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# Data-Driven Model Identification and Control of the Inertial Systems

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## Abstract

In the synthesis of the control algorithm for complex systems, we are often faced with imprecise or unknown mathematical models of the dynamical systems, or even with problems in finding a mathematical model of the system in the open loop. To tackle these difficulties, an approach of data-driven model identification and control algorithm design based on the maximum stability degree criterion is proposed in this paper. The data-driven model identification procedure supposes the finding of the mathematical model of the system based on the undamped transient response of the closed-loop system. The system is approximated with the inertial model, where the coefficients are calculated based on the values of the critical transfer coefficient, oscillation amplitude and period of the underdamped response of the closed-loop system. The data driven control design supposes that the tuning parameters of the controller are calculated based on the parameters obtained from the previous step of system identification and there are presented the expressions for the calculation of the tuning parameters. The obtained results of data-driven model identification and algorithm for synthesis the controller were verified by computer simulation.

## Keywords

Data-Driven Model Identification, Controller Tuning, Undamped Transient Response, Closed-Loop System Identification, PID Controller

## 1. Introduction

Almost all electrical drivers, power converters, technological processes in industrial applications are controlled based on the proportional-integral-derivative (PID) control algorithm [1] [2]. This algorithm is widely used in different industrial applications, due to its simplicity, feasibility and the advantages that offer to automatic control systems, such as good performance and robustness in case of uncertainties and disturbances.

One of the difficulties in automatic control systems is the problem of synthesis of the PID control algorithm, which supposes the procedure of calculation of the tuning parameters according to the dynamics of the control object [2] [3]. The incorrect tuning of the PID controller can lead to bad performance and in the worst case can lead to the instability of the system. In this case, ensuring the closed loop system stability is one of the most important aspects in the synthesis of the control algorithm [4] [5].

For the last decades have been developed a big variety of PID tuning techniques in continuous—time, discrete time and frequency domain [6] [7]. In general, these methods can be grouped into the following categories as: experimental tuning methods, analytical tuning methods and optimization tuning methods, which are based on the algorithms from artificial intelligence domain [6] [8].

The experimental methods do not require to be known preliminary the mathematical model that describes the dynamics of the control object. These methods are based on some simple assumptions for calculation of the tuning parameters. One of the most known and used experimental methods for tuning the typical controllers is Ziegler-Nichols method, which supposes the calculation of the tuning parameters based on the undamped transient response of the closed loop system and it does not require to be known the mathematical model of the control object [2] [9] [10] [11]. Due to its simplicity, it is used as self-tuning method of P, PI or PID controller, though this method is characterized by the drawbacks as:

- It is recommended for slow processes;
- It does not offer the procedure for synthesis of the controller with imposed performance;
- It provides the oscillating step response of the system;
- It does not take into account the requirements related to the system stability reserve;
- This method does not offer the procedure for optimization of the tuning parameters.

The analytical methods require to be known the mathematical model of the control object and based on some algorithms for calculation of the tuning parameters and graphical representation in the time, or frequency domain, it can offer the designed closed loop system satisfactory performance and good robustness [12] [13] [14]. The disadvantages of these methods are the necessity of using a big volume of calculations and the mathematical model of the control object should be given, or should be obtained based on the identification procedure.

Nowadays, there are developed many identification methods and procedures of the mathematical model, allowing with high accuracy to identify different structures of object models with or without time delay, with high or low order inertia. Most of these methods offer procedures for approximation of the dynamics of control object with transfer function with first and second-order inertia and these methods in most cases are applied in the open loop systems [2] [15] [16] [17]. The existence of the procedure, which will permit to obtain the mathematical model of the control object in the closed loop system essentially will simplify the procedure of tuning the typical controllers [18].

Another category of tuning techniques is related to optimization methods and artificial intelligence approaches such as genetic algorithms, fuzzy logic control, neural networks [18]. These approaches are widely used in the case of optimization problems of tuning parameters, especially for the case, when it is needed to control the nonlinear process and offer good performance and robustness of the system. However, these methods usually involve operator implication in formation the initial data set and hardly can be realized as self-tuning methods of the controller.

This paper provides a new method for experimental identification, inspired by the Ziegler and Nichols tuning method when the closed loop control system is marginally stable. The new procedure of tuning the PID controller ensures to the system critically damped step response.

Hereinafter, Section 2 introduces the basic principles of the PID controller and tuning methods. Section 3 presents the algorithm for mathematical model identification of the control object in the closed loop system and it is exposed the method for tuning the PID controller. Section 4 presents the study case of identification the mathematical model of the DC motor, and the study case of synthesis of the PID controller.

## 2. Basic Principles of the PID Control Algorithm and Tuning Methods

### 2.1. PID Control Algorithm

The PID control algorithm is widely used in different industrial applications and it can be easily implemented for various control problems. The PID controller as input receives the error signal $e(t)$ and provides command signal—$u(t)$, where the typical structure of the PID controller is given by the transfer function [8]:

$$H_{\text{PID}}(s) = k_p + \frac{k_i}{s} + k_d s, \tag{1}$$

where $k_p$—is the proportional tuning parameter, $k_i$—integral tuning parameter, $k_d$—derivative tuning parameter of the PID controller.

In **Figure 1**, it is presented the structural scheme of the automatic control system.

