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The Source of Tension in the Measurements of the Hubble Constant

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Abstract

Through an analytical approach, we show that the Hubble constant is not unique and has two distinct values. The first of these values is consistent with the measurements by Riess *et al.*, while the second value is consistent with the measurements by the Planck Collaboration. This is a new alternative approach that does not depend on the standard ΛCDM model and its constraints. Our analysis shows that the tension is due to a geometric mismatch in the comparison of the measurements which is equal to the temporal diameter of the surface of last scattering. Since the calculated values are essentially identical to the corresponding measured values, we conclude that the non-congruency of the ending point of the Riess *et al.* measurement and the starting point of the Planck Collaboration measurement, on the surface of last scattering, is the source of tension in the measurements. Further, the surprising consistency of the calculated values of the Hubble constant with the corresponding measured values confirms both the extreme fidelity of the measurements and the validity of the proposed approach.

Keywords

Cosmology, Hubble Constant, Cosmology: Observations

1. Introduction

Since the discovery of the expansion of the universe by Edwin Hubble [1], there have been considerable efforts in measuring the expansion rate. The refinements in these measurements have been continuing unabated, culminating in 2011 with the measurements that resulted in the discovery of the accelerating expansion, Perlmutter [2], Riess [3], Schmidt [4]. Ever since, many teams have continued their measurements of the Hubble constant, H_0. In particular the work of two teams, one in the US, Reiss, *et al.* [5] [6] [7] [8], Reid, *et al.* [9], and one in

Europe, by the Planck Collaboration, Aghanim, *et al.* [10], Ade, *et al.* [11] [12] has given rise to what is being called tension in the measurements of the Hubble constant. Reiss *et al.*, using the cosmic distance ladder approach, has measured values of the order of $H_0 = 73.48 \pm 1.66\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, while the Planck Collaboration, using CMB temperature fluctuations power spectra, has measured values of the order of $H_0 = 67.66 \pm 0.42\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. There are significant differences in these measured values. Other investigators consider the discrepancy in the measurements to be probably due to systematic errors but in spite of continuing improvements in the measurement methodology and precision, it has not been possible to explain the cause of this tension in the measurements of H_0 via the standard ΛCDM model. There are discussions of needs for possibly new physics, Sutter [13], Greene and Perlmutter [14], Freedman [15].

In this paper we present a new alternative approach that does not depend on the standard ΛCDM model and its constraints. Our goal is to analytically explore and show the source of the Hubble tension. We first use Planck's radiation law, Goldin [16], Anderson [17], to calculate the total energy radiated as photons from the surface of last scattering. Then we calculate the total input energy by summing up the quanta of energies of the waves emitted at the big bang. We consider this total input energy to be equal to the total energy of the photons released at the surface of last scattering. Based on the conservation of energy, this equality of total energies yields a quadratic equation in terms of the Hubble time, t_{H_0}. The solution of this equation yields two values for the Hubble time, t_{H_0}, resulting in the following two values for the Hubble constant, $H_{01} = 73.23\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ and $H_{02} = 68.56\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. These calculated values of H_{01} and H_{02} are remarkably consistent with the corresponding reported measured values by Reiss *et al.* and by the Planck Collaboration. Based on these consistencies we conclude a geometric mismatch to be the cause of the tension between the two cited measured values. The mismatch is due to the non-congruency of the ending point of measurement by Reiss *et al.* and the starting point of measurement by the Planck collaboration, on the surface of last scattering.

In Section 2, we evaluate the total energy of the photons using Planck's radiation law. In Section 3 we evaluate the total energy of emitted waves. In Section 4 we calculate the two values for the Hubble constant. Discussion and conclusions are presented in Sections 5 and 6.

