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Structural-Identification Aspects of Decision-Making in Systems with Bouc-Wen Hysteresis

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Abstract

Considering the structural analysis problem of systems properties with Bouc-Wen hysteresis (BWH), various approaches are proposed for the identification of BWH parameters. The applied methods and algorithms are based on the design of parametric models and consider a priori information and the results of data analysis. Structural changes in the BWH form a priori. Methods for the Bouc-Wen model (BWM) identification and its structure estimation are not considered under uncertainty. The study's purpose is the analysis the structural problems of the Bouc-Wen hysteresis identification. The analysis base is the application of geometric frameworks (GF) under uncertainty. Methods for adaptive estimation parameters and structural of BWM were proposed. The adaptive system stability is proved based on vector Lyapunov functions. An approach is proposed to estimate the identifiability and structure of the system with BWH. The method for estimating the identifiability degree based on the analysis of GF is considered. BWM modifications are proposed to guarantee the system's stability and simplify its description.

Keywords

Structure, Framework, Identification, Structural Identifiability, Bouc-Wen Hysteresis, Nonlinearity, Adaptation

1. Introduction

Various models are used to describe hysteresis [1]. The Bouc-Wen model (BWM) is widely used to describe hysteresis. BWM is proposed by R. Bouc [2] and is generalized by Y.K. Wen [3] (system S_{BW})

$$m\ddot{x}+c\dot{x}+F\left(x,z,t\right)=f\left(t\right), \tag{1}$$

$$F(x,z,t) = \alpha kx(t) + (1-\alpha)kdz(t), \tag{2}$$

$$\dot{z} = d^{-1}\left(a\dot{x} - \beta|\dot{x}||z|^{n} sign(z) - \gamma\dot{x}|z|^{n}\right), \tag{3}$$

where $m > 0$ is mass, $c > 0$ is damping, $F(x,z,t)$ is the recovering force, $d > 0$, $n > 0$, $k > 0$, $\alpha \in (0,1)$, $f(t)$ is exciting force, a,β,γ are some numbers. Equation (3) is the BWM.

Many modifications of BWM [4] are proposed. Each model considers the features of the considered object. The BWM successful application depends on the identification of its parameters. The solution of the nonlinear Equation (3) is the main problem of BWM identification. The methods of identification and control Bouc-Wen hysteresis systems are given in [5] [6]. Adaptive modelling methods [6] are used to analyze the state of structural dynamics objects. An approach to the BWH identification based on analysis of a priori information and some heuristic procedures is proposed in [7]. Adaptive algorithms are proposed in [8] [9] for the BWM parameters estimation with the data forgetting [10]. It is assumed that are available for measurement $\ddot{x}$ and z, and $\dot{x}$, x are obtained by integration. The approach to adaptive identification [11] [12] is based on the least-squares method application and correction of the gain matrix. Change areas $\dot{x}$, x and the parameter n value are set. The adaptive observers use for the BWH identification is considered in [13]. The analysis of other approaches to the BWH parameters identification is given in [4] [14] [15] [16] [17]. Most procedures are based on measuring derivatives x. This possibility does not always exist when solving practical problems.

Examples [18] are known when BWM parameters estimations do not coincide with results obtained for other inputs. Such examples speak about the ambiguity of identification, which causes the instability of the model. Explain it with the fact that the Bouc-Wen model should be stable and ensure the adequacy of a physical process [19]. Requirements for BWM [19]: 1) adequacy of the mathematical model to the physical process; 2) BWM stability. Stability conditions impose restrictions on the changing area of model parameters. The choice of parameters belonging to the stability domain does not always give the adequate BVM [16] under uncertainty. Therefore, the approach [18] to the hysteresis; 3) parameters identification based on the BWH approximation of polynomial is proposed.

So, the analysis of publications shows that many algorithms and procedures for the BWM parameters identification propose. Proposed models consider features of the system. As a rule, the area of the BWM parameters changing sets a priori. Some parameters, such as n, are considered known. It is often assumed that derivatives of the system are measured. This situation does not always occur in practice and gives to the non-realizability of algorithms. The structure choice of the system (1)-(3) (structural identification (SI)) is the use result of the researcher's knowledge and intuition. This approach does not always give an adequate choice of the BWM structure under uncertainty. Often the structural identification problem is reduced to the parametric identification problem [20] [21]. This approach is laborious under uncertainty.

So, we see that BWH structural identification problem has not been developed. The S_{BW}-system modeling effectiveness depends on the choice of its parameters due to its initial instability. The input perturbation choice is significant for obtaining adequate results. The incorrect choice of input can lead to a system's non-identifiability. These problems require a solution for BWH. Some results on these problems are presented in [22].

The systematic approach proposed in this work gives the problem solve of identification systems with Bouc-Wen hysteresis. It includes: 1) the method for the input affect estimation on the S_{BW}-system identifiability; 2) a hierarchical immersion method that allows you to decide on the BWH structure under uncertainty; 3) the adaptive identification of BWH parameters based on input-output data; 4) the method for estimating the identifiability degree based on the analysis of GF and the phase portrait of the S_{BW}-system.

Work structure: 1) problem statement; 2) the method of S_{BW}-system adaptive identification; 3) modifications of the S_{BW}-system to simplify it and ensure stability; 4) the estimation method of structural identification and identifiability S_{BW}-systems; 5) the properties analysis of the input $f(t)$, which guarantees the SI and identifiability of the S_{BW}-system; 6) BMW modifications guarantee its stability.