In **Figure 1**, $H_{\text{PID}}(s)$ is the transfer function of the PID controller, described by the transfer function (1) and $H(s)$ is the transfer function of the control object:

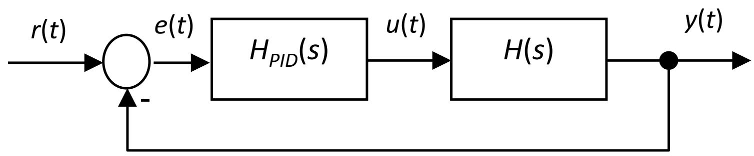


**Figure 1.** Structural scheme of the automatic control system.

$$H(s) = \frac{k}{(T_1 s+1)(T_2 s+1)\cdots(T_n s+1)} = \frac{k}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} = \frac{B(s)}{A(s)}, \quad (2)$$

where $T_0, T_1, \cdots, T_{n-1}, T_n$ are time constants; $a_0, a_1, \cdots, a_{n-1}, a_n$ are the parameters of the characteristic equation and depend on the internal proprieties of the control object; $k$ is the static gain; $n$ is the order of the characteristic equation $A(s)$.

The static gain of the system is calculated [2]:

$$k = \lim_{t\to\infty} \frac{\Delta y}{\Delta u} = \lim_{t\to\infty} \frac{y_{st} - y_{\text{initial}}}{u - u_{\text{initial}}}, \quad (3)$$

where $y_{st}$ is the steady-state output, $y_{\text{initial}}$ is the initial value of the output, $u(t)$—input signal, $u_{\text{inital}}$ is the initial value of the input signal.

In the **Figure 1**, the $r(t)$ is the reference signal, $y(t)$—output signal, $u(t)$—command signal and $e(t)$—error signal:

$$e(t) = r(t) - y(t). \quad (4)$$

According to the Equation (1) and Equation (2) the characteristic equation of the closed loop system with PID controller can be presented by:

$$A(s) = \frac{1}{k}\left(a_0 s^{n+1} + a_1 s^n + \cdots + a_{n-1}s^2 + a_n s\right) + k_d s^2 + k_p s + k_i. \quad (5)$$

## 2.2. Tuning Methods

Next, there are analyzed two methods of tuning the PID controller:

- The experimental method of tuning the PID controller—Ziegler—Nichols method;
- Graph-analytical method—maximum stability degree method with iterations.

### 2.2.1. Ziegler-Nichols Method

The Ziegler-Nichols method for tuning typical controllers from 1942, when it was proposed for the first time, has a big impact on linear feedback control systems and in implementation of self-tuning methods of typical controllers in different industrial applications [10]. The procedure for tuning the PID controller according to the Ziegler-Nichols method, for now, is accepted as a standard in control system practices. Ziegler and Nichols proposed two empirical methods for tuning the typical controllers. The first method is supposed, that the process is aperiodic and it can be obtained the transient response in the open loop and that is approximated with transfer function with inertia first order and time delay. According to this approximation, there are given the analytical expressions for the calculation of the tuning parameters of the P, PI and PID controllers [2] [9] [10] [11].

The second method doesn't require to be known the mathematical model of the control object and it offers the rules for determination in experimental way the tuning parameters. This method of tuning is performed for the closed loop systems with PID controller (**Figure 1**), when the integral tuning parameter—$k_i$ and derivative tuning parameter—$k_d$, are settled to the zero value. The proportional tuning parameter—$k_p$ is slowly increased until the system reaches the limit of stability and it determinates the period of the un-amortized oscillations $T_{cr}$ of the system. The value of the proportional tuning parameter, when the system achieves the limit of stability is the critical transfer coefficient—$k_{cr}$. Based on these two parameters $T_{cr}$ and $k_{cr}$, there are given the expressions for calculations of the tuning parameters of the P, PI and PID controllers (**Table 1**).

### 2.2.2. Maximal Stability Degree Method with Iterations

One of the analytical methods, which permit tuning the P, PI and PID controllers and it is taken into accounts the stability degree of the system is the maximum stability degree method with iterations [17].

**Affirmation 1.** If the coefficients of the characteristic equation are known and constant, then the stability degree of the system $J$ obtains the maximum possible value by the [19]:

$$J = \frac{a_1}{(n+1)a_0}, \tag{6}$$

and in this case the real parts of all characteristic equation roots are equal with each other:

$$\alpha_1 = \alpha_2 = \cdots = \alpha_n = J, \tag{7}$$

where $\alpha_1, \alpha_2, \cdots, \alpha_n$ are real parts of characteristic equation roots $p_i = -\alpha_i \pm j\omega_k$, $i,k = \overline{1,n+1}$, $k = \overline{1,m}$.

For the case when the number of the tuning parameters is equal or less then the characteristic equation order, the value of maximum stability degree value can be chosen as arbitrary value and the **1st Affirmation** will not be valid.

