and main types of the geological structures generating these waves.

2. The History of the Development of the Concept of Strain Waves in the Earth and Theoretical Models

In the late 1960s, a great interest of geophysicists all over the world was aroused with regard to the measurement and interpretation of strain in the Earth, which was related to the study of Earth's tides, earthquakes, and other tectonic processes. Special International Conference "A Discussion on the Measurement and Interpretation of Changes of Strain in the Earth", sponsored by the Royal Astronomical Society, which was held in London in 1972, is clear evidence. A range of presentations reported the first data directly related to strain waves on the Earth.

Kasahara [20] demonstrated tiltmeter records, from which the existence of strain in the Earth migrating east-west along the Pacific Honshu coast at an abnormally low velocity (of about 20 km/yr) was explicitly inferred. The revealed spatial correlation between strain migration and the regime of seismic and volcanic activity in adjacent areas was a key to understanding the mechanism triggering stress transfer in the crust. The presentation [21] reported on continuous observations carried out by a dense network of strainmeters arranged along the San-Andreas Fault, central California, and the detection of aseismic creep, moving at a velocity of about 10 km/day or less, which was not constant. The question concerned with the optimal arrangement of different-type strain sensors for creep recording in proximity to faults was also discussed. Frank [22] claimed that the network of strainmeters installed at 45˚ with respect to a fault at a distance of at least 10 km away from it is capable of recording a 1-mm displacement and provides the best record of strain migration.

The formulation of the concept of strain (tectonic) waves in the Earth was mostly developed on the basis of two discoveries made by that time, namely, migration of the foci of large earthquakes along deep faults [1] [14] and global plate tectonics [23]. The concepts on the lithospheric plates bordered by powerful faults and underlain by viscous asthenosphere resulted in the construction of three types of theoretical models of strain waves: 1) layered models (lithosphere-asthenosphere) [3] [24] [25] [26]; 2) layered models with a complemented flexure effect of the rigid lithospheric plate [27] [28]; 3) fault models with a gouge between the fault walls (viscoelastic) [5] [29] [30]. These models were assigned for a description of slow stress waves that correspond to large earthquake migration along transform faults and trenches (depressions).

Accumulation of data of *in-situ* observations and laboratory experiments resulted in the detection of new facts whose explanation was impossible in the framework of the linear elasticity (viscoelasticity or elastoplasticity) theory. It was a circumstance that motivated to search for analogies and to develop new nonlinear mathematical models simulating the deformation of fault-blocky geological media.

The second stage in the development of the theory of strain waves in the Earth is believed to begin with the publication by Nikolaevsky [31], in essence, postulating the sine-Gordon equation for modeling slow solitary waves in the blocky geological medium, whose indications were observed as migration of the geophysical anomalies in proximity to faults [32].

The detected behavior of the spatiotemporal migration of recent deformations in the fault zones [7] [33] and the dynamics of the seismic activity [34] is indicative of a qualitative similarity to the general concepts on excitable active media [35] [36]. This fact may serve as physical motivation for applying "autowave" analogies to the mathematical modeling of the targeted strain and earthquake migration.

2.1. Elastoviscous Models of Slow Wave Processes

In 1969, the American physicist Walter Elsasser, a disciple of Max Born, was the first to introduce the concept on the mechanism triggering stress transfer at the lithosphere-asthenosphere contact. He proposed the model of the interaction between the lithosphere, a solid plate with the thickness h_1 and the elasticity modulus E_1 and the asthenosphere, an underlying fluid layer with the thickness h_2 and the viscosity μ (**Figure 1**). During persistent translational movement of the lithosphere, the depth distribution profile of the velocity has the shape shown by a solid line in **Figure 1**. The velocity v_x of slippage of the lithospheric plate with the length L under the horizontal stress σ_x is calculated by the formula [3] $v_x = \frac{\sigma_x}{\mu} \frac{h_1 h_2}{L}$. On the assumption that during the plate movement the shear stress $\mu v_x / h_2 = (\mu / h_2) \partial u / \partial t$ at the lower plate boundary is equilibrated by the cumulative horizontal stress $h_1 \partial \sigma_x / \partial x$, $\sigma_x = E_1 \partial u / \partial x$ at the plate margin, Elsasser wrote the following equation for the displacement u averaged over the plate thickness [3] [37]:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

$$\alpha = \frac{h_1 h_2 E_1}{\mu}. \tag{2}$$

Equation (1) has the form of the diffusion or heat conduction equation. The key moment of the model is a viscous coupling between the lithosphere and asthenosphere, characterized by the parameter μ / h_2 dependent, in the general case, upon the perturbation wavelength. It follows from a standard solution of the diffusion equation that the average distance for which the perturbation propagates during the time t, is given by the value $x = 2\sqrt{\alpha t}$. The calculated velocity

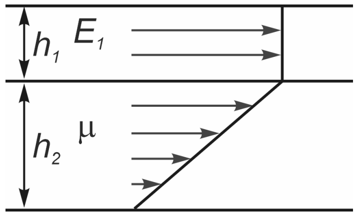

Figure 1. The model of contact interaction of the lithosphere-asthenosphere system [3].

of the plate slippage v_x is of the order of 1 cm/yr, which agrees well with the data of recent GPS measurements performed in different regions worldwide [38] [39].

Being physically "transparent", the Elsasser model was known for simplicity and representativeness and was used to describe the strain and earthquake migration [24] [25], and applied for further explanation of the associated effects [40] [41] [42] [43] [44].

Bott and Dean [24] were the first to apply the Elsasser model (1) - (2) in order to study stress migration at the lithospheric plate boundaries and introduced the term "stress or strain waves". Standard methods were employed to solve the problem aimed to determine the response of an elastic plate of the finite length L to the applied pressure P_0 at one plate margin (**Figure 2**). At $x = 0$ the plate margin was assumed to be fixed and appeared to be the compression boundary. The pressure P_0 was instantaneously "switched on" at the plate margin $x = L$ at the time moment $t = 0$. Equation (1) was solved under the chosen initial: $u = 0$ at $t = 0$ ($0 \le x < L$) and boundary: $u = 0$ at $x = 0$ ($t > 0$) and $\left(\partial u/\partial x\right)_{x=L} = -P_0/E_1$, $t > 0$ conditions chosen.

Assuming that at the plate boundary the pressure varies as $P = P_0 \sin \omega t$ with a period $T = 2\pi/\omega$ and selecting the solution of Equation (1) in the form

$$u(x,t) = -\frac{P_0}{\sqrt{2k}E_1} \mathrm{e}^{-kx} \cos\left(\omega t - kx + \frac{\pi}{4}\right), \quad k = \sqrt{\frac{\omega}{2\alpha}}, \tag{3}$$

the authors derived the following expression for the stress wave velocity along the lithospheric plate

$$v = \frac{\omega}{k} = \sqrt{2\alpha\omega} = 2\sqrt{\frac{\pi E_1 h_1 h_2}{\mu T}}. \tag{4}$$

Modeling has shown that the stress applied at the plate boundary cannot instantaneously affect an entire plate, but is dissipated diffusively in it during the time period of 10^3 - 10^6 years. The viscosity of the asthenosphere is the main factor causing slow stress migration. According to (4), the stress wave velocity is dependent upon the physical properties of the lithosphere and asthenosphere and the wave period, and attains 10 - 100 km/yr at the characteristic parameters of the continental upper mantle.

Anderson [25] generalized the Elsasser model in order to elucidate the mechanism of earthquake migration in the subduction zone and estimated the strain wave velocity along the island arc. The strain wave is generated in the

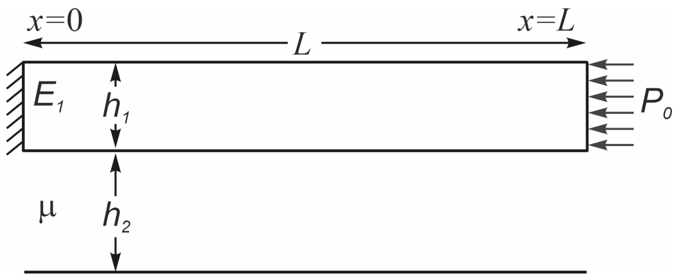

Figure 2. The model of excitement of stress waves in the lithosphere-asthenosphere system [24].

subduction zone and is traveling with velocity v on a fault along the strike of the subducting plate (**Figure 3**). At the model parameters chosen (h_1 = 50 km, h_2 = 200 km, $\mu = 5 \times 10^{18}$ Pa·s, $E_1 = 10^{11}$ Pa), the velocity of the shear strain transfer over the time period t_1 = 1 year equals v = 170 km/yr, and decreases to v = 50 km/yr after passing a distance of 520 km for t_2 = 10 years. These calculations are in agreement with the data on the earthquake migration velocities [14] [45] [46].

Rice [26] modified the Elsasser model substituting the Newtonian rheology of the viscous asthenosphere by the Maxwellian rheology of the viscoelastic body. Introducing this correction was necessary to account for the asthenosphere response to fast loading ($t < \tau = \mu/E_1$) as an elastic body, whereas under the low velocity limit ($t \geq \tau$) the asthenosphere response to loading will only be viscous, as is the case with the Elsasser model.

The viscoelastic model [26] is applicable to mathematically describe slippage at the plate (fault) contact of two types: fault and thrust. Here, the model version (5) - (6) is shown where slippage has a fault pattern, which is convenient for a comparison with the Elsasser model.

$$\frac{\partial u}{\partial t} = \left(\alpha + \beta \frac{\partial}{\partial t}\right)\left\{(1+v)^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right\}, \tag{5}$$

$$\alpha = \frac{h_1 h_2 E_1}{\mu}, \quad \beta = b \approx \left(\frac{\pi}{4}\right)^2 h_1, \tag{6}$$

where b is the effective length of the short-term elastic cohesion; v is the Poisson's ratio; β/α is the relaxation time for the Maxwell model. For the time intervals $t \gg \beta/\alpha \approx 1.5\text{-}15$ years, the viscoelastic model (5) - (6) transforms into the Elsasser viscous model (1) - (2).

In essence, Equation (5) models the strain front propagation in the lithosphere. For long time intervals, the displacement in the plate (**Figure 4**) is written as [26]:

$$u(y,t) = (\Delta u/2)\operatorname{erfc}\left(y/2\sqrt{\alpha t}\right), \tag{7}$$

from which it follows that slippage propagates inside the plate in the form of a diffusion wave.

The viscoelastic model [26] was complemented and applied for analysis of the strain front propagation through the crust and the lithosphere [47]. In particular,

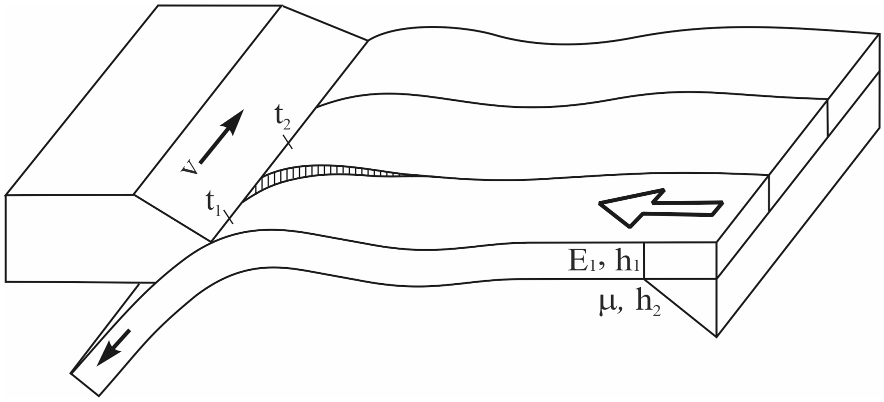

Figure 3. The model of earthquake migration in the subduction zone [25].

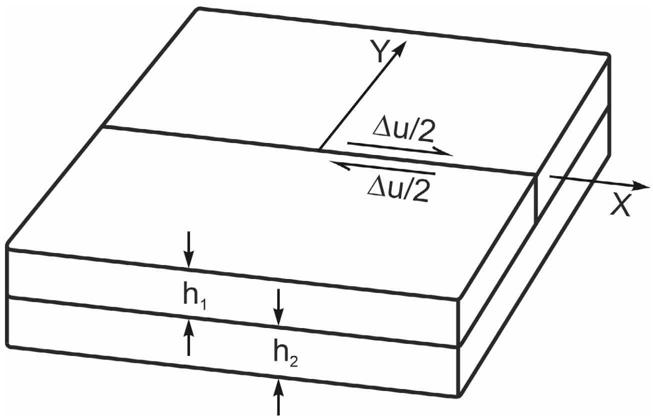

Figure 4. The model of excitement of stress waves due to fault slip and its propagation in the plate [26].

it was shown that the lithosphere-asthenosphere coupling controls the spatiotemporal distribution of a chain of successive large earthquakes along the lithospheric plate (transform fault) boundaries. It also follows from the model that stress is transferred from the boundaries inside the plate which can be one of the possible explanations of intraplate earthquake occurrence and their migration. However, the intraplate earthquake migration pattern is more complicated and is dependent upon the fault system interaction [48]. A 2D model version [47] was further used to simulate the anomalous crustal movement detected from GPS observations in central Honshu, Japan [49].

Being further modified, the Elsasser model accounted for the lateral inhomogeneity of the lithosphere [50]:

$$\frac{\partial u}{\partial t} = \frac{h_2}{\mu} \frac{\partial}{\partial x}\left(h_1 E_1 \frac{\partial u}{\partial x} \right). \tag{8}$$

The most important results of this model are summarized as follows: the lithosphere thinning results in high amplitudes of the diffusion stress, while the stress migration velocity increases due to thickening of the lithosphere. This can explain such a broad range of the earthquake migration velocity values observed in different seismic regions.

Nikolaevsky [27] complemented the Elsasser model with a new element, flexure of the lithospheric plate (see **Figure 5**), which has led to a rigorous mathematical theory of propagation of strain (tectonic) waves. The models developed by other researchers [26] [47] [51] did not account for this explicit effect. Introducing the vertical displacements related to flexure of the lithosphere changes considerably the evolution scenario of tectonic movements. The resultant system of equations for displacements u of the lithosphere along its contact with the asthenosphere (9) and for the vertical displacements η at the asthenosphere-lithosphere contact (10) is written in the following form:

$$\frac{\partial u}{\partial t}=\frac{\alpha}{1-\nu^2}\frac{\partial^2 u}{\partial x^2}-\frac{\partial}{\partial t}\left(h_2\frac{\partial \eta}{\partial x}+\Phi\right)+\left(v_0+w_0\right), \quad (9)$$

$$\frac{\partial \eta}{\partial x}+\frac{E_1 h_1^3}{12\gamma\left(1-\nu^2\right)}\frac{\partial^4 \eta}{\partial x^4}+\frac{\partial^2}{\partial x^2}\left(\frac{N}{\gamma}\frac{\partial \eta}{\partial x}\right)=\frac{2\mu}{\gamma h_2}\left(\frac{\partial \Phi}{\partial t}+w_0\right), \quad (10)$$

where Φ stands for the associated overflows of the asthenosphere matter; $\partial\Phi/\partial t$ is the rate of the flow; v_0 is the stationary velocity of the shear of the lithosphere relative to the asthenosphere; w_0 is the stationary velocity of the asthenosphere; $N=E_1 h_1\left(1-\nu^2\right)^{-1}\partial u/\partial x$ is the compression force acting on the lithosphere; $\gamma=\rho_1 g$ is the specific weight of the lithospheric plate.

A 2D model (9) - (10) describing the lithosphere-asthenosphere interaction due to vertical displacements and low tangential stresses at their contact yields the solutions that are either periodic waves of low intensity (standing and diffusion waves), or solitary waves [27] [28].