Remark 1. The parametric approach does not allow estimating of the BWH structure under uncertainty. The proposed approach is based on the properties analysis of geometric frameworks.

2. Problem Statement

Consider S_{BW}-system. We have information on the input and the output

$$\mathrm{I}_o = \left\{ f(t), x(t), t \in [t_0, t_e] \right\}, \tag{4}$$

where $t_e < \infty$, $f(t), x(t)$ are limited functions of time.

Determine conditions of the system S_{BW} identifiability and structural components of the Bouc-Wen model (3) based on analysis of the sets (4).

Solving this problem is answered to the question: can we get an estimate of the system (3) structure under uncertainty?

Consider the identification of system parameters (1)-(3) by I_o.

3. Adaptive Identification of BWH

3.1. Problem Statement

Consider the system S_{BW}. Let $y = x$ be the output of the system. The set of the experimental data is $\mathrm{I}_o = \left\{ f(t), y(t), t \in J \right\}$, where $J \subset R$ is the specified time interval.

Designate by the parameters vector of the system as $A = [m, c, a, k, \alpha, \beta, \gamma, n]^{\mathrm{T}}$.

Problem: an adaptive observer to design for the evaluation of vector A such that

$$\lim_{t\to\infty}\left|\hat{y}(t)-y(t)\right|\leq\pi_y, \tag{5}$$

where $\hat{y}\in R$ is the output of the adaptive observer, $\pi_y\geq 0$.

Remark 2. The identification effectiveness of the system S_{BW} depends on features of the input $f(t)$. Requirements to $f(t)$ in identification problems are known. The force $f(t)$ must satisfy the constant excitation (CE) condition. This condition is necessary but not enough [23]. The input having the CE property cannot ensure the identifiability of the hysteresis structure. The structural identifiability of hysteresis is possible if $f(t)$ has the S-stabilization property of the system [23].

3.2. Adaptive System Identification

The set I_o has the form (4). Therefore, it is not applicable for estimating the parameters of the system S_{BW}. Design an adaptive observer for the system (1)-(3).

Consider the simplified system (1)-(3) when $d=1$, $a=1$. Substitute $F(x,z,t)$ in (1), and divide it by $s+\mu$, where $\mu>0$ does not coincide with roots of the polynomial $s^2+a_1s+a_2$, $s=d/dt$. Then

$$\dot{x}=a_1x+a_2p_x+a_3p_z+bp_f, \tag{6}$$

$$\begin{aligned}&\dot{p}_x=-\mu p_x+x,\ \dot{p}_f=-\mu p_f+f,\\&\dot{p}_z=-\mu p_z+z,\end{aligned} \tag{7}$$

where $a_1=-(c-\mu m)/m$, $a_2=-(\alpha k-\mu(c-\mu m))/m$, $a_3=-(1-\alpha)k/m$.

Equations (6), (7) contain only measurable variables except z. It complicates the identification process of the system S_{BW} parameters. Apply the model

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_z+\hat{b}p_f \tag{8}$$

to the estimation of the system (6) parameters, where $k_x>0$ is the specified number; $\hat{a}_i(t)$, $i=1,2,3$ and $\hat{b}(t)$ are adjusted parameters.

Designate $e=\hat{x}-x$. Obtain the equation for the identification error from (6), (8)

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_z+\Delta bp_f, \tag{9}$$

where $\Delta b=\hat{b}(t)-b$, $\Delta a_1=\hat{a}_1(t)-a_1$, $\Delta a_2=\hat{a}_2(t)-a_2$, $\Delta a_3=\hat{a}_3(t)-a_3$.

(9) is not solvable as the variable z is unknown in (7). Obtain the current estimation for $z(t)$. Consider model

$$\dot{\hat{x}}_{\bar{z}}=-k_x(\hat{x}_{\bar{z}}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{b}p_f. \tag{10}$$

Determine the misalignment $\varepsilon_z=x-\hat{x}_{\bar{z}}$ and use it for the variable z estimation. Let ε_z is the current estimation z. Apply the model to the estimation z

$$\dot{\hat{z}}=-k_z(\hat{z}-\varepsilon_z)+\tilde{\dot{x}}-\hat{\beta}\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})-\hat{\gamma}\tilde{\dot{x}}\left|\hat{z}\right|^n, \tag{11}$$

where $\tilde{\dot{x}}=(x(t+\tau)-x(t))/\tau$; $k_z>0$ is the given number $\hat{\beta}$, $\hat{\gamma}$ are the hysteresis (3) parameters estimations; τ is the integration step.

Introduce the misalignment $\varepsilon=\hat{z}-\varepsilon_z$ and obtain the equation for ε

$$\dot{\varepsilon}=-k_z\varepsilon+\Delta\dot{x}+\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})+\beta\eta_\beta+\Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n+\gamma\eta_\gamma, \tag{12}$$

$$\eta_\beta=\left|\dot{x}\right|\left|z\right|^n sign(z)-\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z}),\quad \eta_\gamma=\dot{x}\left|z\right|^n-\tilde{\dot{x}}\left|\hat{z}\right|^n,$$

where $\Delta\dot{x}=\tilde{\dot{x}}-\dot{x}$, $\Delta\beta=\beta-\hat{\beta}$, $\Delta\gamma=\gamma-\hat{\gamma}$.