In concordance with the **1st Affirmation**, it is possible to be found the tuning parameters of the PID controller so as to be satisfied the maximum stability degree of the system:

$$J = \eta_{\max} = \max \eta\left(k_p, k_i, k_d\right). \tag{8}$$

**Table 1.** Expressions for calculation the tuning parameters of the P, PI and PID controllers [2].

| Type of controller | $k_p$ | $k_i$ | $k_d$ |
|---|---|---|---|
| P | 0.5 $k_{cr}$ | - | - |
| PI | 0.45 $k_{cr}$ | 1/(0.8 $T_{cr}$) | - |
| PID | 0.75 $k_{cr}$ | 1/(0.6 $T_{cr}$) | 0.1 $T_{cr}$ |

According to the maximum stability degree method with iterations, in the characteristic Equation (5) of the closed loop system with PID controller is introduced the substitution $s = -J$, and the characteristic equation becomes [19]:

$$A(-J) = \frac{1}{k}\left(a_0(-J)^{n+1} + a_1(-J)^n + \cdots + a_{n-1}J^2 - a_n J\right) + k_d J^2 - k_p J + k_i. \quad (9)$$

The Equation (9) derives two times and there are obtained the analytical expressions for calculating the tuning parameters of the PID controller:

$$k_p = \frac{1}{k}\left((-1)^{n+1}(n+1)a_0 J^n + (-1)^n n a_1 J^{n-1} + \cdots + 2a_{n-1}J - a_n\right) + 2k_d J; \quad (10)$$

$$k_i = \frac{1}{k}\left((-1)^n a_0 J^{n+1} - (-1)^n a_1 J^n + \cdots - a_{n-1}J^2 + a_n J\right) - k_d J^2 + k_p J; \quad (11)$$

$$k_d = \frac{1}{2k}\left((-1)^n n(n+1)a_0 J^{n-1} - (-1)^n n(n-1)a_1 J^{n-2} + \cdots - 2a_{n-1}\right). \quad (12)$$

The analytical expressions (10)-(12) permit to calculate the tuning parameters of the PID controller and they represent the dependences of the known parameters of the control object $k$, $a_0, a_1, \cdots, a_n$ and an unknown parameter $J$—stability degree of the system: $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$. According to the 1st **Affirmation**, if the degree of the system is higher than 2nd order, the maximum stability degree value can be calculated based on the Equation (6) [19].

Otherwise, according to the maximum stability degree method with iterations, the value of the stability degree can be varied $J \geq 0$ and based on the dependencies $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$, there are obtained different values of the tuning parameters of the PID controller, that offer the different performance for the automatic control system.

## 3. Synthesis of the PID Control Algorithm Based on the Undamped Transient Response of the Closed Loop System

The procedure for synthesis of the PID controller supposes to be known the mathematical model that approximates the dynamics of the control object, this implies to be used the procedure of experimental identification. The new tuning algorithm of the PID controller is presented in **Figure 2**.

### 3.1. Data-Driven System Identification

In order to solve the problem of mathematical identification in the closed loop system, it is proposed the procedure for experimental identification, based on the undamped transient response of the closed loop control system. According to this procedure, the dynamics of the control object is proposed to be approximated with transfer function with inertia third order:

$$H(s) = \frac{k}{(T_1 s+1)(T_2 s+1)(T_3 s+1)} = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} = \frac{B(s)}{A(s)}, \quad (13)$$

where $T_1, T_2, T_3$ are time constants; $k$ is transfer coefficient, that is calculated according to the Equation (3); $a_0 = T_1 T_2 T_3$, $a_1 = T_1 T_2 + T_1 T_3 + T_2 T_3$, $a_2 = T_1 + T_2 + T_3$, $a_3 = 1$.