2. Evaluation of Total Energy of Scattered Photons Using Planck's Radiation Law

Spectral radiance represents power per steradian per cubic meter. It is defined by Planck's law as,

$$B(\lambda,T) = \frac{2hc^2}{\lambda^5}\left(\mathrm{e}^{\frac{hc}{\lambda kT}-1}\right)^{-1}, \tag{1}$$

where c is the speed of light, $h = 6.62607 \times 10^{-34}\ \mathrm{J \cdot K^{-1}}$ is the Planck's constant, k is the Boltzmann's constant, T represents the temperature at the recombination era, and λ represents the wavelength. A plot of the above relation is presented in **Figure 1** for $T = 3000\ \mathrm{K}$.

Integrating the spectral radiance as defined in Equation (1) over the wavelength from $\lambda = \lambda_P$ to $\lambda = 10^{-5}\ \mathrm{m}$ yields,

$$U(\lambda, T) = \int_{\lambda_P}^{10^{-5}} \frac{2hc^2}{\lambda^5} \left(\mathrm{e}^{\frac{hc}{\lambda kT}} - 1 \right)^{-1} \mathrm{d}\lambda = 1.45511 \times 10^{6}\ \mathrm{W \cdot sr^{-1} \cdot m^{-2}}. \tag{2}$$

In the above relation the wavelength, λ_P, is represented by the Planck length, that is, $\lambda_P = L_P = 1.61623 \times 10^{-35}\ \mathrm{m}$, and $U(\lambda, T)$ represents power per steradian per unit area of the surface of last scattering. It should be noted that we have used $\lambda = 10^{-5}\ \mathrm{m}$, as the cutoff point, for the evaluation of $U(\lambda, T)$ at the present time. If we use a larger value for the cutoff point, for example $\lambda = 10^{-4}\ \mathrm{m}$ or larger, the calculated values of the Hubble constants remain essentially the same.

The surface of last scattering is presumed to have a radius given by

$$r_0 = c\left(t_0 - t_{H_0}\right) = ct_0 \left(1 - \frac{t_{H_0}}{t_0}\right). \tag{3}$$

It should be noted that r_0 defines the radius of the surface of last scattering in the recombination era when photons were becoming free and that it does not represent the distance from the earth to the surface of last scattering. Later we will show the dependence of r_0 and the Hubble parameter on the cosmological redshift and on the age of the universe, at the recombination era, which is assumed to be $\Delta t = 370000$ years. In the above equation t_0 and t_{H_0} represent the age of the universe and the Hubble time, respectively, at the present-time. Thus, considering the surface of last scattering to be the surface of a sphere, the total energy, E_1, of the photons released at the surface of last scattering is given by

$$E_1 = U(\lambda, T)\left(4\pi r_0^2\right)\pi \Delta t = U(\lambda, T)\left(4\pi \left(ct_0\right)^2 \left(1 - \frac{t_{H_0}}{t_0}\right)^2\right)\pi \Delta t. \tag{4}$$

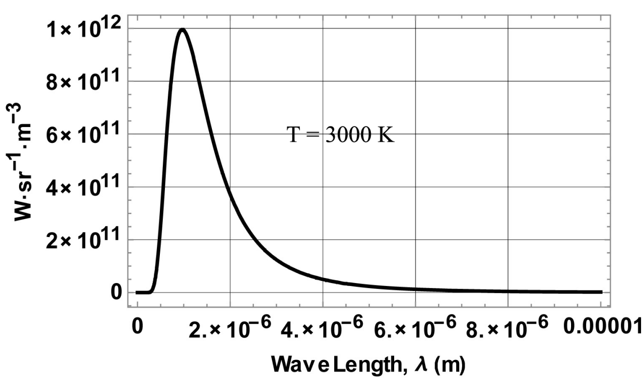

Figure 1. Spectral radiance/wavelength.