Present (8) as

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_{\hat{z}}+\hat{b}p_f, \tag{8a}$$

where

$$\dot{p}_{\hat{z}}=-\mu p_{\hat{z}}+\hat{z}. \tag{13}$$

Then (9) rewrite as

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_{\hat{z}}+\Delta bp_f, \tag{14}$$

and adaptive algorithms describe as

$$\Delta\dot{a}_1=-\gamma_1ex,\Delta\dot{a}_2=-\gamma_2e_x,\Delta\dot{a}_3=-\gamma_3ep_{\hat{z}},\Delta b=-\gamma_bep_f, \tag{15}$$

where $\gamma_i>0, i=1,2,3$; $\gamma_b>0$.

Tuning algorithms for $\Delta\beta$ and $\Delta\gamma$ in (11) have the form

$$\begin{aligned}&\Delta\dot{\beta}=-\chi_\beta\varepsilon\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z}),\\&\Delta\dot{\gamma}=-\chi_\gamma\varepsilon\tilde{\dot{x}}\left|\hat{z}\right|^n,\end{aligned} \tag{16}$$

where $\chi_\beta>0, \chi_\gamma>0$ are parameters ensuring a convergence of algorithms.

Several algorithms are applicable for the indicator n estimation in (11). The effectiveness of their work depends on several factors. The simple algorithm has the form

$$\dot{\hat{n}}=\begin{cases}-\gamma_n\varepsilon\hat{\beta}\left|\hat{z}\right|^{\hat{n}-1}\hat{z}\tilde{\dot{x}}, & \text{if } \left|\dfrac{\varepsilon}{\varepsilon_z}\right|\in[\upsilon_0,\upsilon_1],\\ 0, & \text{if } \left|\dfrac{\varepsilon}{\varepsilon_z}\right|\notin[\upsilon_0,\upsilon_1],\end{cases} \tag{17}$$

where υ_0,υ_1 are set positive numbers, $\gamma_n>0$.

Remark 3. The identification procedure stability is the main problem of the system synthesis with BWM. We propose the method based on adaptive observer application.

3.3. Properties of Adaptive System

We estimate the adaptive system stability by applying Lyapunov vector functions. Consider the subsystem AS_X described by (14), (15). Let

$$\Delta K(t)\triangleq[\Delta a_1(t),\Delta a_2(t),\Delta a_3(t),\Delta b(t)]^{\mathrm{T}},$$

$$V_K(t)\triangleq 0.5\Delta K^{\mathrm{T}}(t)\Gamma^{-1}\Delta K(t), \tag{18}$$

$$V(t)=V_e(t)+V_K(t), \tag{19}$$

where $\Gamma=diag(\gamma_1,\gamma_2,\gamma_3,\gamma_b)$. Next, we give a results generalization [23] [24].

Assumption 1. The input $f(t)$ is constantly exciting and limited.

Theorem 1. Let 1) functions (19), $V_K(t)$ are positive definite and satisfy conditions $\inf_{|e|\to\infty} V_e(e)\to\infty$, $\inf_{\|\Delta K\|\to\infty} V_K(\Delta K)\to\infty$; 2) assumption 1 for the system (1)-(3) is satisfied. Then all trajectories of the system AS_X are limited belong area $\mathrm{G}_t=\{(e,\Delta K):V(t)\le V(t_0)\}$ and the estimation $\int_{t_0}^{t} 2k_x V_e(\tau)\mathrm{d}\tau \le V(t_0)-V(t)$ is fair.

Theorem 1 shows the limitation of adaptive system trajectories. The asymptotical stability ensuring the system demands to impose additional conditions.

Let $P(t)\triangleq\left[x(t)\ p_x(t)\ p_{\hat{z}}(t)\ p_f(t)\right]^{\mathrm{T}}$.

Definition 1. The vector P is constantly excited with a level ν or have property $\mathcal{PE}_\nu$ if $\mathcal{PE}_\nu$: $P(t)P^{\mathrm{T}}(t)\ge \nu I_4$ fairly for $\exists\,\nu>0$ and $\forall t\ge t_0$ on some interval $T>0$, where $I_4\in R^4$ is the unity matrix.

If the vector $P(t)$ has property $\mathcal{PE}_\nu$ then we will write $P(t)\in\mathcal{PE}_\nu$.

The system S_{BW} is stable, and the input $f(t)$ is restricted. Therefore, present the property $\mathcal{PE}_\nu$ for the matrix $B_P(t)=P(t)P^{\mathrm{T}}(t)$ as

$$\mathcal{PE}_{\nu,\bar{\nu}}:\nu I_4\le B_P(t)\le\bar{\nu}I_4,\quad \forall t\ge t_0, \tag{20}$$

where $\bar{\nu}>0$ is some number.

Let the estimation to $V_K(t)$ be fair

$$0.5\beta_l^{-1}(\Gamma)\|\Delta K(t)\|^2\le V_K(t)\le 0.5\beta_1^{-1}(\Gamma)\|\Delta K(t)\|^2, \tag{21}$$

where $\beta_1(\Gamma)$, $\beta_l(\Gamma)$ are minimum and maximum eigenvalues of the matrix Γ.