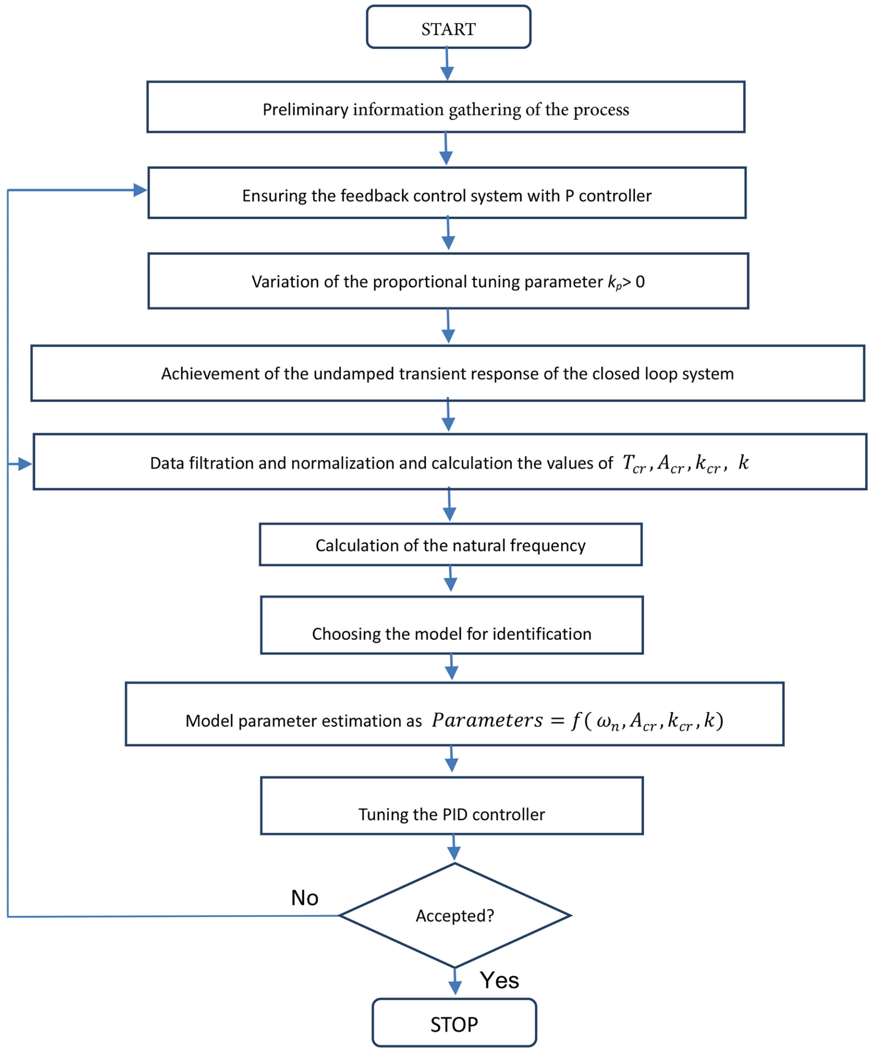


**Figure 2.** The algorithm of data-driven model identification and control.

The characteristic equation of the transfer function (13) is following:

$$A(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3. \tag{14}$$

The coefficients $a_0$, $a_1$, $a_2$, $a_3$ are proposed to be calculated based on the values of the $k_{cr}$—critical transfer coefficient, $A_{cr}$—amplitude of unamortized oscillations and $T_{cr}$—period of the unamortized oscillations, that are obtained for the case, when automatic control system gains the limit of stability. To achieve this state of the system as in the Ziegler-Nichols method, it is given the closed loop system with PID controller (**Figure 1**), the coefficients $k_i$, $k_d$ are settled to the zero, and the proportional tuning parameter $k_p > 0$ is varied, until the system achieves the limit of stability.

If it is known the value of period oscillations, it can be calculated the natural frequency by the [2]:

$$\omega_n = \frac{2\pi}{T_{cr}}. \tag{15}$$

In this way, it is proposed to find the dependencies for the calculation of the parameter of the control object in an experimental way, which depends on the natural frequency, oscillation amplitude and critical transfer coefficient:

$$a_0, a_1, a_2, a_3 = f\left(\omega_n, A_{cr}, k_{cr}\right). \tag{16}$$

The characteristic equation of the closed loop system (13) with critical transfer coefficient is the following:

$$A(s) = \frac{1}{k}\left(a_0 s^3 + a_1 s^2 + a_2 s + a_3\right) + k_{cr}. \tag{17}$$

In case, when the control system achieves the limit of stability, it has the poles placed on the imaginary axe, with real parts equal to zero and it is proposed to make the following substitution in the characteristic Equation (17):

$$s = j\omega_n. \tag{18}$$

Based on the substitution (18), the characteristic Equation (17) will become:

$$\begin{aligned} A(j\omega) &= \frac{1}{k}\left(a_0 (j\omega_n)^3 + a_1 (j\omega_n)^2 + a_2 (j\omega_n) + a_3\right) + k_{cr} \\ &= \left(-a_1\omega_n^2 + a_3 + k_{cr}k\right) + j\left(-a_0\omega_n^3 + a_2\omega_n\right) \\ &= P(\omega) + jQ(\omega). \end{aligned} \tag{19}$$

Next, it is proposed to equal the real and imaginary parts with zero—$P(\omega) = 0$ and $Q(\omega) = 0$, and based on these equalling there are obtained the following expressions:

$$\begin{cases} \omega_n = \sqrt{\dfrac{a_2}{a_0}}; \\ a_1 = \dfrac{k_{cr}k + 1}{\omega_n^2}. \end{cases} \tag{20}$$

The transfer function of the control object (13) can be rewritten in the following form [2]:

$$H(s) = \frac{\alpha\omega_n^2}{(s+\alpha)\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)} = \frac{Y(s)}{X(s)}, \tag{21}$$

where $\xi$ is a damping ratio.

In this case, the unit—step response of the system is given by:

$$Y(s) = \frac{1}{s}H(s), \tag{22}$$

$$y(t) = 1 + C_1 e^{-\alpha t} + C_2 e^{-\xi\omega_n t}\sin\left(\omega_d - \theta\right), \tag{23}$$

where

$$C_1 = -\frac{\omega_n^2}{\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2}, \tag{24}$$

$$C_2 = -\frac{\alpha}{\sqrt{\left(\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2\right)\left(1-\xi^2\right)}}, \tag{25}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{-\xi} + \tan^{-1}\frac{\omega_n\sqrt{1-\xi^2}}{\alpha - \xi\omega_n}, \tag{26}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}. \tag{27}$$

In case, when system achieves the limit of stability there is $\xi = 0$ and the transfer function (21) becomes:

$$H(s) = \frac{\alpha\omega_n^2}{(s+\alpha)(s^2+\omega_n^2)} = \frac{B(s)}{A(s)}. \tag{28}$$

The characteristic equation of the transfer function (28):

$$A(s) = (s+\alpha)(s^2+\omega_n^2)$$

has three poles— $s_1 = -\alpha, s_2 = j\omega_n, s_3 = -j\omega_n$.

According to the Vieta's theorem for calculation the roots of the cubic equation, it is obtained the following dependence:

$$\alpha = \frac{a_1}{a_0}. \tag{29}$$

where $a_0, a_1$ —are parameters of the characteristic Equation (14).