The energy of solitary tectonic waves is supplied from the stationary asthenospheric flow that emerges due to movement of the lithosphere along the asthenosphere thus compensating for viscous losses. This serves as physical backgrounds of the autowave mechanism generating of solitary tectonic waves. The velocity v and the length λ of undamping tectonic waves are governed by the rate of displacement of the lithospheric plates relative to the asthenosphere v_0 (~10 cm/yr), flexure of the lithosphere η (~10 cm) and its thickness h_1 (~100 km):

$$v=\frac{\delta h_1 v_0}{\eta},\quad \lambda=v\frac{\mu}{E_1}=\frac{\delta\mu h_1 v_0}{\eta E_1}. \quad (11)$$

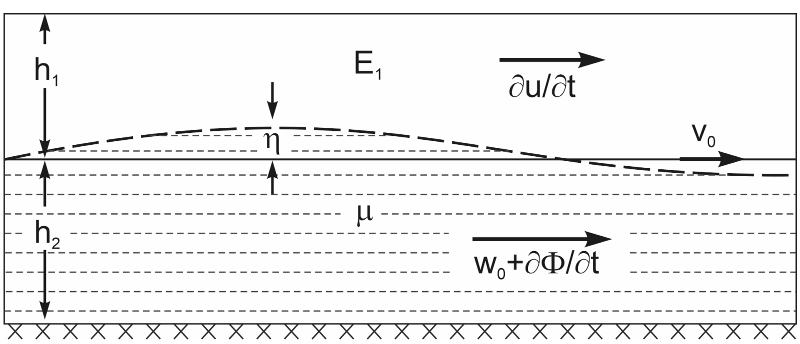

Figure 5. A scheme of excitement of tectonic waves in the lithosphere-asthenosphere system due to flexure of the lithospheric plate [28].

The value of v is approximately equal to 100 km/yr (at the numerical coefficient value $\delta \sim 1$). The solitary wave length decreases with a decrease in the viscosity μ of the asthenosphere or an increase in the elasticity modulus E_1 of the lithosphere, but its velocity remains constant. The calculations have revealed that tectonic waves have a characteristic period of 2, 3, 6, and 11 years at an effective width of about 200 km. Propagation of these solitary waves can explain the migration of seismicity for long distances.

If the vertical displacements of the lithosphere are absent ($\eta = 0$) and the overflows are lacking in the asthenosphere ($\Phi = 0$), then model (9) - (10) at $v^2 \ll 1$ is equivalent to the Elsasser Equation (1), which is the equation of simple horizontal compression-extension of the lithosphere.

Birger [52] proposed a model of stress propagation in the form of diffusion waves. Unlike the Elsasser model, its presents the lithosphere as a thin elastic plate, whereas the asthenosphere is a viscoelastic halfspace, whose transient creep obeys the Andrade rheological law. The stress wave velocity in the elastic lithosphere is equal to:

$$v = \frac{h_1}{\sqrt{3Bt^{1-\gamma}}}, \tag{12}$$

where γ, and B are the rheological parameters, t is the time. The parameter B has the time dimension in the power γ (c^γ). For natural geomaterials, $0 < \gamma < 1$, whereas for the Andrade medium $\gamma = 1/3$. Then the expression for velocity v takes the form [52]:

$$v = \frac{h_1}{\sqrt{3Bt^{2/3}}}. \tag{13}$$

At $\gamma = 1$ model (12) transforms into the Maxwell model, the parameter B has the time dimension (c) and agrees with the Maxwell relaxation time μ/G, and the velocity is defined as $v = h_1 G / \sqrt{3}\mu$. The velocity versus time dependence (13) makes the model [52] fundamentally different from the Elsasser model with the Newtonian asthenosphere: the stresses are monotonously damping at long times.

Note that applying nonlinear rheological models to describe stress transfer in the lithosphere-asthenosphere system was also discussed in [51] [53] [54] [55].

In the diffusive model by Savage [29], the crustal block motion along the transform fault is presented as a flow of edge dislocations. The stress transfer over the fault is determined using the terms of concentration k and a flow q of dislocations. The strain velocity is proportional to the dislocation flow. The diffusive mechanism plays a major role in the dynamics of the dislocation flow, whose kinematic behavior is controlled by the physical properties of the fault zone. The resulting equation takes the form [29]:

$$\frac{\partial q}{\partial t} = -c\frac{\partial q}{\partial x} + D\frac{\partial^2 q}{\partial x^2}, \tag{14}$$

where $c = \partial q / \partial k$ is the velocity, D *is* the diffusion coefficient. The fundamental result is that the mechanism producing the dislocation flow leads to the emergence of "creep waves" along the transform fault, which are stress waves. Based on the calculations performed by Savage, the velocity of such waves along the San Andreas transform fault is estimated at about 10 km/yr and is dependent on the strain amplitude thus increasing with its growth. According to the hypothesis formulated by Savage, creep waves cause an abrupt change of the movement inside the fault and are related to the migration of large earthquakes along the northeastern Pacific margin. The data on the earthquake migration have inferred the north-west to south-east stress front propagation along the San Andreas Fault at a velocity of 30 - 50 km/yr [56].

Ida [30] obtained a solution in the form of "slow-moving deformation pulses" (with a constant velocity $v = Gd/2\mu$ (μ, d are the viscosity and the thickness of the gouge; G is the shear modulus of host rocks) along a fault. The model yields the pulse velocity from 10 - 100 km/yr to 1 - 10 km/day. It was accepted as a perspective model to explain the migration along faults [57].

In the model [5] the crustal blocks (A, a) are bordered by the "soft" weakened transitional zones (B, b), gouges (**Figure 6**), which may be fractured fluid-saturated media with the elasticity moduli much lower than those of the block geomaterial. The authors modified the inelastic Maxwell model and proposed the following relation to connect stress and strain [5]:

$$\kappa \frac{\partial^z \sigma}{\partial t^z} + E\sigma = \kappa E \frac{\partial^z \varepsilon}{\partial t^z}, \tag{15}$$

where σ, ε stand for the stress and strain, E is the elasticity modulus (rigidity), κ is the inelastic parameter, the analog of the viscosity with the dimension g/(cm s^{2-z}). The power z has the value $0 < z < 1$ and at $z = 1$ the parameter κ acquires the dimension of the dynamic viscosity μ, Equation (15) is compatible with the Maxwell model. Given the results of laboratory low-frequency measurements [58] [59], from which it follows that classical viscoelastic models (Newtonian, Maxwellian, Kelvin-Voigt and others) describe inadequately the geomaterial rheology, such replacement appears to be reasonable. The resultant system of equations has the following form [5]:

$$\sigma_n = M \frac{\partial^2 s_n}{\partial t^2}, \tag{16}$$

$$\kappa \frac{\partial^z \sigma_n}{\partial t^z} + E_b \sigma_n = \frac{\kappa E_b}{b} \frac{\partial^z}{\partial t^z}\left[s_{n+1} + s_{n-1} - 2s_n\right]. \tag{17}$$

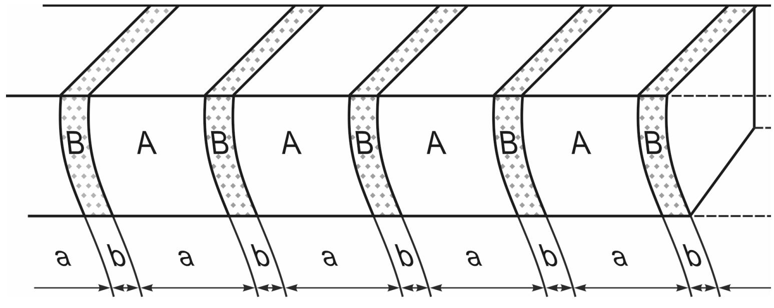

Figure 6. The structural model of the crust [5].

Here, M is the surface density; σ_n is the stress, applied to the n-block; s_n is the displacement of the n-block from the equilibrium position; E_b is the elasticity modulus of the gouge B.

The velocity v of the migration of strain perturbations along a chain of four blocks with a linear dimension ($a + b$), is equal to [5]:

$$v = 2(a+b)\frac{\omega}{\pi}. \tag{18}$$

According to (18), the velocity of a slow stress wave in the blocky geological medium is only determined by the geometric sizes of the blocks a and their oscillation frequency ω. At the block sizes of about 10 km and the frequency of 10^{-5} - 10^{-8} s^{-1} the velocity has the value of (10^{-3} - 10^{-1}) m/s or 30 km/yr - 10 km/day, which is in agreement with numerous observation data. The expression for the strain wave velocity in the lithosphere-asthenosphere system with a deep fault, deduced from the theory [60], has the form [61]:

$$v = \omega\left(\frac{2h_1h_2G_1}{G_2}\right)^{1/2} A_o, \tag{19}$$

and, as well as (18), it contains the wave frequency ω and the geometric sizes of the system (the thickness of the lithospheric plate h_1 and that of the asthenosphere h_2). The velocity is also derived from the non-dimensional relation of the shear moduli of the lithosphere G_1 and the asthenosphere G_2, and the dimensionless coefficient A_0.

To explain the earthquake epicentral migration, the model was proposed that comprised the fluid filtration in the fault zones bordering the lithospheric plates [62]. It was suggested that the fluid supply inside the fault resulted in a decrease of friction thus provoking the movements of the fault walls, an earthquake. Combining the equations of plate motion and fluid filtration, the authors of the model derived a nonlinear parabolic equation whose solution describes the fluid front propagation with a finite velocity $v_* \sim (K/t)^{1/2}$, where $K = k_0p_0/m_0\mu_0$ is the hydraulic diffusion coefficient; k_0, m_0 are the rock permeability and porosity in the fault zone; p_0, μ_0 are the pressure and dynamic viscosity of the fluid in the fault zone. At the characteristic physical-mechanical parameters, velocity v_* is of the order of 30 km/yr, which agrees with the observed velocities of the earthquake epicentral migration along transform faults. The possibility of fluid flow in the fault zones with such a velocity was confirmed in [63], where the mechanism initiating fast hydrocarbon migration in the sedimentary basins from deep reservoirs was proposed, and it was shown that the fluid might have been transited in the form of solitary waves propagating upward along the fault planes with a velocity of 16 - 160 km/yr.

2.2. Sine-Gordon Equation and Tectonic Stress Transfer

The viscoelastic or elastoplastic models [3] [26] [47] [51] [52] cannot describe slow tectonic strain in the form of solitary waves and do not account for the blocky structure of the crust and the lithosphere. Therefore, Nikolaevsky designed an elastic model that simulated the blocky structure of the lithosphere or the crust and accounted for the inertia of kinematically independent microplate rotations or rotational oscillations of the blocks of rocks that compose the crustal bodies in the fault zones. The modification of the Cosserat mechanics has led to the balance of moment of momentum in the form of the sine-Gordon equation [31]. One of the analytical solutions of the sine-Gordon equation is a kink. It is known to date that these kinks appear to be solitons in the strict mathematical sense [64]. The shape of these functions (**Figure 7(b)**, **Figure 7(d)**) is coincident with the records of the geophysical anomalies observed before earthquakes (**Figure 7(a)**, **Figure 7(c)**). In addition, the sine-Gordon soliton may stop and move again not changing its topology which allows modeling of the fault dynamics. Finally, when moving or slipping slowly, the kink radiates linear wave, phonons which may serve as an analog of episodic seismic tremor in subduction zones [65].

The heuristic approach applied for constructing the sine-Gordon equation for the blocky medium has explained slow stress redistribution in the crust due to strain waves (individual jumps or solitary waves), moving with velocities by several orders of magnitude less than the velocities of ordinary seismic waves [31].

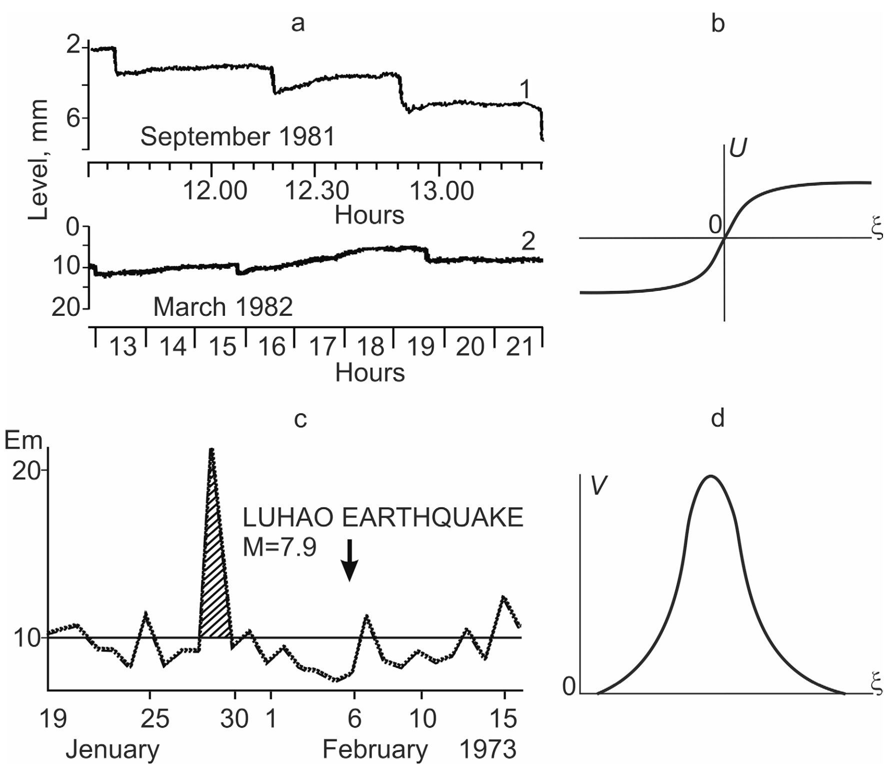

Figure 7. Indirect indications of strain waves along crustal faults. (a) Stepwise fluctuations of the ground water level in the Kim (1) and Asht (2) wells near Ashkhabad [114]; (b) Solution of the sine-Gordon equation U – kink; (c) Radon concentration measured in Guzan before the Luhao earthquake [114]; (d) Solution of the sine-Gordon equation in the shape of a soliton V (the first derivative from function U).

Since then, the sine-Gordon equation has been actively applied for modeling fault dynamics, crustal block motions generating strain waves, and for the interpretation of the observed concomitant seismic and strain effects [2] [65]-[71].

The sine-Gordon equation first obtained when describing dislocations in crystals [72], and then successively used in the ferromagnetism theory, quantum optics, physics of elementary particles and biology [64] [73] [74], has several qualitatively different analytical solutions in the shape of phonons, kinks, breathers, solitary waves, and fast and slow cnoidal waves. In the seismological and geomechanical problems, the evolution of the block and fault systems is different for each of these solutions [75].

It is necessary to emphasize that it does not appear possible to rigorously deduce the sine-Gordon equation from equations of continuum mechanics, and, consequently, slow solitary waves observed in the fault-blocky geological medium, not making the additional assumptions. In geomechanics and seismology, the sine-Gordon equation is not rigorously deduced, as well as the Newton equation is not deduced in classical mechanics, the Maxwell equation in electrodynamics, and the Schrödinger equation in quantum mechanics. These equations are heuristic. It is fair to apply the sine-Gordon equation to geological media, because, as evidenced, its implications are consistent with the experimental results, *i.e.* the equation is a summary of experimental data, which, in turn, ranks it as the law of nature.

The mathematical models describing the mechanisms of crustal block rotation and slippage that are compliant with the sine-Gordon equation, include the rotation angle of fragmented rocks of the massifs [31] [32] [66] [76] or the displacement of the blocks localized along faults [2] [65] [66] [70] [71] [75] as variables. For more detailed descriptions of the mathematical models of strain waves in the Earth compatible with the classical or perturbed sine-Gordon equation, see, for example, an overview [75].