Apply inequalities (20), (21) and obtain estimations for $\dot{V}_e,\dot{V}_K$

$$\dot{V}_e\le -k_xV_e+\frac{\bar{\nu}\beta_l(\Gamma)}{k_x}V_K, \tag{22}$$

$$\dot{V}_K\le -\frac{3}{4}\vartheta\nu\beta_1(\Gamma)V_K+\frac{8}{3}\vartheta V_e, \tag{23}$$

Theorem 2. Let conditions be satisfied 1) positive definite Lyapunov functions $V_e(t)$ and (18) allow the indefinitely small highest limit at $|e(t)|\to 0$, $\|\Delta K(t)\|\to 0$; 2) $P(t)\in\mathcal{PE}_{\nu,\bar{\nu}}$; 3) equality $e\Delta K^{\mathrm{T}}P=\vartheta\left(\Delta K^{\mathrm{T}}B\Delta K+e^2\right)$ is fair in the area $O_v(O)$ with $0<\vartheta$, where $O=\{0,0^{3m}\}\subset R\times R^{3m}\times J_{0,\infty}$, O_v is some neighborhood of the point O; 4) the function $V_K(t)$ satisfies (21); 5) $\dot{V}_\varepsilon,\dot{V}_K$ satisfy the system of inequalities

$$\begin{bmatrix}\dot{V}_e\\ \dot{V}_K\end{bmatrix}\le \underbrace{\begin{bmatrix}-k_x & \dfrac{\bar{\nu}\beta_l(\Gamma)}{k_x}\\ \dfrac{8}{3}\vartheta & -\dfrac{3\nu\vartheta\beta_1(\Gamma)}{4}\end{bmatrix}}_{A_V}\begin{bmatrix}V_e\\ V_K\end{bmatrix}; \tag{24}$$

6) the upper solution for $V_{e,K}(t)=\left[V_e(t)\ V_K(t)\right]^{\mathrm{T}}$ satisfies to the comparison equation $\dot{S}=A_VS$ if

$$V_\rho(t) \le s_\rho(t), \quad \forall(t \ge t_0) \& \left(V_\rho(t_0) \le s_\rho(t_0)\right), \tag{25}$$

where $\rho = e, K$, $S = [s_e \; s_K]^{\mathrm{T}}$, $A_V \in R^{2\times 2}$ is *M*-matrix. Then the system AS_X is exponentially stable with the estimation

$$V_{e,K}(t) \le \mathrm{e}^{A_V(t-t_0)} S(t_0), \tag{26}$$

if

$$k_x \ge \frac{4}{3}\sqrt{\frac{2\bar{\nu}\beta_l(\Gamma)}{\nu\beta_1(\Gamma)}}. \tag{27}$$

Theorem 2 shows that the adaptive system gives the estimates for system (1) parameters. It is fair at the fulfillment of conditions (27). We supposed that the variable $p_{\hat{z}}$ restricted. The boundedness of the variable $\hat{x}_{\bar{z}}$ follows from the boundedness of the system AS_X trajectories.

Consider subsystem AS_Z described by Equations (12), (16). Introduce Lyapunov functions

$$\begin{aligned} &V_{\varepsilon\beta\gamma}(t) = V_\varepsilon(t) + V_{\beta,\gamma}(t), \\ &V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}\left(\Delta\beta(t)\right)^2 + 0.5\chi_\gamma^{-1}\left(\Delta\gamma(t)\right)^2. \end{aligned} \tag{28}$$

Theorem 3. Let 1) functions $V_\varepsilon(t)$, $V_{\beta,\gamma}(t)$ are positive definite and satisfy condition

$$\inf_{|\varepsilon|\to\infty} V_\varepsilon(\varepsilon) \to \infty, \quad \inf_{\|[\Delta\beta,\Delta\gamma]\|\to\infty} V_{\beta,\gamma}(\Delta\beta,\Delta\gamma) \to \infty; \tag{29}$$

2) the function $V_{\varepsilon\beta\gamma}(t)$ has the form (28); 3) the function

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)}, \quad g_1 = \sup_{\varepsilon\in\Omega} \tilde{g}_1(t), \tag{30}$$

exists, where Ω is the definition range of the subsystem AS_Z; 4) $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$; 5) $|\dot{x}| \le \upsilon$, $\upsilon > 0$; 6) the assumption 1 holds for the system (1)-(3). Then all trajectories of the system AS_Z are bounded, belong in the area $\mathrm{G}_\varepsilon = \left\{(\varepsilon,\Delta\beta,\Delta\gamma) : V_{\varepsilon\beta\gamma}(t) \le V_{\varepsilon\beta\gamma}(t_0)\right\}$, and the estimation

$$\begin{aligned} &\int_{t_0}^{t}\left(k_z - \upsilon(\beta+\gamma)g_1\right)V_\varepsilon(\tau)\,\mathrm{d}\tau + \frac{1}{2\left(k_z - \upsilon(\beta+\gamma)g_1\right)(t-t_0)}(\delta_\Delta)^2 \\ &\le V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t) \end{aligned} \tag{31}$$

is fair if

$$k_z > \upsilon(\beta+\gamma)g_1. \tag{32}$$

So, the boundedness of trajectories in the adaptive system is proved. The analysis showed that the subsystem AS_X is asymptotically stable. The prove of trajectories boundedness for the subsystem AS_Z is a more complex problem. The estimation (31) shows that the quality of processes in the AS_Z-system depends on the output derivative of the S_{BW}-system. The following result give more exact estimations for processes in the AS_Z-system.