The value of the amplitude of the unamortized oscillations from the undamped transient response can be calculated according to the step response of the system (23). There is obtained the following expression for calculation the value of oscillations amplitude:

$$A_{cr} = \frac{\alpha}{\sqrt{\alpha^2+\omega_n^2}}. \tag{30}$$

Thus, then it is known the value of the oscillation amplitude, it is possible to calculate the value of *α*:

$$\alpha = \frac{A_{cr}\omega_n}{\sqrt{1-A_{cr}^2}}. \tag{31}$$

In concordance with Equations (20), (29) and (31) the control object's parameters are calculated according to the following expressions:

$$\begin{cases} a_0 = \dfrac{a_1}{\alpha}; \\ a_1 = \dfrac{k_{cr}k+1}{\omega_n^2}; \\ a_2 = a_0\omega_n^2; \\ a_3 = 1. \end{cases} \rightarrow \begin{cases} a_0 = \dfrac{(k_{cr}k+1)\sqrt{1-A_{cr}^2}}{A_{cr}\omega_n^3}; \\ a_1 = \dfrac{k_{cr}k+1}{\omega_n^2}; \\ a_2 = \dfrac{(k_{cr}k+1)\sqrt{1-A_{cr}^2}}{A_{cr}\omega_n}; \\ a_3 = 1. \end{cases} \tag{32}$$

Based on Equations (32), the parameters of the control object can be calculated in dependency of the natural frequency, oscillation amplitude and critical transfer coefficient, that are obtained from the undamped transient response of the closed loop system. In this way, the procedure of identification of the mathematical model can be performed in the closed-loop system.

### 3.2. Data-Driven Control Algorithm Synthesis

It is considered, that control process is described by the transfer function with inertia third order—Equation (13) and the characteristic equation is the following:

$$A(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3. \tag{33}$$

In this case, the system is characterized by the three poles—$s_1$, $s_2$, $s_3$, that can be calculated based on the Vieta's theorem for the calculation the roots of the cubic equation:

$$\begin{cases} s_1 + s_2 + s_3 = -\dfrac{a_1}{a_0}; \\ s_1 s_2 + s_1 s_3 + s_2 s_3 = \dfrac{a_2}{a_0}; \\ s_1 s_2 s_3 = -\dfrac{a_3}{a_0}. \end{cases} \tag{34}$$

The dominant poles are the poles, that are situated more closely to the imaginary axe, and according to the [14], the sum of dominant poles from characteristic Equation (33) is:

$$\Sigma_{\text{dom\_pol}} = s_1 + s_2 \approx -\frac{\dfrac{a_1}{a_0} - \dfrac{a_3}{a_0}}{2}, \tag{35}$$

where $s_1$ and $s_2$ are the dominant poles.

The characteristic Equation (33) has three poles and if it is known the sum of dominant poles (35), based on the Vieta's theorem, it is possible from the system (34) to calculate the production of the dominant poles, that will be equal with:

$$\Pi_{\text{dom\_pol}} = s_1 s_2 \approx -\frac{2a_3}{a_1 + a_3}. \tag{36}$$

The transfer function of the PID controller is given by Equation (1) and in conformity with Vieta's theorem for the second order equation, it is imposed that the zeros of the transfer function of the PID controller to be equal with dominant poles of the control object:

$$\begin{cases} s_1 + s_2 = -\dfrac{k_p}{k_d}; \\ s_1 s_2 = -\dfrac{k_i}{k_d}. \end{cases} \tag{37}$$

Based on Equations (35) and (36) the system of Equations (37) can be rewritten as:

$$\begin{cases} k_p = \dfrac{a_1 - a_3}{2a_0} k_d; \\ k_i = \dfrac{2a_3}{a_1 + a_3} k_d. \end{cases} \tag{38}$$

From (38), it can be observed that tuning parameters for P and I components depend on the parameter of the control process (13), which are known and derivative component that is unknown:

$$k_p, k_i = f\left(k_d, a_0, a_1, a_2, a_3\right). \quad (39)$$

It is proposed to use the maximum stability degree method with iterations [20], in order to obtain the analytical expression for the calculation of the derivative tuning parameter.

Next, based on the maximum stability degree method with iterations, there is obtained the analytical expression for calculation the $k_d$ tuning parameter of the PID controller:

$$k_d = \frac{1}{2k}\left(-12a_0J^2 + 6a_1J - 2a_2\right), \quad (40)$$

where $J$ is the stability degree.

The value of the stability degree is calculated according to the expression [20]:

$$J = \frac{a_1}{4a_0}. \quad (41)$$

Based on Equations (40) and (41), the expression for calculation the $k_d$ tuning parameter is:

$$k_d = \frac{3a_1^2 - 8a_0a_2}{8ka_0}. \quad (42)$$

In this way, the tuning parameters of the PID controller for the case then control process is described by the transfer function with inertia third order are the following:

$$\begin{cases} k_p = \Sigma_{\text{dom\_pol}} \cdot k_d; \\ k_i = \Pi_{\text{dom\_pol}} \cdot k_d; \\ k_d = \dfrac{3a_1^2 - 8a_0a_2}{8ka_0}. \end{cases} \rightarrow \begin{cases} k_p = \dfrac{a_1 - a_3}{2a_0}k_d; \\ k_i = \dfrac{2a_3}{a_1 + a_3}k_d; \\ k_d = \dfrac{3a_1^2 - 8a_0a_2}{8ka_0}. \end{cases} \rightarrow \begin{cases} k_p = \dfrac{\left(a_1 - a_3\right)\left(3a_1^2 - 8a_0a_2\right)}{16ka_0^2}; \\ k_i = \dfrac{3a_1^2a_3 - 8a_0a_2a_3}{4ka_0\left(a_1 + a_3\right)}; \\ k_d = \dfrac{3a_1^2 - 8a_0a_2}{8ka_0}, \end{cases} \quad (43)$$

where the parameters $a_0, a_1, a_2$ are known and there are calculated according to the equations (32).