Let us briefly describe the application of the sine-Gordon equation for modeling specific tectonic activity, which is a short-term slow slip in subduction zones accompanied by Episodic Tremor and Slow (ETS) slip. The velocity of tremor migration along the faults in different subduction zones thousands of kilometers apart is closely similar and equals 10 km/day, on average [77] [78] [79]. The ETS emerges with remarkable temporal regularity, and this time interval spans about 3 - 18 months in different subduction zones [77] [80].

A series of publications by Gershenzon and co-authors [65] [69] [70] [71] has revealed that the sine-Gordon equation is a "suitable instrument" to describe a broad spectrum of the observed features of regular and "slow" earthquakes, the migration of slow slip and seismic tremor.

In [69], the heuristic model of inelastic stress wave propagation along transform faults is proposed based on the qualitative analogy between the movements at the lithospheric plate boundaries and the processes of plastic deformation in crystals (Frenkel-Kontorova model). The plate motion along the fault is suggested to occur due to movement of dislocations along the plate boundary (term dislocation" is used here as presented in the Frenkel-Kontorova model). Dislocations are successively moving by "half-jumps". The elasticity energy is "accumulated" in such dislocations. The average density of dislocations is proportional to the average deformation at the plate boundary, whereas the average velocity of dislocations corresponds to the strain wave velocity.

The model is compliant with the classical sine-Gordon equation:

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = \sin u\,, \tag{20}$$

$$u = 2\pi u'/b\,,\quad t = t'cA/b\,,\quad x = x'A/b\,,$$

where u' is the displacement of the plate in the direction x' (along a fault); t' is the time; b is the distance between asperities on the fault plane; c is the velocity of the compressional seismic wave in the crust with the density ρ; A is the dimensionless empirical coefficient.

Solutions of Equation (20) at $|U| < 1$, $0 \le m \le 1$ are periodic functions:

$$u = \arcsin\left[\pm \mathrm{cn}\left(-\beta\xi\right)\right], \tag{21}$$

$$\varepsilon = \partial u/\partial x = 2\beta \cdot \mathrm{dn}\left(\beta\xi\right),\quad w = \partial u/\partial t = 2\beta U\,\mathrm{dn}\left(\beta\xi\right),$$

$$\xi = x - Ut\,,\quad \beta = \left[m\left(1-U^2\right)\right]^{-1/2},\quad k = \frac{\pi\beta}{2K(m)}\,.$$

Function (21) describes slow cnoidal waves, a succession of pulses with a spatial period $2m\left(1-\beta^2\right)^{1/2}K(m)$, where $K(m)$ is the full elliptic integral of the first kind, m is the modulus of the Yacobian elliptic function. The dimensionless derivatives for the coordinate ε and the time w are the dimensionless deformations and slip velocity; U is the dimensionless velocity of dislocations (in units of c); k is the wave number (in units of A/b).

From model solutions (20), it follows that the inelastic wave velocity value is the exponential function of stresses and varies from a few km/s during an earthquake to 10 km/day and 10 - 100 km/yr over the postseismic and interseismic periods. The calculations show that after an "earthquake" the strain wave velocity is inversely proportional to the time. The aftershock number decreases with time in compliance with this dependence (Omori law). Hence, physical interpretation of the Omori fundamental empirical law is as follows: aftershocks are caused by strain waves that are generated by earthquakes [69].

It is known from theoretical physics that at small perturbations a kink from the sine-Gordon equation is a stable formation that radiates phonons, small- amplitude waves, when moving, and is restored afterwards [81] [82]. This result was used as an analogy for constructing the model of slow slip and episodic tremor [70].

In the framework of the model [70], the main solutions of the perturbed sine-Gordon equation which are solitons (kinks) and unharmonic oscillations (phonons), were interpreted as slip pulses and seismic tremor, respectively. The seismic tremor (phonon radiation) may emerge due to acceleration or deceleration of the slip pulse (kink), the interaction of the slip pulse with large asperities of the plate boundaries, and the impact of the external stress perturbation upon the plate boundaries.

If accounting for only the last factor, then the model complies with the perturbed sine-Gordon equation [70]:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \sin u = \Sigma_S^0 - f \,, \tag{22}$$

where Σ_S^0, f are the external shear stress and the friction force per unit area (in units of $\mu A/(2\pi)$). If searching for a solution in the shape of a traveling wave $u = u(x - Ut) = u(\xi)$ with a dimensionless velocity U, then at $U^2 > 1$, $0 \le m \le 1$ the solution of Equation (22) is a phonon mode with the initial phase of motion ξ_0 [83]:

$$u = 2\arcsin\left[\sqrt{m}\,\mathrm{sn}\left(\frac{\xi - \xi_0}{\sqrt{U^2 - 1}}, m\right)\right]. \tag{23}$$

The model predicts generating of radiation (23) in the frequency range determined by the fault parameters, and exciting of resonant oscillations inside the fault in some depth ranges. The model explains a complicated pattern of tremor migration with a velocity of about 10 km/day and "rapid" tremor (~50 km/h). Tremor is the inner response of the fault to the external impact.

In the case $U^2 < 1$, the solution of Equation (22) will be a localized wave in the shape of a perturbed kink, soliton:

$$u_0(x, t, U, X) = 4\,\mathrm{arctg}\exp\left[\pm\left(\frac{x - \int_0^t U \mathrm{d}t' - X}{\sqrt{1 - U^2}}\right)\right], \tag{24}$$

where X is the initial location (coordinate) of the center of a soliton. The interaction between the phonon and kink, *i.e.* between two modes (23) and (24), is possible, if the terms in the right-hand part of Equation (22) are not equal to zero.

To summarize, the model and the calculations show that shear stresses accumulated in the subduction zone may relax due to nucleation and slow motion of the slip pulse (kink) whose interaction with different-size the asperities produces the radiation (tremor) inside the fault [70] [71].

2.3. Models of Slow Strain Autowave Processes

The autowave concept was applied to describe slow strain processes based on the concepts of global plate tectonics [16] [27] and the synergetics principles [84] [85]. As mentioned in Section 2.1, a fundamental possibility of generating global tectonic autowaves in the lithosphere-asthenosphere system was shown for the first time by Nikolaevsky [27]. The issues related to the autowave processes in the lithosphere and their mathematical modeling were discussed in detail in the monograph [16].

The "autowave" approach has further been developed in the recent years. Kuz'min [7] elaborated the model simulating the formation of autowave deformations in the fault zones and constructed a nonlinear diffusion equation of surface displacements. It was convincingly demonstrated that the spatiotemporal migration of anomalies of recent surface movements in the fault zones is a result of the autowave deformation processes occurred in the active excitable geological medium which is an open system, and in terms of its structure, the migration complies with the Kolmogorov-Petrovsky-Piskunov-Fisher equation. Naturally, the existence of autowave structures is supported by the energy supply due to geodynamic processes on the regional and global scales [33] [86].

On the assumption that the solitary waves of seismicity migration appear to be autosolitons, Spirtus [34] [87] [88] proposed a model of excitable hierarchic-blocky seismic medium (analog of the model of active medium with restoration), where the intensity of seismicity and the "deconsolidation" degree of a characteristic block of the medium are used as variables, whereas the average strain velocity that defines the seismicity level is the governing parameter. The mathematical model of such a medium agrees with a modified FitzHugh-Nagumo model and allows the explanation of the inverse dependence revealed between the shock migration velocity and the energy of seismic events, and accounts for specific features of seismicity migration. The calculations are in agreement with individual manifestations of the foreshock migration and acoustic emission.

The Makarov model [89] rests upon the synergetics methodology: the autowave deformation processes in geological media are ascribed to the phenomenon of self-organization of deformations at different scale levels. The velocities of slow strain wave front propagation in geological media are suggested to be governed by the velocity of defects and damage generation due to dynamic impact on these media, and are regulated by the velocity value of the energy supplied [90]. The physical mechanism of exciting slow strain autowaves suggests the loss of stability of the loaded elastoplastic medium due to movements at the boundaries between the crustal blocks and lithospheric plates. Mathematical model involves the equations of mechanics of deformed solid body, the rheological relations that give the velocities of inelastic strain accumulation, and the cellular automata method. The solutions of the relevant systems of equations yield the stress waves with a broad velocity spectrum.

Advances in the development of the concept of strain waves in the Earth or the concept of wave migration of seismicity are likely to be due to modeling the fault-blocky geological medium in the form of the autowave or autooscillation (stick-slip) system whose properties are mostly governed by the state of its inner parameters.

2.4. The Sine-Gordon Equation and the Development of Sliding Regimes in the Faults

The problem related to the deformation and dynamics in the crustal fault zones implies identifying the processes and determining the parameters that govern the sliding regimes in the faults. Examining strain migration on macro- and mesoscales allowed one to gain insight into the process of localized strain propagation in the form of solitary waves (kinks, solitions) and autowaves. Understanding of physical processes regulating the transition between different deformation regimes suggests elucidating the conditions that enable the transition from the soliton to autowave regime of deformation of the fault-blocky geological medium, or, simply, the transition to diffusion dissipation of stress. Precisely, the problem reduces to answering the question of how does sliding occur in the faults?

Generally accepted concepts suggest that the transition from creep to stick-slip along a fault frequently accompanied by tectonic earthquake is caused by the geometric inhomogeneities of the fault, the friction decrease in separate fault segments, and the pore pressure anomalies. Seismic movements may also be initiated by slow strain waves excited in the crust and the lithosphere.

Stick-slip experiments with rock samples have inferred that the propagation of localized strain in the form of slip waves moving along the block contact always occurs prior to a dynamic slip [2]. The onset of a slip is preceded by the transmittance of wave fronts of different types that are visually observable at the contact of two blocks. Propagation of a slowly moving, with a velocity from 40 to 80 m/s, front is a dominant mechanism producing the break of the contact [91]. The displacement of one block relative to another one, *i.e.* weakening of the contact, does not occur till a slowly moving front crosses an entire surface of the block contact. Stick-slip experiments have also revealed solitary failure fronts propagating at a constant velocity of about 30 - 60 m/s [92]. The existence of slow strain waves that possess the soliton properties is supported by laboratory experiments [93]. Slow autowave perturbations propagating in the form of localized plastic strain fronts were detected from the compression of various rock samples [94]. The main stick-slip effects observed at the contacts of blocks of rocks were reproduced by applying the perturbed sine-Gordon equation [2].

The mathematical models of solitary waves and autowave processes in fault-blocky geological media can be conventionally divided into two types: conservative (for the medium with dispersion) and dissipative (for the medium with diffusion). As mentioned in Section 2.2, the conservative models compliant with the canonic sine-Gordon equation are actively applied in geomechanics and seismology. The dissipative models are involved in describing stick-slip in the blocky excitable medium with elastic coupling [95] [96], slow autowave deformation processes in the geological medium [7], and seismicity migration in the excitable hierarchic-blocky medium [88]. The mathematical models simulating these processes reduce to the FitzHugh-Nagumo and Kolmogorov-Petrovsky-Piskunov-Fisher reaction-diffusion equations and describe the wavefront dynamics. Kinks and solitons are the solutions of the conservative models compatible with the canonic sine-Gordon equation. The dissipative models described by the reaction-diffusion equations yield the solutions in the shape of autowaves.

The properties of the solutions derived from these two types of models are absolutely different. Thus, the capability of solitons to retain their velocity, shape and amplitude is fundamentally due to the lack of dissipation in the medium. Moreover, the solitons with different velocities and amplitudes can exist in the same medium. Conversely, the autowaves propagate in the medium with dissipation and retain their velocity, shape and amplitude due to external energy supply. In the active medium, all autowaves are similar and their characteristics are only dependent upon the medium parameters.

In [75], the model is presented which describes the development and change of the sliding regime along a fault. This model of solitary waves in the fault complies with the perturbed sine-Gordon equation and, unlike the conservative or dissipative block models generating strain waves, accounts for both inertia and dissipation simultaneously, which seems to be more realistic for a description of the deformation regime in the fault-blocky system.

The model includes three most important mechanisms that provide the interaction between the fault walls, namely, friction, geometric inhomogeneities (asperities and "cohesion") and the external load, which govern the process of stick-slip along a fault over a certain time period. The resulting mathematical model is compatible with the perturbed sine-Gordon equation [75]:

$$\frac{\partial^2 U}{\partial \xi^2}-\frac{\partial^2 U}{\partial \eta^2}=\sin U+\alpha\frac{\partial U}{\partial \eta}+\gamma(\xi)\delta(\xi-L)\sin U+\sigma(\eta), \tag{25}$$

$$U=2\pi\frac{u}{a},\quad \xi=\frac{\pi x}{ap},\quad \eta=\frac{\pi\omega_0 t}{p},\quad p^2=\frac{a^2 D_t}{4mgh},\quad \omega_0^2=\frac{D_t}{m},$$
$$\alpha\approx\frac{a\mu}{d\Delta\rho_s(gh)^{1/2}},\quad \gamma=\frac{H}{L}.$$

Here, u is the displacement of blocks positioned periodically along the length of a fault; a is the distance between the block centers; D_t is the tangential contact stiffness; $m = 4\pi r^3\rho_s/3$ is the block mass; ρ_s is the density of the block material; r is the block radius (size); h is the distance between the neighboring block layers; g is the gravity acceleration; μ is the viscosity of the gouge between the blocks; d is the size of the block contact; Δ is the thickness of the gouge; α, γ are the friction and inhomogeneity parameters; H, L denote the height of "cohesions" and the distance between them, normalized to ap/π; $\delta(\xi)$ is the Dirac delta-function; $\sigma(\eta)$ is the function, which reflects the external effect at the contact of the fault walls.

In the right-hand part of Equation (25), the first term corresponds to the "restoring" force emerging due to shear along a sinusoidal surface of the fault walls; the second term corresponds to the friction force proportional to the velocity of relative displacement; and the third one to the corrections introduced for point inhomogeneities located with a spatial period of apL/π. Based on Equation (25), it was shown that strain effects due to friction decrease ($\alpha \ll 1$) at the contacts of inhomogeneous fault walls are capable of generating solitary strain waves proposed to be interpreted as the waves of fault activation propagating at velocity V_α [75]. These waves appear to be the strain ε localized on a mesoscale (26), which propagates along the fault with a dimensionless velocity β, connected with V_α by relation (27) and governing the sliding regime in the fault.

$$\varepsilon = \frac{\partial U}{\partial \xi} = \pm \frac{1}{\left(1-\beta^2\right)^{1/2}} \operatorname{sech}\left(\frac{\xi - \beta\eta}{\left(1-\beta^2\right)^{1/2}}\right), \tag{26}$$

$$\beta = \frac{2r}{\alpha}\left(\frac{\pi r \rho_s}{3D_t}\right)^{1/2} V_\alpha . \tag{27}$$

It follows from the calculations that the velocity profile v of the block located at the surface of the fault walls has the shape of a soliton $v(x,t) = v_{\max} \operatorname{sech}(x - V_\alpha t)$, moving along the fault with velocity V_α. If the value of V_α is small, then v is insufficient and stable slow (creep) slip occurs. At velocities of V_α of about 1 - 10 m/s, we obtain the soliton profile showing the displacement velocity of the fault walls v ~0.1 - 1 m/s and a stepwise profile of the displacement (kink) $u(x,t)$. To conclude, as evidenced from experiments [91], the transmitting solitary wave (26) weakens the contact, which, at constant external load, results in a dynamic slip, the displacement of the fault walls. These waves are, by origin, similar to slip waves observed in numerous stick-slip experiments performed at the contact of blocks of rocks [2]. At specific parameters of the fault, a solitary wave "acquires" the stationary regime with velocity V_α ~10^{-4} - 10^{-1} m/s or 30 km/yr - 10 km/day, which agrees with that of strain waves.