3. Evaluation of the Total Energy of Emitted Waves

In Equation (4) above, E_1 represents the total energy of photons released from the surface of last scattering at the time when the universe had cooled down to a temperature of about 3000 K (corresponding to $z = z_s = 1100$). We consider E_1 to be the output energy. The total input energy is supplied by the energy of the waves emitted at the big bang. To calculate the input energy, we assume that the shortest wavelength, λ_e, of waves emitted at the big bang is equal to the Planck length, L_P. The corresponding wavelength in the recombination era is given as

$$\lambda_o = (1 + z_s)\lambda_e, \tag{5}$$

where z represents the cosmological redshift, Simionat [18]. The value of $z = z_s = 1100$ represents the value of the redshift at the surface of last scattering. It corresponds to the temperature, T, about 3000 K at the recombination era, Fixsen [19]. The number of "observed" waves, n_o, and the number of emitted waves, n_e, are calculated as follows:

$$n_o = \frac{ct_0}{2\lambda_o} = \frac{ct_0}{2(1 + z_s)\lambda_e} \tag{6}$$

$$n_e = \frac{ct_0}{2\lambda_e}, \tag{7}$$

and the wavelength of the n^{th} wave is defined by

$$\lambda_n = \frac{ct_0}{n}, \tag{8}$$

which yields the period of the n^{th} wave as

$$p_n = \frac{\lambda_n}{c} = \frac{t_0}{n}. \tag{9}$$

Considering each wave to be associated with an oscillator, according to Planck each oscillator can absorb or emit a quantum of energy given by

$$\Delta E_n = \frac{h}{p_n}. \tag{10}$$

Thus the total energy emitted by all these oscillators is given by

$$E_2 = \sum_{n_o}^{n_e} \Delta E_n = \sum_{n_o}^{n_e} \frac{h}{p_n} = \frac{h}{t_0}\sum_{n_o}^{n_e} n. \tag{11}$$

4. Evaluation of the Hubble Constants

Considering the principle of conservation of energy, we equate the total output energy, E_1 to the total input energy, E_2. This equality of total input and total output energies yields the following quadratic equation:

$$U(\lambda, T)\left(4\pi(ct_0)^2\left(1 - \frac{t_{H_0}}{t_0}\right)^2\right)\pi\Delta t = \frac{h}{t_0}\sum_{n_o}^{n_e} n, \tag{12}$$

the solutions of which are

$$\frac{t_{H_0}}{t_0} = 1 \pm \beta , \tag{13}$$

where β is defined by

$$\beta = \sqrt{\frac{h}{4\pi^2 (ct_0)^2 U(\lambda,T) t_0 \Delta t} \sum_{n_o}^{n_e} n} \quad . \tag{14}$$

According to the Planck's mission [10], the age of the universe is $t_0 = 13.787 \pm 0.02 \text{ Gy}$. Assuming the age of the universe to be $t_0 = 13.8 \text{ Gy} = 4.3549488 \times 10^{17} \text{ s}$ and the elapsed time between the instant of initiation of the big bang and the release of photons is $\Delta t = 370000 \text{ years}$, for the redshift $z = z_s = 1100$, we obtain the value of β to be

$$\beta = 0.032946 . \tag{15}$$

As seen in Equation (14), the value of β depends on Δt, the age of the universe at the recombination era. It also depends on the cosmological redshift, z, via the number n_o. Substitution of the above value for β into Equations (13) yields the two values for the Hubble time as

$$t_{H_{01}} = t_0 - \beta t_0 = 4.211 \times 10^{17} \text{ s} \tag{16}$$

$$t_{H_{02}} = t_0 + \beta t_0 = 4.498 \times 10^{17} \text{ s} . \tag{17}$$

Substitution for $\frac{t_{H_{01}}}{t_0}$ from Equation (16) for $\frac{t_{H_0}}{t_0}$ in Equation (3) yields the radius of the surface of last scattering as

$$r_0 = \beta c t_0 . \tag{18}$$

This relation implies that the term (βt_0) in Equations (16) and (17) represents the temporal radius of the surface of last scattering. Thus $t_{H_{01}}$ in Equation (16) is equal to the age of the universe minus the temporal radius of the surface of last scattering, while $t_{H_{02}}$ in Equation (17) is equal to the age of the universe plus the temporal radius of the surface of last scattering. These two Hubble times render the following two Hubble constants, respectively:

$$H_{01} = \frac{1}{t_{H_{01}}} \frac{\text{MegaParsec}}{1000} = 73.23 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \tag{19}$$

$$H_{02} = \frac{1}{t_{H_{02}}} \frac{\text{MegaParsec}}{1000} = 68.56 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \quad . \tag{20}$$