## 4. Study Cases

### 4.1. Identification of the Mathematical Model of the DC Motor

In automatic control systems, the DC motor is frequently used as an actuator element.

Next, it is done the experimental identification of the mathematical model of the DC motor and in this study it was chosen the 2342L012 series Coreless encoder motor with technical parameters [20]:

- $J$ rotor inertia, equal to 5.7 gcm$^2$;

- $k_e$ back EMF constant, equal to 1.4 mV/min;
- $k_m$ torque constant, equal to 13.4 mNm/A;
- $R$ terminal resistance, equal to 1.9 Ω;
- $k_n$ speed constant, equal to 713 min$^{-1}$/V;
- $L$ rotor inductance, equal to 65 μH.

The input and output values of the DC motor are:

- Input value $U_a$: supply voltage;
- Output value $\dot{\theta} = \omega$: rotor shaft speed;
- Output value $\theta$: rotor shaft position.

The transfer function in the open loop, in which the control value is the rotational speed of the rotor shaft and the input is a voltage applied to the armature windings $U_a$ is:

$$\frac{\dot{\theta}(s)}{U_a(s)} = \frac{k_m}{(Ls+R)(Js+b)+k_e k_m}. \tag{44}$$

According to the technical parameters, the analytical model of the DC motor is:

$$H(s) = \frac{\dot{\theta}(s)}{U_a(s)} = \frac{641.885}{0.00000177s^2 + 0.052s + 1}. \tag{45}$$

Based on the proposed procedure of identification in the closed loop, the automatic control system was brought to the critically regime and the experimental curve is presented in **Figure 3**.

The presented data were normalised and based on the undamped transient response of the closed loop system, there are obtained the following parameters:

$$k_{cr} = 5,\ A_{cr} = 0.1185,\ T_{cr} = 0.0011\ \text{s},\ \omega_n = 5712.$$

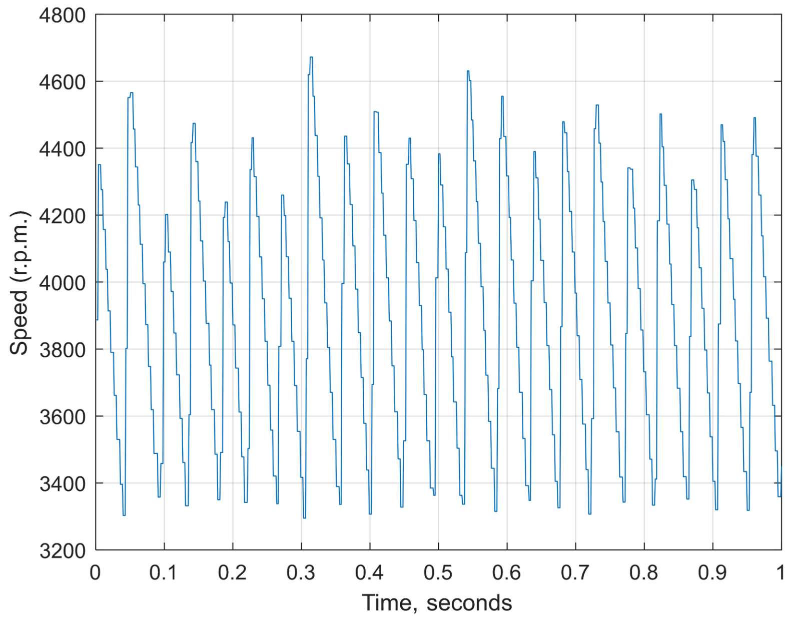


**Figure 3.** Undamped transient response of the system.

Based on Equation (32) there are calculated the parameters of the control object:

$$a_0 = 2.698\text{e}-10,\ a_1 = 1.839\text{e}-07,\ a_2 = 0.008802,\ a_3 = 1.$$

The identified transfer function is the following:

$$H(s) = \frac{1}{2.698\text{e}-10s^3 + 1.839\text{e}-07s^2 + 0.008802s + 1} = \frac{B(s)}{A(s)}. \quad (46)$$

Next in **Figure 4**, it is presented the comparison between the step response of the model of object analytical identified (transfer function (45))—curve 1; step response of the model of object identified by the proposed method (transfer function (46))—curve 2; the experimental curve of speed motor variation at the 5800 rpm—curve 3.