It is likely that elucidating the conditions which allow the transition from the model of solitary waves in the conservative medium with low "friction" (soliton-like behavior of the system) to the model of solitary waves in the active medium with diffusion (autowave behavior of the system) requires analyzing equations derived from (25) in any limit case.

Let us consider the possible deformation regimes of the fault-blocky medium under strong friction condition. The physical model of such structure can be shown as a set of blocks positioned periodically along both fault walls (**Figure 8(a)**) [97]. In the merely general case, separate blocks may even produce vertical oscillation motions (**Figure 8(b)**) or pendulum oscillation motions, when the lower parts of the blocks are fixed, whereas the upper ones are non-equilibrated (**Figure 8(c)**). High friction hampers the displacement of one fault wall relative to another one, but the neighboring blocks in the fault body are sticking out by one another and the blocks at the opposite fault wall. Such block behavior seems realistic, if taking into account that part of the blocks in the fault body may be contracted, whereas the other part may be unloaded due to differences in the roughness of the block contacts or different effective viscosity of the interblock gouge.

If "cohesions" are completely absent at the fault wall surface ($\gamma = 0$) and friction is high ($\alpha > 1$), then in Equation (25), the term with the first time variable that corresponds to dissipative losses exceeds considerably the inertial term with

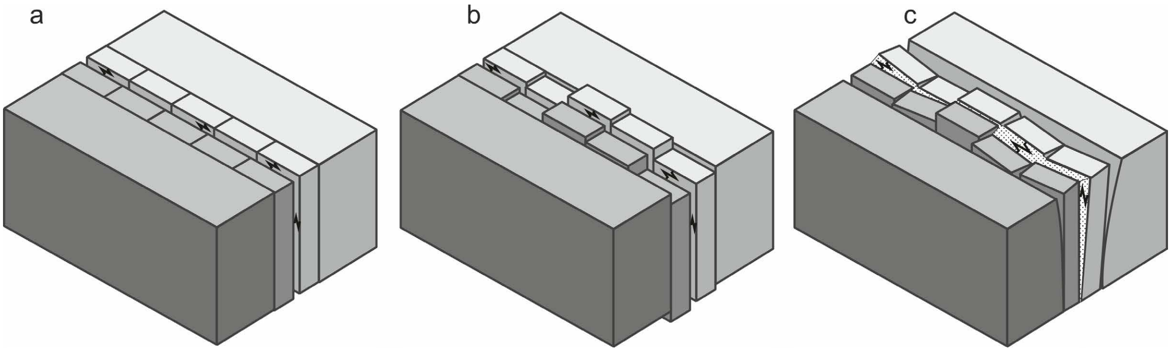

Figure 8. A scheme illustrating the structure and location of the blocks in the fault body [97]. (a) The blocks positioned periodically along the fault walls; (b) Vertical oscillation motions of separate blocks; (c) Pendulum oscillation motions.

the second time variable, and it can be neglected. In this case, the perturbed sine-Gordon Equation (25) transforms into equation [97]:

$$\frac{\partial^2 U}{\partial \xi^2} = \sin U + \alpha \frac{\partial U}{\partial \eta} + \sigma(\eta), \tag{28}$$

which is coincident, in terms of its structure, with the equations describing for example, autowaves in active media with dissipation and energy supply [98] [99] or excitable waves in the reaction-diffusion systems [100].

It is known from theoretical physics that weak damping ($\alpha \ll 1$), due to soliton moving in the medium with "friction", may be compensated by the energy supplied to a soliton from the external source. Such stationary solitary waves in the medium with low "friction" are not largely different in their properties from the solitons in the conservative systems [101]. However, this difference is still more increasing with the growth of dissipation, *i.e.* with "friction" increase in the system. The similarity between the solitary waves in active media with diffusion and solitons will be persistent until a certain critical damping value is exceeded. After that, the transition of the system from the soliton (25) to the autowave (28) regime occurs. This results in abrupt changes of the properties of the medium, which is manifested, in the first place, in the response of the medium to the interaction of slow solitary strain waves. When two autowaves collide, their annihilation, *i.e.* mutual vanishing, or transformation into the autowave of different type (static or pulsating autosoliton) occurs [102]. Conversely, solitons restore and retain their shape after colliding and continue moving at the same velocities and in the same orientations, as prior to the interaction.

If a kink and antikink (images of strain wavefronts) are moving reciprocally with equal velocity V_e, at which the energy losses due to dissipation are equal to the energy supplied to the kink, then, following [83], and using the notation, the expression can be written for velocity $V_e = \left(1 + (4\alpha/\pi\sigma)^2\right)^{-0.5}$. This case demonstrates the classical autowave behavior of the medium.

If the external source $\sigma(\eta)$ is lacking and the "restoring" force $\sin U$ (asperity of the fault walls) is not taken into account, then the transition of the system to the ordinary diffusion regime occurs, and Equation (28) takes the form of the classical diffusion equation:

$$\frac{\partial U}{\partial \eta} = \alpha^{-1}\frac{\partial^2 U}{\partial \xi^2}, \tag{29}$$

which, when substituting the dimensionless values by the physical parameters integrated in initial Equation (25), may be written as:

$$\frac{\partial u}{\partial t} = \kappa\frac{\partial^2 u}{\partial x^2}, \tag{30}$$

$$\kappa = \frac{a^2}{2\pi}\frac{D_t}{m}\frac{d\Delta\rho_s}{\mu} \quad \text{or} \quad \kappa \approx \frac{a^2 d\Delta\rho_s}{2\pi}\frac{\omega_0^2}{\mu}, \tag{31}$$

where κ is the stress diffusion coefficient.

The equations of this type, presented in Section 2.1, were earlier applied for modeling stress transfer along the lithosphere-asthenosphere contact and for describing the strain and earthquake migration.

At the sinusoidal variations of the load with a period $T = 2\pi/\omega$ velocity V_d of the strain wave is defined as:

$$V_d = \sqrt{2\kappa\omega} = 2\sqrt{\frac{\pi\kappa}{T}} \approx \sqrt{\frac{\Delta D_t}{\mu T}} = \sqrt{\frac{D_t}{\delta T}} = a\omega_0\Lambda, \tag{32}$$

where $\delta = \mu/\Delta$ is the specific viscosity of the contact; Λ is the dimensionless coefficient. At the diffusion coefficient value κ = 0.01 - 1.0 m^2/s [26] [103] and characteristic parameters of the crust derived from the calculations using formulas (31) - (32), the velocity of the diffusion wave $V_d = 8 \times 10^{-5}$ - 2.5×10^{-4} m/s (2.5 - 10 km/yr), which is comparable with the velocity of slow strain (tectonic) waves of about 1 - 100 km/yr [75]. Hence, the localized strain transfer (moving of a kink) is changed by the diffusion dissipation of stress.

We have gradually come to an understanding that frictional movement along the surface of the block contact or along crustal faults is accompanied by slip waves of different types against the background of creep [104] [105]. Such slip waves may exist in the form of solitary waves (slip pulses), periodic waves or wave fronts. Examining the model of solitary waves in the fault that is compliant with the perturbed sine-Gordon equation has revealed the physical conditions which allow a possible transition of the system from the soliton to the autowave regime, or the regime that prevents the localized strain transfer, when the diffusion dissipation of stress occurs.

In summary, the development of sliding regimes in the faults is directly related to slow dynamics in the geological medium, *i.e.* the wave processes considerably slower than seismic ones. Slow dynamics of deformed fault zones includes the localized strain transfer in the form of solitary waves and autowaves, the generation of strain waves of different types and various-scale wavefronts. Slow dynamics is governed by the crustal block interaction and their synchronization in time.

3. Methods of Strain Wave Detection and Main Observations

Geological media are modeled by elastic, quasielastic, plastic and viscoelastic bodies which experience the deformation that affects their volume (extension and compression) and shape (shear) due to external force. If applying a force to the medium, a wave always emerges in it. In physics, the most common definitions of the wave motions are as follows. Waves are defined as changes in the state of the medium (perturbations) that propagate in the medium and carry the energy. The main property of all the waves, irrespective of their origin, is that the energy is transferred in the wave without the matter transfer.

The definition given by Whitham [106] can likely be considered the most reasonable one to describe the wave geodeformation processes and analyze the *in-situ* measurement results on the wave manifestation of strain migration: "A wave is any identifiable signal, transmitted from one part of the medium to another one with a specific velocity. This signal may be a perturbation of any kind: the maximum of any value or its abrupt variation on condition that this perturbation is clearly distinguishable and its location can be detected at any time moment given. This ***signal*** may be distorted, may change its value and velocity, but ***should remain identifiable***". For example, when examining the wave motions of crustal strain, the strain maximum was chosen to be the "maximum of any value" [107].

Strain wave propagation in the geological medium is accompanied by various seismic, hydrogeological, electrokinetic, geochemical and other effects. The methods of strain wave detection are divided into indirect, detecting wave variations in geophysical fields due to temporal stress-state variations of the geological medium, and direct that are immediately recording strain migration.

3.1. Indirect and Direct Observations of Strain Waves in the Earth

Indirect evidences of strain waves are as follows: targeted migration of large earthquakes [108]; displacements of seismic wave velocity anomalies (temporal variations of seismic wave velocities, travel times and time misfits, and different parameters of the seismotectonic process) [109] [110]; wandering of aseismic bands in the Earth's mantle [16]; oscillation movements of the seismic reflectors [85] [111] [112]; migration of geophysical anomalies (radon, electrokinetic signals) [113] [114] [115]; and episodic tremor and slow-slip migration along subduction zones [116].

Direct indications of strain waves include wave fluctuations of the ground water level; migration of slopes and strain at the surface. Direct methods studying temporal variations of crustal strain involve deformographic [4] [6] [107] [110] [117] [118] [119], hydrogeodynamic [11] [12] and geodetic measurements [7] [33] [86], including methods of strain measurements using laser long ranging [120] and data from continuous GPS observations [10] [121].

3.2. Laboratory Experiments

In 1949, when investigating the Portevin–Le Chatelier effect, McReynolds [122] made a fundamental discovery of slow waves, which accompanied a discrete deformation: a kink-type step was moving along the tested sample with a velocity from 0.5 to 80 cm/s (**Figure 9(a)**). The possibility of generating slow waves in metals (McReynolds' slow wave) with velocities of about a few cm/s was confirmed in 1966 by other researchers [123] [124]. A change in the shape and a decrease in the amplitude of the strain pulse (**Figure 9(b)**), *i.e.* the dispersion and dissipation, which are the main properties of the wave process, were also detected.

In the same 1966, the first results of stick-slip experiments at the contact of blocks of rocks were published and this effect was proposed to be used as an analog of the earthquake source [125]. Experiments with samples of Westerly granite yielded the results (**Figure 9(c)**) [126], which were qualitatively consistent with the data for metals. Later, stick-slip experiments performed under biaxial

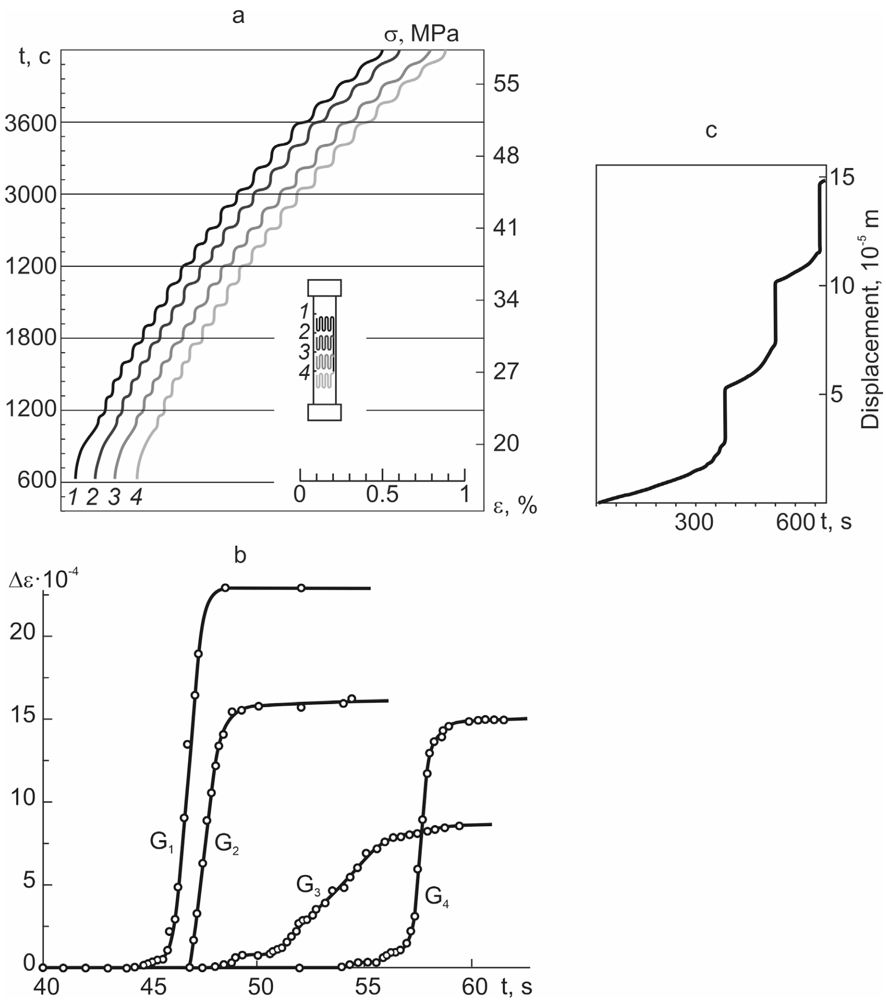

Figure 9. Evolution of deformation and displacement in metals and rocks. (a) Discreteness of deformation ε along the length of aluminum sample depending on stress σ. Curve numbers (1 - 4) correspond to the number of pre-cut faults (1 - 4) of the sample [122]; (b) A change of deformation $\Delta\varepsilon$ in time, observed in four pre-cut faults of aluminum sample [123]; (c) A change in the displacement due to stick-slip in the sample of Westerly granite [126].

compression of Tsukuba granite samples recorded the evolution of displacement due to stick-slip with high accuracy (**Figure 10**), and determined the maximum slip velocity from 1 to 40 cm/s [127] which is practically equal to the McReynolds' velocity of slow waves in metals.

The most pronounced result of numerous stick-slip experiments with rock samples and various materials is that the stress wave propagating along the block contact always emerges prior to a dynamic slip, the final stage of each stick-slip cycle (**Figure 10**). Strain waves of different types are generated at the border between solid bodies due to their relative displacement [2]. The system of two contacting blocks of rocks may probably serve as a simplified generator of strain waves.

Stick-slip experiments have also detected solitary failure fronts moving at a constant velocity of at least about 5% from the velocity of ordinary shear waves $V = 0.05\,V_s \approx 30$ - 60 m/s [92]. The localized strain propagating in the form of slow strain waves of 10^{-2} m length with a velocity of 10^{-5} - 10^{-4} m/s was detected from the compression of various rock samples (sylvinite, marble, and sandstone) [94] and metals [128].

Physical modeling first detected slow strain waves in the shear zone, which was formed in the elastoviscoplastic blocky medium [129]. Spatiotemporal dynamics of strain waves in the shear zone is governed by its inner fault-blocky structure, whereas the average velocity of strain waves attains (0.65 - 4.65) $\times 10^{-3}$ m/s depending on the level of stresses accumulated in the zone.