Theorem 4. Let 1) positive definite Lyapunov functions $V_{\beta,\gamma}(t)$ and $V_\varepsilon(t)$

exist and have the indefinitely small higher limit at $\left\|\left[\Delta\beta(t),\Delta\gamma(t)\right]\right\|\to 0$ to $|\varepsilon(t)|\to 0$; 2) $P(t)\in \mathcal{PE}_{\nu,\bar{\nu}}$; 3) such $c_1>0, c_2>0$ exist that conditions

$$\begin{gathered}\varepsilon\Delta\gamma\tilde{x}|\hat{z}|^n = c_2\left[(\Delta\gamma)^2\left(\tilde{x}|\hat{z}|^n\right)^2+\varepsilon^2\right],\\ \varepsilon\Delta\beta|\tilde{x}||\hat{z}|^n\, sign(\hat{z}) = c_1\left[(\Delta\beta)^2\left(|\tilde{x}||\hat{z}|^n\right)^2+\varepsilon^2\right]\end{gathered} \tag{33}$$

are satisfied in the area $O_v(O)$, where $O=\{0,0^2\}\subset R\times R^2\times J_{0,\infty}$, O_v is some neighbourhood of the point O; 4) inequality $(\varepsilon-\varepsilon_z)^{2n}\geq c_z$ holds for almost all t where $c_z\geq 0$; 5) such $\pi_x\geq 0$ and $\omega>0$ exist that $(\tilde{x})^2\geq \pi_x$ and $|\varepsilon-\varepsilon_z|\leq \omega|\varepsilon|$; 6) the function

$$g_2(t)=\sup_{\varepsilon\in\Omega}\frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)},\quad g_2=\sup_{\varepsilon\in\Omega}\tilde{g}_2(t) \tag{34}$$

exists, where Ω the subsystem AS_Z definition domain; 7) $\dot{V}_\varepsilon$, $\dot{V}_{\beta,\gamma}$ satisfy the system of inequalities

$$\begin{bmatrix}\dot{V}_\varepsilon\\ \dot{V}_{\beta,\gamma}\end{bmatrix}\leq \underbrace{\begin{bmatrix}-\left(k_z-2\tilde{\upsilon}g_1-\omega\upsilon g_2\right) & \lambda\chi\omega\upsilon\\ c & -\dfrac{d_s}{2}\end{bmatrix}}_{A_\varepsilon}\begin{bmatrix}V_\varepsilon\\ V_{\beta,\gamma}\end{bmatrix}+\underbrace{\begin{bmatrix}\dfrac{1}{2k_z}\\ 0\end{bmatrix}}_{B_\varepsilon}(\delta_\Delta)^2; \tag{35}$$

8) the upper solution for $V_{\varepsilon,\beta,\gamma}=\left[V_\varepsilon(t)\,V_{\beta,\gamma}(t)\right]^{\mathrm{T}}$ satisfies to the equation

$$\dot{\tilde{S}}=A_\varepsilon\tilde{S}+B_\varepsilon(\delta_\Delta)^2 \tag{36}$$

if

$$V_{\tilde{\rho}}(t)\leq \tilde{s}_{\tilde{\rho}}(t),\quad \forall(t\geq t_0)\,\&\left(V_{\tilde{\rho}}(t_0)\leq \tilde{s}_{\tilde{\rho}}(t_0)\right), \tag{37}$$

where $\tilde{\rho}=\varepsilon,\beta,\gamma$, $\tilde{S}=\left[\tilde{s}_\varepsilon\ \tilde{s}_{\beta,\gamma}\right]^{\mathrm{T}}$, $A_\varepsilon\in R^{2\times 2}$ is M-matrix. Then the system AS_Z is exponentially dissipative with the estimation

$$V_{\varepsilon,\beta,\gamma}(t)\leq \mathrm{e}^{A_\varepsilon(t-t_0)}\tilde{S}(t_0)+(\delta_\Delta)^2\int_{t_0}^{T}\mathrm{e}^{A_\varepsilon(t-\tau)}B_\varepsilon\mathrm{d}\tau, \tag{38}$$

if $\left(k_z-2\tilde{\upsilon}g_1-\omega\upsilon g_2\right)d_s>2c\lambda\chi\omega\upsilon$, $k_z>2\tilde{\upsilon}g_1-\omega\upsilon g_2$, $d_s>0$,

$$\bar{\chi}=\min\left(\chi_\beta,\chi_\gamma\right),\ \bar{c}=\min\left(c_1,c_2\right),\ \chi=\max\left(\chi_\beta,\chi_\gamma\right),\ d_s=\chi\pi_x\bar{c}c_z. \tag{39}$$

M-matrix is considered in [25].

So, the system AS_Z is exponentially dissipative. The dissipativity area depends on the informational set I_o of the S_{BW}-system.

3.4. Simulation Results

Consider the system (1)-(3) with parameters $n=1.5$, $c=2$, $m=1$, $\beta=0.5$, $\alpha=0.7$, $k=0.6$, $d=a=1$. Parameters are selected based on simulation. The exciting force $f(t)=2-2\sin(0.15\pi t)$. The system is modeled with initial conditions $x(0)=1$, $\dot{x}(0)=0$, $z(0)=1$. Form the set I_o. The system phase portrait and output of the hysteresis shown in **Figure 1**.

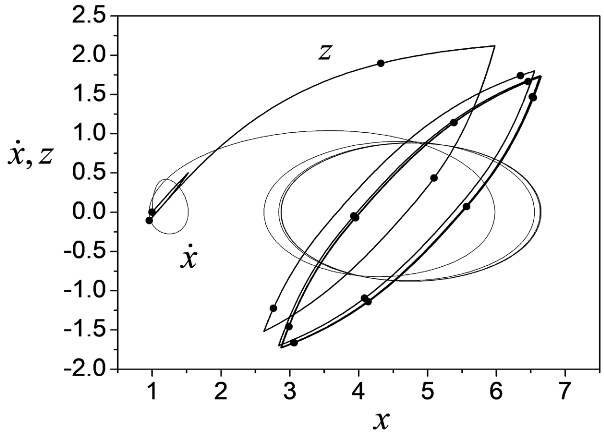

Figure 1. System phase portrait and hysteresis change.