### 4.2. PID Control Algorithm Synthesis

It is supposed that control process is described by the transfer function:

$$H(s) = \frac{1}{(2s+1)(5s+1)(10s+1)} = \frac{1}{100s^3 + 80s^2 + 17s + 1} = \frac{B(s)}{A(s)}, \quad (47)$$

The automatic control system with P controller was simulated and it was obtained the undamped transient response of the closed-loop system and according to the identification algorithm there are obtained the following parameters of the model:

$$k_{cr} = 12.5993,\ T_{cr} = 15.2394\ \text{s},\ A_{cr} = 0.874,\ \omega_n = 0.4123.$$

According to the obtained values—the natural frequency, oscillation amplitude, critical transfer coefficient and based on the Equations (32), there are calculated the parameters of the model:

$$\begin{cases} a_0 = \dfrac{(k_{cr}k+1)\sqrt{1-A^2}}{A\omega_n^3} = 108.7; \\ a_1 = \dfrac{k_{cr}k+1}{\omega_n{}^2} = 80.43; \\ a_2 = \dfrac{(k_{cr}k+1)\sqrt{1-A^2}}{A\omega_n} = 18.39; \\ a_3 = 1. \end{cases} \quad (48)$$

Thus, the data-driven identified model is the following:

$$H(s) = \frac{1}{108.7s^3 + 80.43s^2 + 18.39s + 1} = \frac{B(s)}{A(s)}. \quad (49)$$

Next, it is done the comparison between original step response of the control object and step response of the identified model, that are presented in **Figure 5**.

Next, to the model of object (49) is tuned the PID controller based on Equations (43):

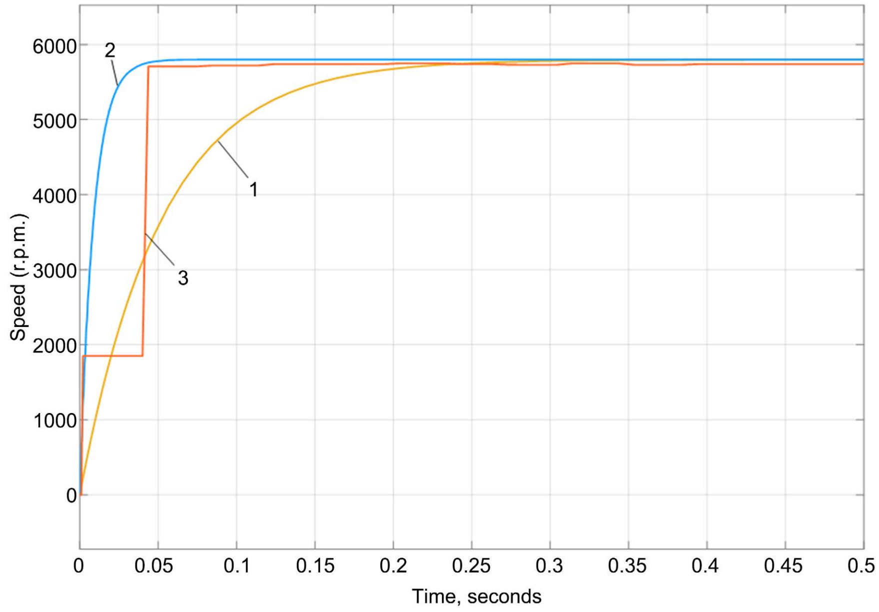


**Figure 4.** Comparison of the DC motor output: 1—step response of the system with transfer (45); 2—step response of the system with transfer function (46); 3—experimental curve of DC motor speed variation.

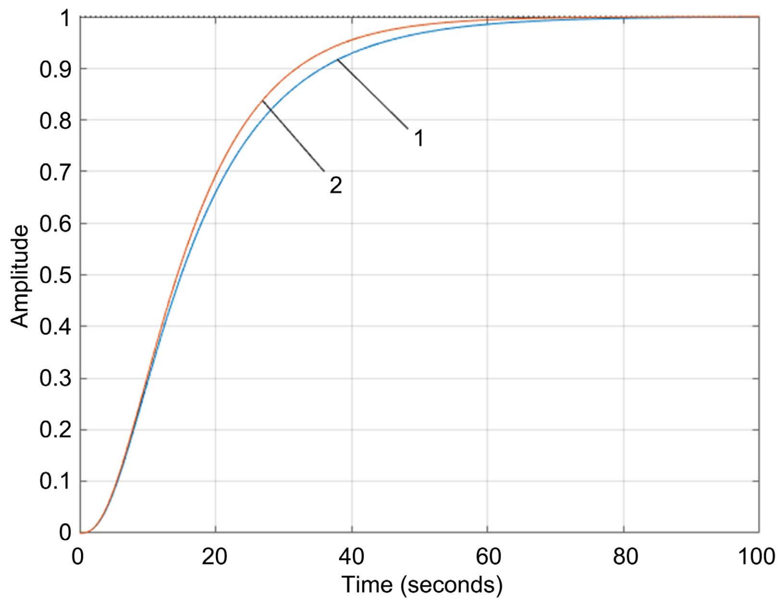


**Figure 5.** Comparison of the step responses of the control object: 1—step response of the system with transfer (47); 2—step response of the identified model (49).

$$\begin{cases} k_p = \dfrac{a_1 - a_3}{2a_0} k_d = 3.9193; \\ k_i = \dfrac{2a_3}{a_1 + a_3} k_d = 0.0963; \\ k_d = \dfrac{3a_1^2 - 8a_0 a_2}{8ka_0} = 1.4313. \end{cases}$$

The obtained results of tuning the PID controller were compared with maximum stability degree method with iterations (MSMI), where the tuning parameters were calculated according to the relations (10)-(12). For comparison, it was used the Ziegler-Nichols, where the tuning parameters were calculated according to **Table 1** and it was used the parametrical optimization, based on the PID Tuner Toolbox from MATLAB. The obtained tuning parameters and obtained performances of the automatic control system ($t_r$—rise time, $t_s$—settling time, $\sigma$—overshoot) are presented in **Table 2**.