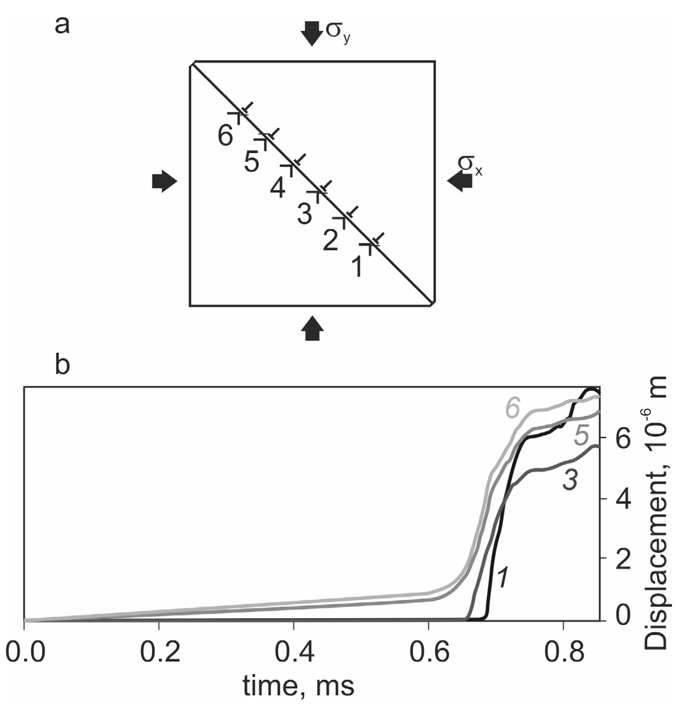

Figure 10. A scheme of stick-slip experiment under biaxial compression (a) and evolution of the displacement at different contact points of granite blocks (b). The numbers of displacement curves (1 - 6) (b) correspond to the numbers of sensors (1 - 6) (a), installed near the block contact [127].

3.3. Field Observations

The slow migration of deformations of the Earth's crust was first discovered in the 1960s using water-tube tiltmeters [130]. Then, when analyzing data from borehole strain gauges from five observatories located in the northeast of the Japanese island arc (Honshu Island), migration of the shear strain maximum at a velocity of 20 - 40 km/year was found [107] [117]. Mathematical modeling [131] showed for this region a good agreement between calculations based on the generalized Elsasser model and field observations of the "migration of crustal movements" and confirmed the effectiveness of ideas about the migration of deformations in the form of slow waves. Higher migration velocity of crustal deformations with amplitudes of less than 1 cm were recorded along the Japanese islands using horizontal displacements at 900 points of Japan's National GPS network GEONET from 1994 to 2001 [132]. At the same time, for each GPS station, the effects of earthquakes and the interaction of lithospheric plates were excluded. The observational network of strain gauges installed in the Kyushu region recorded the migration of deformations from the subduction zone towards the continent at a velocity of 90 - 140 km/year [6]. Thus, deformation measurements in various regions of the world revealed the migration of strain at a velocity of about 10 - 140 km/year [4] [5] [6] [9].

It should be noted that as early as 1979 in China, using a sensitive laser strain gauge, the movement of a strain wave was recorded and it was proposed to use this effect as a precursor of strong earthquakes [133]. The results of monitoring the migration of crustal deformations and their anomalous behavior before strong earthquakes in Japan were also discussed in [134].

Johnston and Linde [104] give examples of registration of "slip waves", which generate coherent deformations over several kilometers of the San Andreas fault surface. The amplitude of the "slip wave" is about 1 cm, and the deformations exceed 10^{-7}. The authors also note that the main difficulties in recording slip waves propagating along active faults are associated either with an insignificant amplitude of these waves (<1 cm) or with a very large period, as a result of which these waves cannot be distinguished against the background of "tectonic noise". At the same time, it is argued that if these waves traveled at higher speeds, then they could be easily recorded by downhole strain gauges, but slip waves with a velocity of the order of km/year cannot be detected in such a network.

4. Properties of Slow Strain Waves

Strain waves have kinematic and dynamic characteristics: velocity, frequency and wavelength, amplitude (attenuation) and waveform. Of greatest interest are two parameters of waves, velocity and attenuation. The entire spectrum of observed strain wave velocities varies from a few km/year to 10 km/day. There are even data on strain migration (seismic tremor) at a velocity of 15 - 150 km/h [78] [79]. Slow strain waves have ultra-low Hz frequencies and long km. The main feature of these waves is that their velocity is less than the velocity of seismic waves by 6 - 7 orders of magnitude, but more than the rate of the relative movement of lithospheric plates by 10^6 - 10^7 times. Do slow strain waves have characteristics that are inherent in the usual wave process and, in particular, dispersion and attenuation?

The studies of crustal strain in the Tohoku region (northeastern Japan) have established that the migration of the maximum shear deformation has a velocity dispersion: for a period of 5.1 years, the phase velocity of migration was 40 km/year and, with an increase in the period to 5.8 years, decreased to 20 km/year. In this case, the wavelength in the first case is 210 km, and in the second, 110 km [107]. Waves of tectonic stresses with approximately the same period (5 - 7 years) were revealed by instrumental seismic observations in seismically active and aseismic regions [135].

In the course of seismic and deformational observations in Central California, Southern California, and the Kopetdag region have detected a strong dispersion of velocity $v \sim T^{-1}$ and damping decrements (0.3 - 1.0) of strain waves were established [136]. For Southern California, the velocity of slow strain waves with a period of 3 years was 40 km/year; in Central California, it was 20 km/year with a period of 5 years [17] there is a normal dispersion.

The dispersion pattern of strain wave propagation was also noted by other researchers [11]. Moreover, "wave forms of migrating strain are pronouncedly absorbed during the motion" [4]. According to our calculations, the absorption coefficient of the strain wave (shear strain maximum migrating from the coast deep into the Tohoku region [117] at a frequency of 6.0×10^{-9} Hz is (10^{-11} - 10^{-10}) m^{-1}, on average. It is 10^3 - 10^4 times less than the absorption of ordinary seismic waves in the seismological frequency range (0.01 - 1 Hz), which allows strain waves to propagate for long distances.

Strain waves are compressional and shear, as are the ordinary seismic (elastic) waves [41] [137], and, also, solitary waves (kinks, solitons) [114]. Slow strain waves are observed as shear strain migration [117] and as global compression-extension waves [138] [139]. Hydrogeological methods have recorded two kinds of strain wave perturbations: the motion of the front of stepwise stress variations [114] and the migration of solitary waves of compression-extension [12].

The strain wavelength is dependent upon the propagation velocity and oscillation frequency of the wave source. In the general case, the order of the strain wave velocity is defined by the period of the wave T and a characteristic size L of crustal blocks $v \sim LT^{-1}$.

As mentioned, the propagation velocity of strain wave equals (10^{-4} - 10^{-1}) m/s, *i.e.* varies in a broad range. As for the frequency of these waves, two groups have thus far been known, comparatively low- and high-frequency. The former have frequencies of the order of 10^{-10} - 10^{-9} Hz [140] [141], whereas the latter is 10^{-8} - 10^{-7} Hz [136] [139]. The wavelengths of low-frequency strain waves are equal to a few hundred or even a few thousand kilometers, while those of high-frequency waves attain to a few tens of km. For example, the wavelength for a frequency of 10^{-8} Hz is 60 - 70 km. Another property, the velocity of earthquake migration waves, is the function of their magnitude, the less are the magnitudes of earthquakes examined in the migration chain, the higher are the migration wave velocities [18] [142] [143].

5. Types of Geostructures Generating Strain Waves

The most probable types of geostructures generating strain waves are subduction, collision active rifting and transform fault zones. These intense sources of different tectonic origin share a common property, being a zone of the geoblock and lithospheric plate interaction. As pointed out earlier, the laboratory experiments have inferred that strain waves are also generated at the contacts of blocks of rocks.

The revealed orientation of shear strain migration from the ocean toward the coast should be considered as being of significant importance. This common trend was first inferred in the Japanese Island Arc area, where the east-west migration was observed, and on the opposite Pacific coast, in the Western Cordilleras, where strain migrated from south to north [4]. Later evidences are available which show the strain migration orienttation from subduction zones toward the continent. For example, a slow migration of the maximum vertical crustal strain (vertical displacements) at a velocity of about 10 km/yr was observed in subduction zones near the Tohoku District (northeastern Japan) and the Izu Peninsula (central Japan), where the Pacific and Philippine plates subduct beneath the Eurasian plate [144]. All these data have led to the assumption that subduction zones, where the oceanic plates subduct beneath the continental ones, appear to be one of the possible sources of strain waves.

Moreover, the studies of seismicity dynamics along the northern boundary of the Amurian plate have shown a remarkable result [145]. Here, the migration of epicenters of weak earthquakes ($2 \leq M \leq 4$) was initiated by the strain wave front moving east-west at an average velocity of 2.7 km/day. This wave is modulated by a slow strain wave process at a velocity of the order of 10 - 20 km/yr, which is nucleated in the Japan-Kuril-Kamchatka subduction zone.

In the Pacific subduction zones, the earthquake foci migration was detected along the subducting plates toward the surface at a velocity of 65 - 260 km/yr [46] [146] and in the opposite direction inside the Earth's interiors at a velocity of 90 - 130 km/yr [146]. The direct three-component deformographic measurements have established that the strain propagating both along and across the subducting plate is generated in the subduction zone [6].

The seismicity manifestation in southern Middle Asia can be explained by strain waves excited due to oscillation regime of collision between the Eurasian and Indian lithospheric plates in the Pamirs and the Tien Shan junction zone [18]. The compression at the boundary between the Indostan and Eurasian lithospheric plates in the Himalayan collision zone is the source of "fast" and "slow" plastic flow waves initiating earthquakes in central and eastern Asia. Fast ("decade") and slow ("century") waves have periods of 7 - 18 and 68 - 133 years, and move at velocities of 12 - 45 and 1 - 7 km/yr, respectively [140].

In the Baikal Rift zone, north of China, the main groups of strain waves that initiate recent seismic fault activation in central Asia, have been distinguished. Their wavelengths range between 130 - 2000 km, whereas their velocities vary from 7 to 95 km/yr [147] [148], which agrees well in terms of the value and the propagation direction with the data on the earthquake migration in China [140].

The block and microplate rotation in extensive tension zones of the crust (lithosphere) may also be the source of strain waves. Rotational block motions in the geological medium due to tectonic processes or earthquakes were observed by many researchers. Therefore, the rotational block motion in the fault-related zones is considered to be one of the main physical mechanisms generating strain waves [32].

Propagation of slow strain waves at a velocity of 40 - 50 km/yr was recorded along transform faults at the lithopheric plate boundaries in southern California and the Kopet Dag region [110]. They move along a narrow, about 100 km wide, "corridor" [17]. The assumption was also made that seismicity variations along the Pacific and North American plate boundary in the San Andreas transform fault zone, California, are related to slowly traveling strain waves [43].

Large interplate earthquakes are preceded by the phase of increase in the seismic activity. Migration of weaker shocks toward the main shock epicenter is observed during the preparation process of earthquakes with $M \geq 7.0$. Migration velocities of "background" earthquakes vary from 4 to 250 km/yr [149] [150]. The shock migration in the zone of impending large earthquake is observed in a broad magnitude range irrespective of seismotectonics of the regions, and the migration velocity is linked to the velocity of interplate motions [150].

Retrospective analysis of the largest Tohoku earthquake (March, 11, 2011, $M_w = 9.0$) has identified two foreshock sequences migrating at velocities of 2 - 5 and 10 km/day along the oceanic trench axis toward the epicenter. A slow slip was observed to propagate in the distinguished zone of weak earthquake migration along the surface of the Pacific lithospheric plate toward the main shock area [151]. The earthquake sequences migrating at a velocity of 2 - 10 km/day were detected during an April, 1, 2014, $M_w = 8.1$, Iquique, Chile, earthquake that occurred near the subduction zone at the eastern boundary of the Nazca plate [152]. The physical mechanism of the foreshock migration is plausibly a slow slip at the plate surface similar to a slip at the contact of blocks of rocks observable in stick-slip experiments.

6. Current Status and Outlook

In the recent 50 years, the crustal strain migration has been detected and its wave pattern has been revealed, and consequently, the existence of strain waves in the Earth has been proven using direct and indirect methods by researchers from different countries in various regions of the world. The studies of targeted earthquake migration, the fundamental basis of the concept of strain waves in the Earth, have recently been activated at a new qualitative level [46] [48] [141] [142] [143] [145] [147] [149] [150] [153]-[163]. All mentioned above is due to the creation of unique databases, the developing of new techniques for the study of the seismic process on a planetary scale, and applying modern information-computational technologies for a wealth of basic data processing and analysis. Being amalgamated, this yields obtaining new estimates of the earthquake migration parameters in order to construct the wave models of the seismotectonic regime of the Earth, and examines the features of the velocity distribution pattern of seismicity migration at different energy scale levels.

The fundamental property of the geological medium, its blocky and layered structure, is considered to be the physical background of the concept of strain waves in the Earth. Different-type and various-scale strain waves are generated at the contacts of solid bodies, the block, and plate interfaces due to their reciprocal motion [2]. On large spatiotemporal scales, the pattern of displacements caused by slow deformations of rock masses is determined by the properties of interblock contacts rather than those of the block material. Slow strain waves are a specific mechanism of energy transfer in blocky media [164].

The concepts of the blocky structure of the crust were first formulated based on the data of geodetic observations and were published in 1920-1930 by Japanese scientists (for example, see [165] [166]). It was also established that the crust and the lithosphere appear to be a set of blocks and plates, whose sizes L form a discrete hierarchic row: $L = 70$, 120, 500, 1200, and 3200 km [167].

The types of geostructures generating strain waves, and the depth levels (lithospheric layers) where these waves propagate, have been identified. From the viewpoint of two-stage plate tectonics [168], two scales of propagation of tectonic waves exist on the continents, the crustal and lithospheric. The seismicity migration is suggested to be related to the crustal scale, since the major portion of the earthquake foci is clustered in the continental crust. Accumulation of new data allowed distinguishing already three depth scales of strain waves, covering an entire lithosphere, the upper brittle lithospheric layer, and the crust [169]. The fault-blocky structure of the upper elastic layer of the lithosphere and the crust generates waves with different frequencies and velocities depending on the sizes of the blocks that compose these layers-levels.

Conventionally, the strain wave processes may be quantitatively divided into two groups: global tectonic waves with velocities of 1 - 100 km/yr and strain waves in faults with velocities of 1 - 10 km/day. It follows from models [24] [29], that elastic interactions between neighboring fault segments will dominate at a velocity of strain waves of the order of 100 km/yr and more, while at lower velocities the viscoelastic properties of the lower crust and upper mantle play a major role in a slow strain front motion, for example, after the largest earthquakes [170].

Fast seismicity migration at a velocity of 1 - 10 km/day was observed in different regions of the world [19] [160]. Strain waves along crustal faults are manifested in the migration of the geochemical and geophysical anomalies with similar velocities [114]. The velocity of aseismic creep along the San Andreas Fault, central California, attains about 10 km/day [21] [171]. A slow slip and episodic tremor migrate in subduction and transform fault zones with a similar velocity of 1 - 10 km/day [79] [151] [152] [172]. The data are available on the migration of the gravity and magnetic anomalies at velocities from 200 to 1200 km/yr (0.7 - 4.0 km/day), which are coincident in the occurrence time and the displacement direction [173]. Surprisingly good agreement of the migration velocities of the aforementioned anomalies and their velocity correlation with seismicity migration (2.7 km/day) [160] may imply that a single internal source of perturbations of the stress state of the crust is moving that initiates tectonomagnetic, gravity and seismic effects. The crustal strain migration in the form of slow waves may likely be this source.

The agreement between the velocities of the geophysical anomalies and the migration velocities of weak earthquakes, creep, strain, and Episodic Tremor and Slow (ETS) slip is a fundamental result. This is where the main breakthrough in the physics of earthquakes can be expected.