Consider the identification of the system parameters. Determine the parameter μ of the system (13) using the transient process analysis for $\tilde{e}$ and $t < 9.85\,\text{s}$. Calculate Lyapunov exponents (LE) [26]. The estimation for the maximum LE is −0.9. Therefore, we set $\mu = 0.8$. Initial conditions in (7) are equal to zero.

Adaptive system work results presented in **Figures 2-4**. Parameters k_x, k_z are equal to 2.5 and 0.75. The tuning process of AS_X-systems (the model (8)) parameters shown in **Figure 2**. **Figure 3** shown the model (11) parameters tuning.

Show the modification of identification errors e, ε in **Figure 4**. We see that the accuracy of obtained estimations depends on the numbers of tuned parameters and the level $\dot{x}$ and properties $f(t)$. Obtained results confirm statements of theorems 3, 4. The AS_Z-system work results influence the tuning processes in the AS_X-system. Gain coefficients in (15), (16) and (17) are $\chi_\beta = 0.0000002$, $\chi_\gamma = 0.0000002$, $\gamma_4 = 0.00005$ $\gamma_1 = 0.0002$, $\gamma_2 = 0.00001$, $\gamma_3 = 0.00002$. The hysteresis output estimation shown in **Figure 5**.

So, simulation results confirm the exponential dissipativity of the designed system.

4. Modification S_{BW}-Systems

Various modifications of BWM have been proposed (see, for example, [4] [5] [26] [27] [28]). They reflect the features and properties of the control object. System (1)-(3) is the basis for modifications. The BWM modification proposes for the case of asymmetric hysteresis in [29]. The model has the form

$$\dot{z} = \left(a - \left(\beta + \gamma sign\left(z\dot{x}\right)\right)\left|z\right|^n\right)\dot{x}\,. \tag{40}$$

The BWM modifications set is based on the introduction of new multipliers in (3) [4] [30]. They reflect requirements to the system. BWM considering the degradation and clamping of reinforced concrete structures has the form [30]

$$\dot{z} = \frac{h(z,\varepsilon)}{1+\delta_\eta}\left[\left(\dot{x} - 1 - \delta_\nu\right)\left(\beta\left|\dot{x}\right|\left|z\right|^{n-1} + \gamma\left(\dot{x}\left|z\right|\right)^n\right)\right], \tag{41}$$

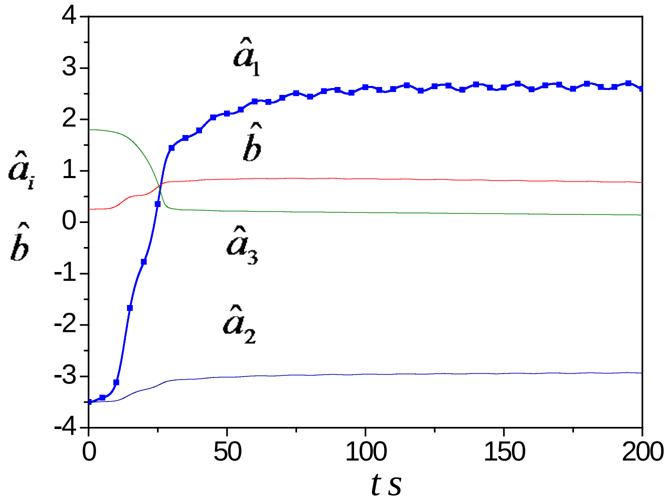

Figure 2. Tuning of model (8) parameters.

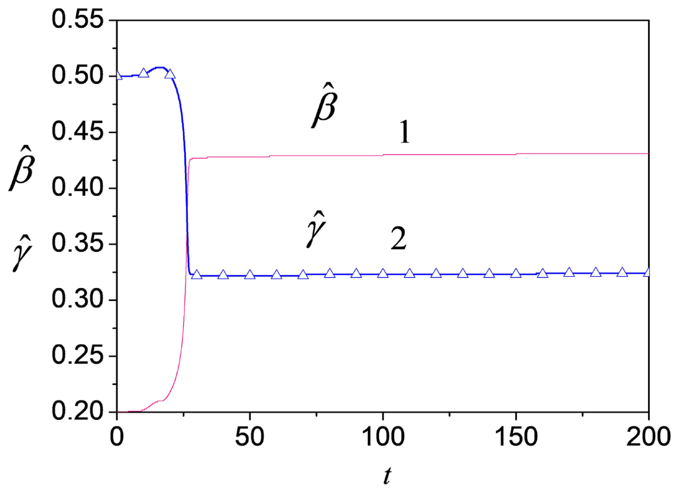

Figure 3. Tuning of model (11) parameters: 1 is tuning $\hat{\beta}$, 2 is tuning $\hat{\gamma}$.