In **Figure 6**, there is presented the computer simulation of the automatic control system with PID controller tuned by: the proposed method in this paper—curve 1, maximum stability degree method with iterations—curve 2, Ziegler—Nichols method—curve 3, parametrical optimization—curve 4.

From **Figure 6**, it was observed that in case of using the proposed method for

**Table 2.** Tuning parameters and the performance of the automatic control system.

| No. | Tuning method | $k_p$ | $k_i$ | $k_d$ | $t_s$ | $t_r$ | $\sigma$, % |
|---|---|---|---|---|---|---|---|
| 1 | MSD method with dominant poles allocation | 1.43 | 0.096 | 3.91 | 30.5 | 30.5 | 0.0 |
| 2 | MSMI | 1.74 | 0.12 | 3.91 | 37.57 | 18.72 | 3.33 |
| 3 | Ziegler-Nichols | 9.44 | 0.109 | 1.52 | 171.79 | 6.55 | 56.75 |
| 4 | Parametrical optimization | 3.29 | 0.245 | 9.64 | 10.98 | 27.5023 | 6.39 |

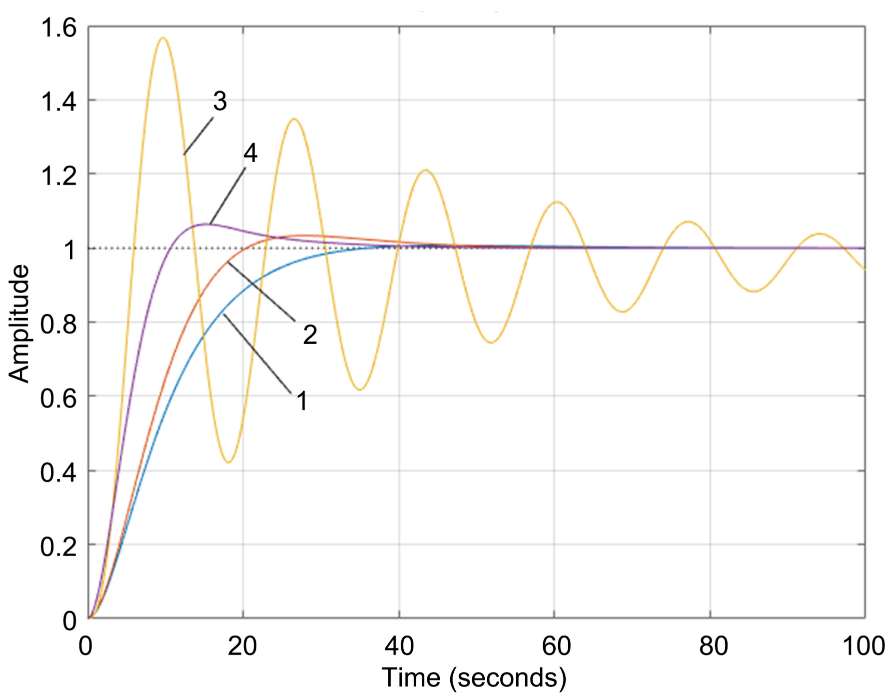


**Figure 6.** Transient responses of the automatic control system: 1—PID controller tuned by MSD method with dominant poles allocation; 2—PID controller tuned by MSMI; 3—PID controller tuned by Ziegler-Nichols method; 4—PID controller tuned by parametrical optimization.

tuning the PID controller, it is obtained the critically damped step response without overshoot. In the case of using the Ziegler-Nichols method, it is obtained oscillated transient response. In the case of using MSMI and parametrical optimization method, there are obtained the underdamped transient responses of the automatic control system.

## 5. Conclusions

Synthesis of the control algorithm is one of the main problems that concerned engineers during the design of automatic control systems. The solution to this problem supposes to be known some key parameters of the control process, or the mathematical model, that approximates the dynamics of the control process. Frequently the mathematical model of the control process is obtained in the experimental, or analytical way in the open loop. This implies being involved with the operator for system identification in the open loop. The Ziegler-Nichols method is become so used, due to the fact that does not require to be known the mathematical model of the control process and the procedure of tuning the controller supposes the extraction of some key parameters from step response of the closed-loop control system, when system is marginally stable.

In this paper, it is presented the procedure for data-driven model identification and control of the inertial systems. According to this procedure the first step involves being done the system identification, based on the undamped step response of the closed-loop system. The algorithm for mathematical identification supposes to be achieved the limit of stability of the closed-loop system and based on the parameters, that are extracted from the undamped step response, there are presented some simple expressions for calculation the parameters of the model.

In the second step, there are presented the expressions for calculation of the tuning parameters of the PID controller. These expressions were obtained based on the value of the maximum stability degree criterion and the tuning procedure of the PID controller ensures the system the critically damped step response.

The procedure of the data driven identification and control was verified by computer simulation and gave good performances in the case of tuning the PID controller, and as further research, the proposed tuning method can be extended to the frequency domain.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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