Another example of the unilateral movement of natural anomalies is the migration of seismic and volcanic activity. In 1976, the French researcher Blot pointed out that earthquakes and volcano eruptions that occurred in the subduction zone are "aligned" in a chain of events, whose "switch-on" velocity is about 300 km/yr (~1 km/day). The researcher emphasized that volcanic activity on the continental margins is also a result of "tectonic process", strain transfer along the subducting lithosphere, which is manifested in the earthquake [174].

Vikulin and co-authors [142] [143] have deduced a spatial correlation between the migration of seismic and volcanic activity in the most tectonically active regions on Earth, namely, the Pacific margin, the Alpine-Himalayan Belt, and the Mid-Atlantic Ridge. The correlation of mud volcano explosions and earthquakes with the strain front propagation in Sakhalin Island was reported by Saprygin [175]. The space and time correlation of volcano eruptions and the largest earthquakes ($M \geq 8$) was discussed in [176].

To conclude, the first speculations on the spatial correlation between strain migration and thereby seismic and volcanic activity [20] were confirmed by numerous studies and were changed for confidence, that seismicity and volcano eruptions are governed by an internal wave process and appear to be its manifestation at the surface.

We may assert that, in essence, based on the concept of strain waves in the Earth, a new research direction is announced in the Earth Sciences: "Slow dynamics of strain processes", the goal of which is to study sufficiently slower than seismic, processes of localized strain transfer in the form of solitary waves and autowaves, and to examine different-type strain waves and various-scale wavefronts, slow perturbations of geodynamic fields, the sliding regimes in the fault zones and the earthquake migration.

Particular interest with regard to slow wave motions (tectonic and strain waves) in the blocky structures of the crust and the lithosphere is increasingly aroused. From the viewpoint of many researchers, strain waves allow the persistence of equilibrium between natural processes in the crust, and are manifested in variations of not only strain fields, but also other geophysical fields.

The goal of further studies in this field of recent geodynamics is to search for and accumulate new facts, indications, and manifestations of slow strain wave processes, and to reveal spatiotemporal trends in the patterns of earthquake migration and occurrence of geophysical anomalies.

The main objectives include the designing of specific sensors for strain waves in the Earth capable of efficient recording of these waves, the development of adequate models of wave geodynamic processes, and the mathematical modeling of transfer of tectonic stresses and effects related to strain migration in the geological medium.

Investigation of strain waves on the Earth is difficult, first of all, because of the ambiguity of the observed data interpretation. We need to learn how to properly understand the seismic, geodetic, geochemical, geoelectric, and other measurement results. It is well known from the history of science that no observations can be free of theory [177], and any interpretation is governed to a considerable extent by the adopted theoretical concept.

To date, numerous results are available, which confirm the wave pattern of dynamics of slow strain processes and are evidence of progress made in the understanding of the nature of seismicity migration, and the mechanisms responsible for the redistribution and transfer of tectonic stresses.

In the nearest future, the studies of slow strain waves may cardinally change the existing concepts on the physics of the seismic process and the fault interaction, and can help understand the mechanisms of energy exchange between the geophysical fields and the Earth's shells, and reveal new predictive indications of the seismic hazard.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Richter, E.F. (1958) Elementary Seismology. W.H. Freeman and Company, New York, 768 p.

[2] Bykov, V.G. (2008) Stick-Slip and Strain Waves in the Physics of Earthquake Rupture: Experiments and Models. *Acta Geophysica*, **56**, 270-285. https://doi.org/10.2478/s11600-008-0002-5

[3] Elsasser, W.M. (1969) Convection and Stress Propagation in the Upper Mantle. In: Runcorn, S.K., Ed., *The Application of Modern Physics to the Earth and Planetary Interiors*, Wiley, New York, 223-246.

[4] Kasahara, K. (1979) Migration of Crustal Deformation. *Tectonophysics*, **52**, 329-341. https://doi.org/10.1016/0040-1951(79)90240-3

[5] Bella, F., Biagi, P.F., Caputo, M., Della Monica, G., Ermini, A., Manjgaladze, P., Sgrigna, V. and Zilpimian, D. (1990) Very Slow-Moving Crustal Strain Disturbances. *Tectonophysics*, **179**, 131-139. https://doi.org/10.1016/0040-1951(90)90362-C

[6] Harada, M., Furuzawa, T., Teraishi, M. and Ohya, F. (2003) Temporal and Spatial Correlations of the Strain Field in Tectonic Active Region, Southern Kyusyu, Japan. *Journal of Geodynamics*, **35**, 471-481. https://doi.org/10.1016/S0264-3707(03)00008-5

[7] Kuz'min, Y.O. (2012) Deformation Autowaves in Fault Zones. *Izvestiya*, *Physics of the Solid Earth*, **48**, 1-16. https://doi.org/10.1134/S1069351312010089

[8] Reuveni, Y., Kedar, S., Moore, A. and Webb, F. (2014) Analyzing Slip Events along the Cascadia Margin Using an Improved Subdaily GPS Analysis Strategy. *Geophysical Journal International*, **198**, 1269-1278. https://doi.org/10.1093/gji/ggu208

[9] Yoshioka, S., Matsuoka, Y. and Ide, S. (2015) Spatiotemporal Slip Distributions of Three Long-Term Slow Slip Events Beneath the Bungo Channel, Southwest Japan, Inferred from Inversion Analyses of GPS Data. *Geophysical Journal International*, **201**, 1437-1455. https://doi.org/10.1093/gji/ggv022

[10] Kaftan, V.I. and Tatarinov, V.N. (2022) Registration of Slow Deformation Waves according to GNSS Observations. *Doklady Earth Sciences*, **505**, 489-495. https://doi.org/10.1134/S1028334X22070091

[11] Barabanov, V.L., Grinevsky, A.O., Kissin, I.G. and Milkis, M.R. (1988) Manifestations of Strain Waves in the Hydrogeological and Seismic Regimes of the Zone of the Frontal Kopetdag Fault. *Izvestiya Akademii Nauk SSSR*, *Seriya Fizika Zemli*, **5**, 21-31. (In Russian)

[12] Kissin, I.G. (2008) Hydrogeological Effects of Strain Waves in the Crust. *Geofizicheskie Issledovaniya*, **9**, 43-52. (In Russian)

[13] Bykov, V.G. (2005) Strain Waves in the Earth: Theory, Field Data, and Models. *Russian Geology and Geophysics*, **46**, 1158-1170.

[14] Mogi, K. (1968) Migration of Seismic Activity. *Bulletin of the Earthquake Research Institute*, **46**, 53-74.

[15] Vilkovich, E.V., Guberman, S.A. and Keilis-Borok, V.I. (1974) Waves of Tectonic Strain in Large Faults. *Doklady Akademii Nauk SSSR*, **219**, 77-80. (In Russian)

[16] Malamud, A.S. and Nikolaevsky, V.N. (1989) Earthquake Cycles and Tectonic Waves. Donish, Dushanbe, 142 p. (In Russian)

[17] Nevsky, M.V., Artamonov, A.M. and Riznichenko, O.Yu. (1991) Strain Waves and Seismicity Energy. *Doklady Akademii Nauk SSSR*, **318**, 316-320. (In Russian)

[18] Nersesov, I.L., Lukk, A.A., Zhuravlev, V.I. and Galaganov, O.N. (1990) On Propagation of Strain Waves in the Crust of the Southern Regions of Central Asia. *Izvestiya Akademii Nauk SSSR*, *Seriya Fizika Zemli*, **5**, 102-112. (In Russian)

[19] Barabanov, V.L, Grinevsky, A.O., Belikov, V.M. and Ishankuliev, G.A. (1994) On the Migration of Crustal Earthquakes. In: Nikolaev, A.V., Ed., *Dynamic Processes in Geo-*

physical Medium, Nauka, Moscow, 149-167. (In Russian)

[20] Kasahara, K. (1973) Earthquake Fault Studies in Japan. *Philosophical Transactions of the Royal Society of London A*: *Mathematical*, *Physical and Engineering Sciences*, **274**, 287-296. https://doi.org/10.1098/rsta.1973.0055

[21] King, C.-Y., Nason, R.D. and Tocher, D. (1973) Kinematics of Fault Creep. *Philosophical Transactions of the Royal Society of London A*: *Mathematical*, *Physical and Engineering Sciences*, **274**, 355-360. https://doi.org/10.1098/rsta.1973.0063

[22] Frank, F.C. (1973) Dislocation Models for Fault Creep Processes. *Philosophical Transactions of the Royal Society of London A*: *Mathematical*, *Physical and Engineering Sciences*, **274**, 351-354. https://doi.org/10.1098/rsta.1973.0062

[23] Isacks, B., Oliver, J. and Sykes, L.R. (1968) Seismology and the New Global Tectonics. *Journal of Geophysical Research*, **73**, 5855-5899. https://doi.org/10.1029/JB073i018p05855

[24] Bott, M.H.P. and Dean, D.S. (1973) Stress Diffusion from Plate Boundaries. *Nature*, **243**, 339-341. https://doi.org/10.1038/243339a0

[25] Anderson, D.L. (1975) Accelerated Plate Tectonics. *Science*, **187**, 1077-1079. https://doi.org/10.1126/science.187.4181.1077

[26] Rice, J.R. (1980) The Mechanics of Earthquake Rupture. In: Dziewonski, A.M. and Boschi, E., Eds., *Physics of the Earth's Interior*, Italian Physical Society, Bologna, 555-649.

[27] Nikolaevsky, V.N. (1983) Mechanics of Geomaterials and Earthquakes. In: Cherepanov, G.P., Ed., *Science and Technics Results*, *Mechanics of Deformed Solid Body*, Vol. 15, VINITI, Moscow, 149-230. (In Russian)

[28] Nikolaevsky, V.N. and Ramazanov, T.K. (1985) Theory of Fast Tectonic Waves. *Applied Mathematics and Mechanics*, **49**, 462-469. (In Russian) https://doi.org/10.1016/0021-8928(85)90035-8

[29] Savage, J.C. (1971) A Theory of Creep Waves Propagating along a Transform Fault. *Journal of Geophysical Research*, **76**, 1954-1966. https://doi.org/10.1029/JB076i008p01954

[30] Ida, Y. (1974) Slow-Moving Deformation Pulses along Tectonic Faults. *Physics of the Earth and Planetary Interiors*, **9**, 328-337. https://doi.org/10.1016/0031-9201(74)90060-0

[31] Nikolaevsky, V.N. (1995) Mathematical Modeling of Solitary Deformation and Seismic Waves. *Doklady Akademii Nauk*, **341**, 403-405.

[32] Nikolaevskiy, V.N. (1996) Geomechanics and Fluidodynamics. Kluwer, Dordrecht, 349 p.

[33] Kuz'min, Y.O. (1989) Modern Geodynamics of Fault Zones in Sedimentary Basins, and Earthquake Preparation Processes. In: Sadovsky, M.A., Ed., *Earthquake Forecasting*, Vol. 11, Donish, Moscow, Dushanbe, 52-60. (In Russian)

[34] Spirtus, V.B. (2008) Investigation of Geosolitons in the Crimea-Black Sea Region in the Models of FitzHugh-Nagumo Type. *Geophysical Journal*, **30**, 91-100. (In Russian)

[35] Zykov, V.S. (1984) Modeling of Wave Processes in Excitable Media. Nauka, Moscow, 168 p. (In Russian)

[36] Vasil'ev, V.A., Romanovskii, Y.M. and Yakhno, V.G. (1979) Autowave Processes in Distributed Kinetic Systems. *Soviet Physics Uspekhi*, **22**, 615-639. https://doi.org/10.1070/PU1979v022n08ABEH005591

[37] Elsasser, W.M. (1971) Two-Layer Model of Upper-Mantle Circulation. *Journal of*

Geophysical Research, **76**, 4744-4753. https://doi.org/10.1029/JB076i020p04744

[38] Bird P. (2003) An Updated Digital Model of Plate Boundaries. *Geochemistry*, *Geophysics*, *Geosystems*, **4**, Article No. 1027. https://doi.org/10.1029/2001GC000252

[39] Kreemer, C., Blewitt, G. and Klein, E.C. (2014) A Geodetic Plate Motion and Global Strain Rate Model. *Geochemistry*, *Geophysics*, *Geosystems*, **15**, 3849-3889. https://doi.org/10.1002/2014GC005407

[40] Baranov, B.V. and Lobkovsky, L.I. (1980) Shallow-Focus Seismicity in the Rear Area of the Kuril Island Arc and Its Relation to the Zavaritsky-Benioff Zone. *Doklady Akademii Nauk SSSR*, **255**, 67-71. (In Russian)

[41] Baranov, B.V., Vikulin, A.V. and Lobkovsky, L.I. (1989) Shallow-Focus Seismicity in the Rear Area of the Kuril-Kamchatka Island Arc and Its Relation to the Strongest Earthquakes in the Underthrust Zone. *Vulkanologiya i Seismologiya*, **6**, 73-84. (In Russian)

[42] Rydelek, P.A. and Sacks, I.S. (1988) Asthenospheric Viscosity Inferred from Correlated Land-Sea Earthquakes in North-East Japan. *Nature*, **336**, 234-237. https://doi.org/10.1038/336234a0

[43] Press, F. and Allen, C. (1995) Patterns of Seismic Release in the Southern California region. *Journal of Geophysical Research*: *Solid Earth*, **100**, 6421-6430. https://doi.org/10.1029/95JB00316

[44] Pollitz, F.F., Burgmann, R. and Romanowicz, B. (1998) Viscosity of Oceanic Asthenosphere Inferred from Remote Triggering of Earthquakes. *Science*, **280**, 1245-1249. https://doi.org/10.1126/science.280.5367.1245

[45] Kuznetsov, I.V. and Keilis-Borok, V.I. (1997) The Interrelation of Earthquakes of the Pacific seismic Belt. *Doklady Akademii Nauk*, **355**, 869-873.