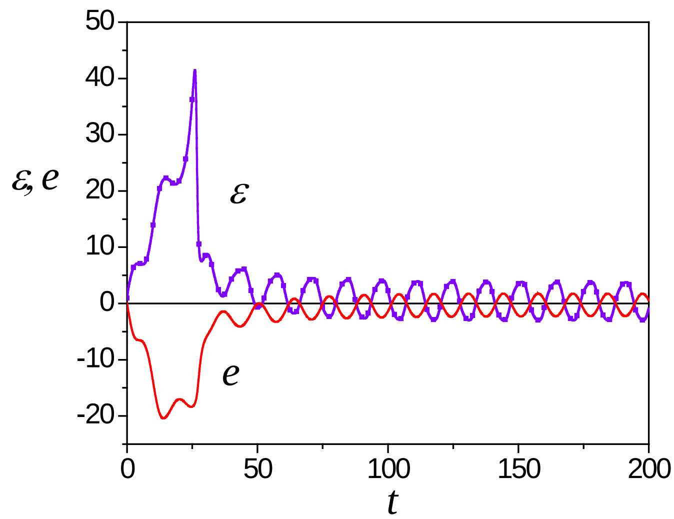

Figure 4. Outputs modification of systems AS_x, AS_z.

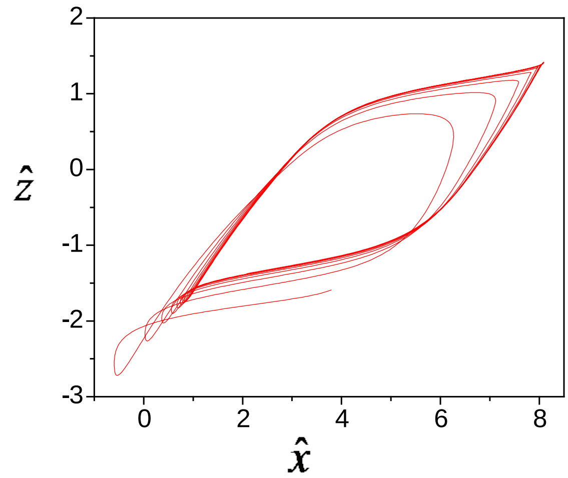

Figure 5. Hysteresis estimation at adaptation of AS_{BW} -system.

where δ_η and δ_ν are parameters reflecting the decrease in rigidity and strength of the structure, $h(z,\varepsilon)$ considers the pinching effect.

The analysis showed that the last term in (3) is responsible for "fine-tuning" the hysteresis in the saturation or switching areas. If this is not critical for the object, then by selecting the parameters of the SW-system, this term in Equation (3) can compensate. In addition, some modifications simplify and increase the system (1)-(3) stability. They have the form [32]

$$\mathcal{M}_{\rho\omega\mu\upsilon\beta n}: \dot{z} = -\rho z|\dot{x}|^{\omega} + a|\dot{x}|^{\mu} sign(\dot{x}) - \beta|\dot{x}|^{\upsilon}|z|^{n} sign(z), \tag{42}$$

$$\mathcal{M}_{\mu\beta n}: \dot{z} = a|\dot{x}|^{\mu} sign(\dot{x}) - \beta|\dot{x}||z|^{n} sign(z), \tag{43}$$

$$\mathcal{M}_{\mu\upsilon\beta n}: \dot{z} = a|\dot{x}|^{\mu} sign(\dot{x}) - \beta|\dot{x}|^{\upsilon}|z|^{n} sign(z). \tag{44}$$

The introduction in (42) of the linear component of z increases the feasibility of the BWM and the S_{BW} -system stability. As the system is nonlinear, the function $|\dot{x}(t)|^{\omega}$ introduces to ensure the required hysteresis state. It guarantees a change z in the specified boundaries. Parameters $\rho > 0, \omega \geq 0$ are some numbers.

A comparison of the models (42)-(44) and BWM is shown in **Figure 6**. The representation allows comparing model properties by generalized indicators in the "minimum-maximum" space. Notation in **Figure 6**: z is model (3), z1 is model $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$, z2 is model $\mathcal{M}_{\mu\beta n}$, z3 is model $\mathcal{M}_{\mu\upsilon\beta n}$; ♦ is average value; — is median; ○ is the extreme value (end of the "saturation" region).

So, BWM modifications are considered. The application of proposed models depends on the object properties. The parameters influence analysis of models (42)-(44) give in [31].

5. Theoretical Foundations of SI

5.1. Preliminary

The modern direction of structural identification is based on the parametric

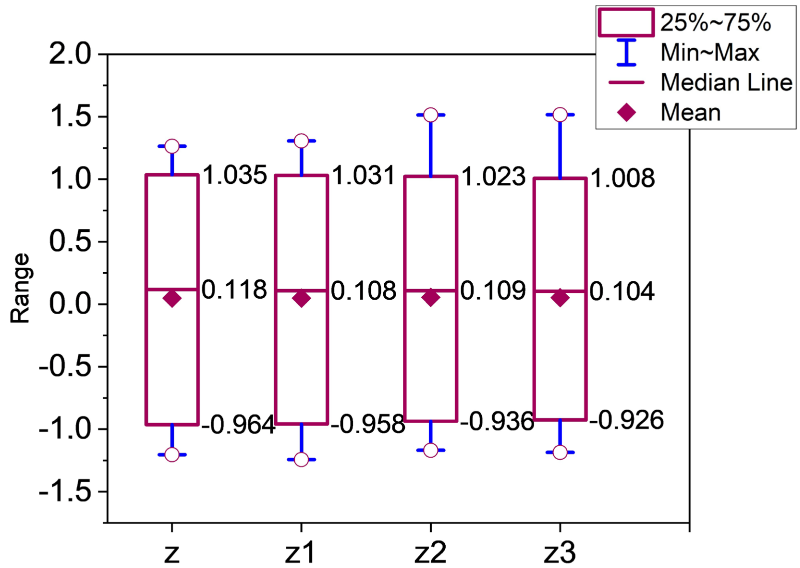

Figure 6. Comparison of hysteresis models (3), (42)-(44).

paradigm. It is explained by the formation and development of the theory of identification. Nonlinear systems SI methods are based on the approximation of nonlinearity by parametric models (see, for example, [32] [33] [34] [35] [36]). This approach leads to levelling of the nonlinearity structure. The second direction of structural identification is related to the analysis of geometric frameworks (GF). GF reflect the state of the system nonlinear part. It is the new direction in the identification theory. This approach proposes in [22] [37]. The statement of this approach gives below.