The above calculated value of H_{01} is remarkably consistent with the Reiss *et al.* reported measured value of $H_0 = 73.48 \pm 1.66 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. As well, the above calculated value of H_{02} is very much consistent with the Planck Collaboration reported measured value of $H_0 = 67.66 \pm 0.42 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. It should be noted that we assumed the age of the universe $t_0 = 13.8 \text{ Gy}$ based on trial and error such that the calculated values for both H_{01} and H_{02} would be consistent with the corresponding measured values. As shown in **Table 1**, the calculated values for the Hubble constants are sensitive to both the cosmic age of

Table 1. Sensitivity of the Hubble constants to cosmic age of the universe.

$km \cdot s^{-1} \cdot Mpc^{-1}$	$\Delta t = 370{,}000$ yr		$\Delta t = 400{,}000$ yr	
	$t_0 = 13.8$ Gyr	$t_0 = 14.0$ Gyr	$t_0 = 13.8$ Gyr	$t_0 = 14.0$ Gyr
H_{01}	73.23	72.17	73.20	72.14
H_{02}	68.56	67.60	68.59	67.63

the universe, Δt, in the era of recombination and to the cosmic age of the universe, t_0, at the present time. It appears that they are more sensitive to the changes in t_0 than to the changes in Δt. It is auspicious that the assumed age for the universe, t_0, is consistent with the age of the universe as calculated by the Plank's mission.

5. Discussion

The two calculated values of the Hubble constant are remarkably consistent with the corresponding measured values by Riess *et al.* and the Planck Collaboration, respectively. Based on these consistencies, we surmise that the two measured values are different because the measurement by Riess *et al.* is a one phase process. It uses the cosmic distance ladder approach, which involves looking back in time toward the instance of the birth of the universe at the big bang. Its measurement ends at a point on the surface of last scattering.

But the measurement by the Planck Collaboration is a two phase process. It uses the CMB temperature fluctuations power spectra. It starts its measurement from a point on the surface of last scattering. But, by the time of recombination, the universe has already aged by $\Delta t = 370000 \text{ years}$. Thus, because CMB is isotropic, the first phase of the Planck Collaboration measurement involves looking back in time from a point on the surface of last scattering toward the instance of the birth of the universe at the big bang. Then in its second phase of measurement it looks forward in time from the instance of the birth of the universe at the big bang toward the present time. That is why the Planck Collaboration measures the Hubble time as the cosmic age of the universe, t_0, plus the radius of the surface of last scattering, βt_0. Therefore there exists a temporal discontinuity equal to the temporal diameter of the surface of last scattering, $2\beta t_0$, between the Reiss *et al.* and the Planck Collaboration measurements. That is, the point at which the Reiss *et al.* measurement meets the surface of last scattering is temporally $2\beta t_0$ away from the point at which the Planck Collaboration starts its measurement at the surface of last scattering. This temporal discontinuity (or geometric mismatch), that is, the non-congruency of these ending and starting measurement points, is the cause of the tension between the two cited measured values.

6. Conclusion

The surprising consistency of the calculated values of the Hubble constant with the corresponding measured values confirms both the validity of the proposed approach and the extreme fidelity of the measurements. We conclude a geometric mismatch to be the cause of the tension between the two measurement methodologies. The mismatch is due to the non-congruency of the ending point of measurement by Reiss *et al.* and the starting point of measurement by the Planck collaboration, on the surface of last scattering. Further, as a direct consequence of the foregoing results, we conclude that improvements and refinements in measurement methodologies and precision, unless we consider the Hubble time to be the same as the age of the universe, cannot lead to finding a single-valued Hubble constant.

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Conflicts of Interest

The author states no conflict of interest.

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