[46] Molchanov, O.A. and Uyeda, S. (2009) Upward Migration of Earthquake Hypocenters in Japan, Kurile-Kamchatka and Sunda Subduction Zones. *Physics and Chemistry of the Earth*, *Parts A*/*B*/*C*, **34**, 423-430. https://doi.org/10.1016/j.pce.2008.09.011

[47] Lehner, F.K., Li, V.C. and Rice, J.R. (1981) Stress Diffusion along Rupturing Plate Boundaries. *Journal of Geophysical Research*: *Solid Earth*, **86**, 6155-6169. https://doi.org/10.1029/JB086iB07p06155

[48] Liu, M., Stein, S. and Wang, H. (2011) 2000 Years of Migrating Earthquakes in North China: How Earthquakes in Midcontinents Differ from Those at Plate Boundaries. *Lithosphere*, **3**, 128-132. https://doi.org/10.1130/L129.1

[49] Takahashi, K. and Seno, T. (2005) Diffusion of Crustal Deformation from Disturbances Arising at Plate Boundaries—A Case of the Detachment Beneath the Izu Peninsula, Central Honshu, Japan. *Earth*, *Planets and Space*, **57**, 935-941. https://doi.org/10.1186/BF03351873

[50] Albarello, D. and Bonafede, M. (1990) Stress Diffusion across Laterally Heterogeneous Plates. *Tectonophysics*, **179**, 121-130. https://doi.org/10.1016/0040-1951(90)90361-B

[51] Dragoni, M., Bonafede, M., and Boschi, E. (1982) Stress Relaxation in the Earth and Seismic Activity. *La Rivista del Nuovo Cimento*, **5**, 1-34. https://doi.org/10.1007/BF02740828

[52] Birger, B.I. (1989) Propagation of Stresses in the Earth's Lithosphere. *Izvestiya Akademii Nauk SSSR*, *Seriya Fizika Zemli*, **12**, 3-18. (In Russian)

[53] Melosh, H.J. (1976) Nonlinear Stress Propagation in the Earth's Upper Mantle. *Journal of Geophysical Research*, **81**, 5621-5632.

https://doi.org/10.1029/JB081i032p05621

[54] Rydelek, P.A. and Sacks, I.S. (1990) Asthenospheric Viscosity and Stress Diffusion: A Mechanism to Explain Correlated Earthquakes and Surface Deformations in NE Japan. *Geophysical Journal International*, **100**, 39-58. https://doi.org/10.1111/j.1365-246X.1990.tb04566.x

[55] Mitlin, V.S. and Nikolaevsky, V.N. (1990) Nonlinear Diffusion of Tectonic Stresses. *Doklady Akademii Nauk SSSR*, **315**, 1093-1096. (In Russian)

[56] Malin, P.E. and Alvarez, M.G. (1992) Stress Diffusion along the San Andreas Fault at Parkfield, California. *Science*, **256**, 1005-1007. https://doi.org/10.1126/science.256.5059.1005

[57] Vilkovich, E.V. and Shnirman, M.G. (1982) Waves of Migration of Epicenters (Examples and Models). In: Keilis-Borok, V.I. and Levshin, A.L., Eds., *Mathematical Models of the Earth's Structure and Earthquake Prediction. Computational Seismology*, **14**, 27-37. (In Russian)

[58] Liu, H.-P., Anderson, D.L. and Kanamori, H. (1976) Velocity Dispersion due to Anelasticity; Implications for Seismology and Mantle Composition. *Geophysical Journal of the Royal Astronomical Society*, **47**, 41-58. https://doi.org/10.1111/j.1365-246X.1976.tb01261.x

[59] Caputo, M. (1979) Which Is the Correct Stress Strain Relation for the Anelasticity of the Earth's Interior? *Geophysical Journal of the Royal Astronomical Society*, **59**, 227-230. https://doi.org/10.1111/j.1365-246X.1979.tb02563.x

[60] Nikolaevsky, V.N. and Ramazanov, T.K. (1986) Generation and Propagation of Tectonic Waves along Deep Faults. *Izvestiya Akademii Nauk SSSR*, *Seriya Fizika Zemli*, **10**, 3-13. (In Russian)

[61] Nevsky, M.V., Morozova, L.A. and Fyuz, G.S. (1989) Long-Period Deformation Waves. In: Sadovsky, M.A., Ed., *Discrete Properties of Geophysical Medium*, Nauka, Moscow, 18-33. (In Russian)

[62] Barenblatt, G.I., Keilis-Borok, V.I. and Monin, A.S. (1983) Filtration Model of the Earthquake Sequence. *Doklady Akademii Nauk SSSR*, **269**, 831-834.

[63] Revil, A. and Cathles, L.M. (2002) Fluid Transport by Solitary Waves along Growing Faults: A Field Example from the South Eugene Island Basin, Gulf of Mexico. *Earth and Planetary Science Letters*, **202**, 321-335. https://doi.org/10.1016/S0012-821X(02)00784-7

[64] Scott, A.C. (2007) The Nonlinear Universe: Chaos, Emergence, Life. Springer, Berlin, 365 p.

[65] Gershenzon, N.I., Bambakidis, G., Hauser, E., Ghosh, A. and Greager, K.C. (2011) Episodic Tremors and Slip in Cascadia in the Framework of the Frenkel-Kontorova Model. *Geophysical Research Letters*, **38**, Article No. L01309. https://doi.org/10.1029/2010GL045225

[66] Garagash, I.A. (1996) Microdeformations in a Prestressed Discrete Geophysical Medium. *Doklady Akademii Nauk*, **347**, 324-327.

[67] Mikhailov, D.N. and Nikolaevskii, V.N. (2000) Tectonic Waves of the Rotational Type Generating Seismic Signals. *Izvestiya*, *Physics of the Solid Earth*, **36**, 895-902.

[68] Vikulin, A.V. and Ivanchin, A.G. (1998) Rotational Model of the Seismic Process. *Tikhookeanskaya Geologiya*, **17**, 95-103. (In Russian)

[69] Gershenzon, N.I., Bykov, V.G. and Bambakidis, G. (2009) Strain Waves, Earthquakes, Slow Earthquakes, and Afterslip in the Framework of the Frenkel-Kontorova Model. *Physical Review E*, **79**, Article ID: 056601.

https://doi.org/10.1103/PhysRevE.79.056601

[70] Gershenzon, N.I. and Bambakidis, G. (2014) Model of Deep Nonvolcanic Tremor Part I: Ambient and Triggered Tremor. *Bulletin of the Seismological Society of America*, **104**, 2073-2090. https://doi.org/10.1785/0120130234

[71] Gershenzon, N.I. and Bambakidis, G. (2015) Model of Deep Nonvolcanic Tremor Part II: Episodic Tremor and Slip. *Bulletin of the Seismological Society of America*, **105**, 816-830. https://doi.org/10.1785/0120140225

[72] Kontorova, T.A. and Frenkel, Y.I. (1938) To the Theory of Plastic Deformation and Twinning. *Journal of Experimental and Theoretical Physics*, **8**, 89-95. (In Russian)

[73] Braun, O.M. and Kivshar, Yu.S. (2004) The Frenkel-Kontorova Model: Concepts, Methods, and Applications. Springer, Berlin, 472 p. https://doi.org/10.1007/978-3-662-10331-9

[74] Aero, E.L., Bulygin, A.N. and Pavlov, Y.V. (2009) Solutions of the Three-Dimensional Sine-Gordon Equation. *Theoretical and Mathematical Physics*, **158**, 313-319. https://doi.org/10.1007/s11232-009-0025-3

[75] Bykov, V.G. (2014) Sine-Gordon Equation and Its Application to Tectonic Stress Transfer. *Journal of Seismology*, **18**, 497-510. https://doi.org/10.1007/s10950-014-9422-7

[76] Garagash, I.A. and Nikolaevsky, V.N. (2009) Cosserat Mechanics in Earth Sciences. *Computational Continuum Mechanics*, **2**, 44-66. (In Russian)

[77] Obara, K. (2002) Nonvolcanic Deep Tremor Associated with Subduction in Southwest Japan. *Science*, **296**, 1679-1681. https://doi.org/10.1126/science.1070378

[78] Shelly, D.R., Beroza, G.C. and Ide, S. (2007) Non-Volcanic Tremor and Low-Frequency Earthquake Swarms. *Nature*, **446**, 305-307.https://doi.org/10.1038/nature05666

[79] Beroza, G.C. and Ide, S. (2011) Slow Earthquakes and Nonvolcanic Tremor. *Annual Review of Earth and Planetary Sciences*, **39**, 271-296. https://doi.org/10.1146/annurev-earth-040809-152531

[80] Lowry A.R. (2006) Resonant Slow Fault Slip in Subduction Zones Forced by Climatic Load Stress. *Nature*, **442**, 802-805. https://doi.org/10.1038/nature05055

[81] Peyrard, M. and Kruskal, M.D. (1984) Kink Dynamics in the Highly Discrete Sine-Gordon System. *Physica D: Nonlinear Phenomena*, **14**, 88-102. https://doi.org/10.1016/0167-2789(84)90006-X

[82] Braun, O.M. and Kivshar, Y.S. (1998) Nonlinear Dynamics of the Frenkel-Kontorova Model. *Physics Reports*, **306**, 1-108. https://doi.org/10.1016/S0370-1573(98)00029-5

[83] McLaughlin, D., and Scott, A. (1978) A Multisoliton Perturbation Theory. In: Lonngren, K. and Scott, A., Eds., *Solitons in Action*, Academic Press, New York, 201-256. https://doi.org/10.1016/B978-0-12-455580-8.50015-9

[84] Androsov, I.V., Zhadin, V.V. and Potashnikov, I.A. (1989) Spatial-Temporal Structure of Earthquake Migration and Seismic Belts. *Doklady Akademii Nauk SSSR*, **306**, 1339-1342. (In Russian)

[85] Bazavluk, T.A. and Yudakhin, F.N. (1993) Strain Waves in the Crust of the Tien Shan from Seismological Data. *Doklady Akademii Nauk*, **329**, 565-570. (In Russian)

[86] Sidorov, V.A. and Kuz'min, Y.O. (1989) Spatial-Temporal Characteristics of Modern Dynamics of the Geophysical Medium in Seismically Active and Aseismic Regions. In: Sadovsky M.A., Ed., *Discrete Properties of Geophysical Medium*, Nauka, Moscow, 33-47. (In Russian)

[87] Spirtus, V.B. (2010) Possibilities of Biophysical Models of FitzHugh-Nagumo Type

in Mapping of Two-Dimensional Migration of Seismicity. *Geophysical Journal*, **32**, 134-143. (In Russian)

[88] Spirtus, V.B. (2011) Features of the Dynamics of Seismic Activity in the Models of FitzHugh-Nagumo Type. *Geophysical Journal*, **3**, 57-63. (In Russian)

[89] Makarov, P.V. and Peryshkin, A.Y. (2017) Slow Motions as Inelastic Strain Autowaves in Ductile and Brittle Media. *Physical Mesomechanics*, **20**, 209-221. https://doi.org/10.1134/S1029959917020114

[90] Makarov, P.V. (2007) Evolutionary Nature of Structure Formation in Lithospheric Material: Universal Principle for Fractality of Solids. *Russian Geology and Geophysics*, **48**, 558-574. https://doi.org/10.1016/j.rgg.2007.06.003

[91] Rubinstein, S.M., Cohen, G. and Fineberg, J. (2004) Detachment Fronts and the Onset of Dynamic Friction. *Nature*, **430**, 1005-1009. https://doi.org/10.1038/nature02830

[92] Nielsen, S., Taddeucci, J. and Vinciguerra, S. (2010) Experimental Observation of Stick-Slip Instability Fronts. *Geophysical Journal International*, **180**, 697-702. https://doi.org/10.1111/j.1365-246X.2009.04444.x

[93] Sharon, E., Cohen, G. and Fineberg, J. (2001) Propagating Solitary Waves along a Rapidly Moving Crack Front. *Nature*, **410**, 68-71. https://doi.org/10.1038/35065051

[94] Zuev, L.B., Barannikova, S.A., Zhigalkin, V.M. and Nadezhkin, M.V. (2012) Laboratory Observation of Slow Movements in Rocks. *Journal of Applied Mechanics and Technical Physics*, **53**, 467-470. https://doi.org/10.1134/S0021894412030200

[95] Cartwright, J.H.E., Hernandez-Garcia, E. and Piro, O. (1997) Burridge-Knopoff Models as Elastic Excitable Media. *Physical Review Letters*, **79**, 527-530. https://doi.org/10.1103/PhysRevLett.79.527

[96] Morales, J.E.M., James, G. and Tonnelier, A. (2016) Solitary Waves in the Excitable Burridge-Knopoff Model. Technical Report RR-8996, INRIA Grenoble Rhône-Alpes, Montbonnot-Saint-Martin, 26 p.

[97] Bykov, V.G. (2020) Development of Sliding Regimes in Faults and Slow Strain Waves. *Physical Mesomechanics*, **23**, 271-278. https://doi.org/10.1134/S1029959920030121

[98] Kerner, B.S. and Osipov, V.V. (1989) Autosolitons. *Soviet Physics Uspekhi*, **32**, 101-138. https://doi.org/10.1070/PU1989v032n02ABEH002679

[99] Davydov, V.A., Zykov, V.S. and Mikhailov, A.S. (1991) Kinematics of Autowave Structures in Excitable Media. *Soviet Physics Uspekhi*, **34**, 665-684. https://doi.org/10.1070/PU1991v034n08ABEH002462

[100] Ivanitskii, G.R. (1999) Biophysics at the Turn of the Century: Autowaves. *Biofizika*, **44**, 773-795.

[101] Kivshar, Y.S. and Malomed, B.A. (1989) Dynamics of Solitons in Nearly Integrable Systems. *Reviews of Modern Physics*, **61**, 763-915. https://doi.org/10.1103/RevModPhys.61.763

[102] Kerner, B.S. and Osipov, V.V. (1991) Autosolitons: Localized Highly Nonequilibrium Regions in Homogeneous Dissipative Systems. Nauka, Moscow, 200 p.

[103] Scholz, C.H. (2002) The Mechanics of Earthquake and Faulting. 2nd Edition, Cambridge University Press, Cambridge, 471 p.

[104] Johnston, M.J.S. and Linde, A.T. (2002) Implications of Crustal Strain during Conventional, Slow, and Silent Earthquakes. In: Lee, W.H.K., Kanamori, H., Jennings, P.C. and Kisslinger C., Eds., *International Handbook of Earthquake and Engineering Seismology, Part A. International Geophysical Series*, Vol. 81, Academic Press, Amsterdam, 589-605. https://doi.org/10.1016/S0074-6142(02)80239-X

[105] Putelat, T., Dawes, J.H.P. and Champneys, A.R. (2017) A Phase-Plane Analysis of

Localized Frictional Waves. *Proceedings of the Royal Society A*: *Mathematical*, *Physical and Engineering Sciences*, **473**, Article ID: 20160606. https://doi.org/10.1098/rspa.2016.0606

[106] Whitham, G.B. (1974) Linear and Nonlinear Waves. Wiley, New York, 636 p.

[107] Ishii, H., Sato, T. and Takagi, A. (1978) Characteristics of Strain Migration in the Northeastern Japanese Arc. (I)—Propagation Characteristics. *Science Reports of the Tohoku University*, *Series 5*, *Geophysics*, **25**, 83-90.

[108] Stein, R.S., Barka, A.A. and Dieterich, J.H. (1997) Progressive Failure on the North Anatolian Fault since 1939 by Earthquake Stress Triggering. *Geophysical Journal International*, **128**, 594-604. https://doi.org/10.1111/j.1365-246X.1997.tb05321.x

[109] Lukk, A.A. and Nersesov, I.L. (1982) Variations in Time of Different Parameters of the Seismotectonic Process. *Izvestiya Akademii Nauk SSSR*, *Seriya Fizika Zemli*, **3**, 10-27. (In Russian)

[110] Nevsky, M.V., Morozova, L.A. and Zhurba, M.N. (1987) The Effect of Propagation of Long-Period Deformation Perturbations. *Doklady Akademii Nauk SSSR*, **296**, 1090-1093. (In Russian)

[111] Bazavluk, T.A. and Yudakhin, F.N. (1998) Temporal Variations of Exchange-Forming Inhomogeneities in the Earth's Crust, Tien Shan. *Doklady Earth Sciences*, **362**, 987-989.

[112] Bormotov, V.A. and Bykov, V.G. (2001) Seismological Monitoring of the Deformation Process. *Geology of Pacific Ocean*, **16**, 981-994.

[113] Rudakov, V.P. (1992) Mapping of Geodeformation Processes of Seasonal (Annual) Periodicity in the Dynamics of the Subsoil Radon Field. *Doklady Akademii Nauk*, **324**, 558-561. (In Russian)

[114] Nikolaevskiy, V.N. (1998) Tectonic Stress Migration as Nonlinear Wave Process along Earth Crust Faults. In: Adachi, T., Oka, F. and Yashima, A., Eds., *Localization and Bifurcation Theory for Soils and Rocks*, Balkema, Rotterdam, 137-142.

[115] Firstov, P.P., Makarov, E.O. and Glukhova, I.P. (2017) Peculiarities of Subsoil Gas Dynamics before the *M*7.2 Zhupanovo Earthquake of January 30, 2016, Kamchatka. *Doklady Earth Sciences*, **472**, 196-199. https://doi.org/10.1134/S1028334X17020015

[116] Schwartz, S.Y. and Rokosky, J.M. (2007) Slow Slip Events and Seismic Tremor at Circum-Pacific Subduction Zones. *Reviews of Geophysics*, **45**, RG3004. https://doi.org/10.1029/2006RG000208

[117] Ishii, H., Sato, T. and Takagi, A. (1980) Characteristics of Strain Migration in the Northeastern Japanese Arc. (II)—Amplitude Characteristics. *Journal of the Geodetic Society of Japan*, **26**, 17-25.

[118] Ishii H., Sato T., Tachibana, K., Hashimoto, K., Murakami, E., Mishina, M., Miura, S., Sato, K. and Takagi, A. (1983) Crustal Strain, Crustal Stress and Microearthquake Activity in the Northeastern Japan Arc. *Tectonophysics*, **97**, 217-230. https://doi.org/10.1016/0040-1951(83)90149-X

[119] Bella, F., Bella, R., Biagi, P.F., Della Monica, G., Ermini, A. and Sgrigna, V. (1987) Tilt Measurements and Seismicity in Central Italy over a Period of Approximately Three Years. *Tectonophysics*, **139**, 333-338. https://doi.org/10.1016/0040-1951(87)90108-9

[120] Milyukov, V., Mironov, A., Kravchuk, V., Amoruso, A. and Crescentini, L. (2013) Global Deformations of the Eurasian Plate and Variations of the Earth Rotation Rate. *Journal of Geodynamics*, **67**, 97-105. https://doi.org/10.1016/j.jog.2012.05.009

[121] Kaftan, V.I. (2021) An Analysis of Ground Movements and Deformations from

13-Year GPS Observations before and during the July 2019 Ridgecrest, USA Earthquakes. *Journal of Volcanology and Seismology*, **15**, 97-106. https://doi.org/10.1134/S0742046321010115

[122] McReynolds, A.W. (1949) Plastic Deformation Waves in Aluminum. *Transactions of the American Institute of Mining and Metallurgical Engineers*, **185**, 32-45. https://doi.org/10.1007/BF03398071

[123] Dillon, O.W. (1966) Waves in Bars of Mechanically Unstable Materials. *Journal of Applied Mechanics*, **33**, 267-274. https://doi.org/10.1115/1.3625037

[124] Kenig, M.J. and Dillon, O.W. (1966) Shock Waves Produced by Small Stress Increments in Annealed Aluminum. *Journal of Applied Mechanics*, **33**, 907-916. https://doi.org/10.1115/1.3625201

[125] Brace, W.F. and Byerlee, J.D. (1966) Stick-Slip as a Mechanism for Earthquakes. *Science*, **153**, 990-992. https://doi.org/10.1126/science.153.3739.990

[126] Scholz, C., Molnar, P. and Johnson, T. (1972) Detailed Studies of Frictional Sliding of Granite and Implications for the Earthquake Mechanism. *Journal of Geophysical Research*, **77**, 6392-6406. https://doi.org/10.1029/JB077i032p06392

[127] Ohnaka, M. (2013) The Physics of Rock Failure and Earthquakes. Cambridge University Press, Cambridge, 270 p. https://doi.org/10.1017/CBO9781139342865

[128] Zuev, L.B. and Danilov, V.I. (2003) Slow Autowave Processes during Deformation of Solids. *Physical Mesomechanics*, **6**, 75-94. (In Russian)

[129] Bornyakov, S.A., Panteleev, I.A. and Tarasova, A.A. (2016) Dynamics of Intrafault Deformation Waves: Results of Physical Simulation. *Doklady Earth Sciences*, **471**, 1316-1318. https://doi.org/10.1134/S1028334X16120175

[130] Asada, T. (1982) Earthquake Prediction Techniques. Their Application in Japan. University of Tokyo Press, Tokyo, 312 p.

[131] Sato, K. (1989) Numerical Experiments on Strain Migration. *Journal of the Geodetic Society of Japan*, **35**, 27-36. (In Japanese with English Abstract).

[132] Ito, T. and Hashimoto, M. (2001) Migrating Crustal Deformation from GEONET Observations. In: *AGU Fall Meeting Abstracts*, Vol. 2001, American Geophysical Union, Washington DC, G31A-0122.

[133] Lu, D. (1980) Stress Wave, Motion of Strain Wave and Slow Earthquake. *Scientia Sinica*, **23**, 1428-1434.

[134] Takemoto, S. (1995) Recent Results Obtained from Continuous Monitoring of Crustal Deformation. *Journal of Physics of the Earth*, **43**, 407-420. https://doi.org/10.4294/jpe1952.43.407

[135] Gamburtseva, N.G., Lyuke, E.I., Nikolaevsky, V.N., Oreshin, S.I., Pasechnik, I.P., Peregontseva, V.E., and Rubinshtein, K.D. (1982) Periodic Variations in Seismic Wave Parameters during Scanning of the Lithosphere by Strong Explosions. *Doklady Akademii Nauk SSSR*, **266**, 1349-1353. (In Russian)

[136] Nevsky, M.V. (1994) Ultra-Long-Period Strain Waves at the Boundaries of Lithospheric Plates. In: Nikolaev, A.V., Ed., *Dynamic Processes in Geophysical Medium*, Nauka, Moscow, 40-55. (In Russian)

[137] Lursmanashvili, O.V. (1977) Temporal-Spatial Distribution of Strong Earthquakes in the Caucasus, and Potential Interrelation between Earthquakes through Plastic Waves. *Reports of the Academy of Sciences of the Georgian SSR*, **87**, 601-604. (In Russian)

[138] Lyuke, E.I., An, V.A. and Pasechnik, I.P. (1988) Detection of the Front of a Tectonic

Global Wave during Seismic Scanning of the Earth. *Doklady Akademii Nauk SSSR*, **301**, 569-573. (In Russian)

[139] Saprygin, S.M., Vasilenko, N.F. and Soloviev, V.N. (1997) Propagation of the Wave of Tectonic Stresses through the Eurasian Plate in 1978-1983. *Russian Geology and Geophysics*, **38**, 701-709.

[140] Wang, S. and Zhang, Z. (2005) Plastic-Flow Waves ("Slow-Waves") and Seismic Activity in Central-Eastern Asia. *Earhquake Research in China*, **19**, 74-85.

[141] Sherman, S.I. (2007) New Data on Regularities of Fault Activation in the Baikal Rift System and the Adjacent Territory. *Doklady Earth Sciences*, **415**, 794-798. https://doi.org/10.1134/S1028334X07050303

[142] Vikulin, A.V., Vodinchar, G.M., Gusyakov, V.K., Melekestsev, I.V., Akmanova, D.R., Dolgaya, A.A. and Osipova, N.A. (2011) Migration of Seismic and Volcanic Activity in the Stress-State Zones of the Most Geodynamically Active Megastructures of the Earth. *Bulletin of Kamchatka State Technical University*, **17**, 5-15. (In Russian)

[143] Vikulin, A.V., Melekestsev, I.V., Akmanova, D.R., Ivanchin, A.G., Vodinchar, G.M., Dolgaya, A.A. and Gusyakov, V.K. (2012) Information-Computational System for Modeling of Seismic and Volcanic Processes as a Foundation of Research on Wave Geodynamic Phenomena. *Computational Technologies*, **17**, 34-54. (In Russian)

[144] Miura, S., Ishii, H. and Takagi, A. (1989) Migration of Vertical Deformations and Coupling of Island Arc plate and Subducting Plate. In: Cohen, S.C. and Vanííek, P., Eds., *Slow Deformation and Transmission of Stress in the Earth*, *Geophysical Monograph Series*, Vol. 49, American Geophysical Union, Washington DC, 125-138. https://doi.org/10.1029/GM049p0125

[145] Bykov, V.G. and Trofimenko, S.V. (2016) Slow Strain Waves in Blocky Geological Media from GPS and Seismological Observations on the Amurian Plate. *Nonlinear Processes in Geophysics*, **23**, 467-475. https://doi.org/10.5194/npg-23-467-2016

[146] Molchanov, O.A. (2011) Underlying Mechanism of Precursory Activity from Analysis of Upward Earthquake Migration. *Natural Hazards and Earth System Sciences*, **11**, 135-143. https://doi.org/10.5194/nhess-11-135-2011

[147] Sherman, S.I. and Gorbunova, E.A. (2008) Wave Nature of Activation of Faults in Central Asia on the Basis of Seismic Monitoring. *Physical Mesomechanics*, **11**, 115-122. (In Russian)

[148] Gorbunova, E.A. and Sherman, S.I. (2012) Slow Deformation Waves in the Lithosphere: Registration, Parameters, and Geodynamic Analysis (Central Asia). *Russian Journal of Pacific Geology*, **6**, 13-20. https://doi.org/10.1134/S181971401201006X

[149] Pustovitenko, B.G., and Porechnova, E.I. (2008) On the Processes of Formation of Focal Zones of Strong Earthquakes. *Geophysical Journal*, **30**, 73-90. (In Russian)

[150] Novopashnina, A.V. and San'kov, V.A. (2015) Migration of Seismic Activity in Strike-Slip Zones: A Case Study of the Boundary between the North American and Pacific Plates. *Russian Journal of Pacific Geology*, **9**, 141-153. https://doi.org/10.1134/S1819714015020050

[151] Kato, A., Obara, K., Igarashi, T., Tsuruoka, H., Nakagawa, S. and Hirata, N. (2012) Propagation of Slow Slip Leading up to the 2011 M_w 9.0 Tohoku-Oki Earthquake. *Science*, **335**, 705-708. https://doi.org/10.1126/science.1215141

[152] Kato, A. and Nakagawa, S. (2014) Multiple Slow-Slip Events during a Foreshock Sequence of the 2014 Iquique, Chile M_w 8.1 Earthquake. *Geophysical Research Letters*, **41**, 5420-5427. https://doi.org/10.1002/2014GL061138

[153] Di Giovambattista, R., and Tyupkin, Y. (2001) Cyclic Migration of Weak Earthquakes between Lunigiana Earthquake of October 10, 1995 and Reggio Emilia Earthquake of October 15, 1996 (Northern Italy). *Journal of Seismology*, **5**, 147-156. https://doi.org/10.1023/A:1011497601121

[154] Levina, E.A. and Ruzhich, V.V. (2015) The Seismicity Migration Study Based on Space-Time Diagrams. *Geodynamics & Tectonophysics*, **6**, 225-240. https://doi.org/10.5800/GT-2015-6-2-0178

[155] Stepashko, A.A. (2013) The Structure of the Lithospheric Mantle of the Siberain Craton and Seismodynamics of Deformation Waves in the Baikal Seismic Zone. *Geodynamics & Tectonophysics*, **4**, 387-415. (In Russian) https://doi.org/10.5800/GT-2013-4-4-0108

[156] Novopashina, A.V. and Lukhneva, O.F. (2020) Methodical Approach to Isolation of Seismic Activity Migration Episodes of the Northeastern Baikal Rift System (Russia). *Episodes*, **43**, 947-959. https://doi.org/10.18814/epiiugs/2020/020058

[157] Novopashina, A.V., and Lukhneva, O.F. (2021) The Propagation Velocity of Seismic Activity Migrating along the Directions of the Geodynamic Forces Prevailing in the Northeastern Baikal Rift System, Russia. *Annals of Geophysics*, **64**, SE436. https://doi.org/10.4401/ag-8654

[158] Ruzhich, V.V., Kocharyan, G.G. and Levina, E.A. (2016) Estimated Geodynamic Impact from Zones of Collision and Subduction on the Seismotectonic Regime in the Baikal Rift. *Geodynamics & Tectonophysics*, **7**, 383-406. (In Russian) https://doi.org/10.5800/GT-2016-7-3-0214

[159] Trofimenko, S.V., Bykov, V.G. and Merkulova, T.V. (2015) Seismicity Migration in the Zone of Convergent Interaction between the Amur Plate and the Eurasian Plate. *Journal of Volcanology and Seismology*, **9**, 210-222. https://doi.org/10.1134/S0742046315030069

[160] Trofimenko, S.V., Bykov, V.G. and Merkulova, T.V. (2017) Space-Time Model for Migration of Weak Earthquakes along the Northern Boundary of the Amurian Microplate. *Journal of Seismology*, **21**, 277-286. https://doi.org/10.1007/s10950-016-9600-x

[161] Žalohar, J. (2018) The Omega-Theory: A New Physics of Earthquakes. In: *Developments in Structural Geology and Tectonics*, Vol. 2, Elsevier, Amsterdam, 570 p.

[162] Zhao, G. and Yao, L. (1995) Earthquake Migration in East Asia Mainland (I)—The Migration of Huge Earthquakes and Volcanic Activity from West Pacific Trench to the Chinese Mainland. *Acta Seismologica Sinica*, **8**, 541-549. https://doi.org/10.1007/BF02650521

[163] Zhao, G., Wu, Z. and Liu, J. (2020) The Types, Characteristics and Mechanism of Seismic Migration. *Journal of Geomechanics*, **26**, 13-32. (In Chinese with English Abstract).

[164] Goldin, S.V. (2002) Destruction of the Lithosphere and Physical Mesomechanics. *Fizicheskaya Mezomekhanika*, **5**, 5-22. (In Russian)

[165] Imamura, A. (1930) On the Chronic Block-Movements in the Kyoto-Osaka District. *Japanese Journal of Astronomy and Geophysics*, **7**, 93-101.

[166] Imamura, A. (1931) On the Northward Movement of Crustal Deformation along the Western Boundary of the Kwanto Plain. *Proceeding of the Imperial Academy*, **7**, 315-318. https://doi.org/10.2183/pjab1912.7.315

[167] Sadovsky, M.A., Bolkhovitinov, L.G. and Pisarenko, V.F. (1987) Deformation of Geophysical Medium and Seismic Process. Nauka, Moscow, 100 p. (In Russian)

[168] Lobkovsky, L.I. (1988) Geodynamics of Spreading and Subduction Zones, and Two-Level Plate Tectonics. Nauka, Moscow, 294 p. (In Russian)

[169] Sherman, S.I. (2014) Seismic Process and the Forecast of Earthquakes: Tectonophysical Conception. Academic Publishing House "Geo", Novosibirsk, 359 p. (In Russian)

[170] Sanders, C.O. (1993) Interaction of the San Jacinto and San Andreas Fault Zones, Southern California: Triggered Earthquake Migration and Coupled Recurrence Intervals. *Science*, **260**, 973-976. https://doi.org/10.1126/science.260.5110.973

[171] Nason, R.D. (1969) Preliminary Instrumental Measurements of Fault Creep Slippage on the San Andreas Fault, California. *Earthquake Notes*, **40**, 7-10. https://doi.org/10.1785/gssrl.40.1.7

[172] Brudzinski, M.R. and Allen, R.M. (2007) Segmentation in Episodic Tremor and Slip All along Cascadia. *Geology*, **35**, 907-910. https://doi.org/10.1130/G23740A.1

[173] Bykov, V.G. and Trofimenko, S.V. (2017) Slow Strain Waves from Seismological and Geophysical Observations. *Geophysical Research Abstracts*, **19**, EGU2017-2380.

[174] Blot, C. (1981) Earthquakes at Depth Beneath Volcanoes, Forerunners of Their Activities. Application to White Island, New Zealand. *Journal of Volcanology and Geothermal Research*, **9**, 277-291. https://doi.org/10.1016/0377-0273(81)90040-8

[175] Saprygin, S.M. (2013) To the Problem of Seismicity of the Sakhalin Faults. *Tikhookeanskaya Geologiya*, **32**, 73-77. (In Russian)

[176] Yamashina, K. (1989) Volcanic Eruptions and Crustal Deformation in Subduction Zones. *Journal of the Geodetic Society of Japan*, **35**, 257-261.

[177] Shapere, D. (1982) The Concept of Observation in Science and Philosophy. *Philosophy of Science*, **49**, 485-525. https://doi.org/10.1086/289075