5.2. Problem Statement

Consider dynamic system

$$\begin{aligned} \dot{X} &= AX + \varphi(y)I + Bu, \\ y &= C^{\mathrm{T}} X, \end{aligned} \tag{45}$$

where $u \in R$, $y \in R$ are input and output system; $A \in R^{q \times q}$, $B \in R^q$, $I \in R^q$ $C \in R^q$; $\varphi(y)$ is the scalar nonlinear function belonging to the class of the hysteresis $\mathcal{F}_h$; $I = [0, 0, \cdots, 0, 1]^{\mathrm{T}}$. We suppose that A is the Hurwitz matrix.

Various assumptions are made about the structure of the function $\chi = \varphi(y)$. They determine by the level of a priori information. Under a priori definiteness, apply the methods based on linearization [38]. In the absolute stability study of nonlinear systems, suppose [28]

$$\chi \in \mathcal{F}_\varphi = \left\{ \varphi(\xi)\xi \geq \xi^2, \xi \neq 0, \varphi(0) = 0 \right\}, \tag{46}$$

where $\xi \in R$ is the nonlinearity input. ξ is a linear combination of the state variables (the vector X). The sector condition is used for approximation of function χ

$$\chi \in \mathcal{F}_\varphi = \left\{ \gamma_1 \xi^2 \leq \varphi(\xi)\xi \leq \gamma_2 \xi^2, \xi \neq 0, \varphi(0) = 0, \gamma_1 \geq 0, \gamma_2 < \infty \right\}, \tag{47}$$

Static nonlinearity often applies in control systems. Therefore, next, we consider the static (algebraic) functions which describe a hysteresis. For system (45), we have a set of the data

$$\mathrm{I}_o = \{u(t), y(t), t \in J = [t_0, t_k]\}. \tag{48}$$

Problem: determine a form and parameters of function $\varphi(y) \in \mathcal{F}_h$ based of the analysis and a processing of the set I_o.

The problem solution is based on the formation of the set $\mathrm{I}_{N,g}$ contained data about $\varphi(y)$.

5.3. Formation of Set $\mathrm{I}_{N,g}$

The differentiation operation applies to $y(t)$ and designates the obtained variable as x_1. Generate informational the set $\mathrm{I}_{ent} = \{\mathrm{I}_o, x_1\}$. Select the data $\mathrm{I}_g \subset \mathrm{I}_{ent}$ subset described the particular solution (steady state) of the system (45). The mathematical model

$$\hat{x}_1^l(t) = H^{\mathrm{T}}\left[1\ u(t)\ y(t)\right]^{\mathrm{T}} \tag{49}$$

applies to obtain $\mathrm{I}_g = \mathrm{I}_{ent} \setminus \mathrm{I}_{tr}$. Model (49) is determined on the time gap $J_g = J \setminus J_{tr}$ and gives the linear component x_1 estimation. $H \in R^3$ is a parameters model vector. The choice of an interval J_g depends on the value of criterion $Q(e)$.

Determine a vector H as

$$\min_H Q(e)\Big|_{e=\hat{x}_1^l - x_1} \to H_{opt}, \tag{50}$$

where $Q(e) = 0.5e^2$.

Apply the model (49) and determine the forecast for the variable $x_1(t)$ $\forall t \in \mathrm{I}_g$. Compute the error $e(t) = \hat{x}_1^l(t) - x_1(t)$. $e(t)$ depends on nonlinearity $\varphi(y)$ in the system (45). Obtain set

$$\mathrm{I}_{N,g} = \{y(t), e(t), t \in J_g\}, \tag{51}$$

which we will use next. We will apply the designation $y(t)$, supposing that $y(t) \in \mathrm{I}_{N,g}$.

The further problem solution is based on the analysis of frameworks $\mathcal{S}_{ey}$, $\mathcal{S}_{ek}$ which reflect the state of the nonlinearity.

Remark 4. Choice of the model (49) structure is one of the stages of structural identification. Simulation results show that the model (49) is used in identification systems of plants with static nonlinearity. For other classes of nonlinearity, this problem demands further research.

5.4. Frameworks $\mathcal{S}_{ey}$, $\mathcal{S}_{ek}$

Go into space $\mathcal{P}_{ye} = (y, e)$ and construct the phase portrait $\mathcal{S}$ of the system (45). The framework $\mathcal{S}_{ey}$ corresponds to a phase portrait $\mathcal{S}$ [37]. $\mathcal{S}_{ey}$ describes function $\Gamma_{ey}: \{y\} \to \{e\}$ $\forall t \in J_g$. $\mathcal{S}_{ey}$ must have a closed form. This property $\mathcal{S}_{ey}$ differs from frameworks $\mathcal{S}_{ek}$. $\mathcal{S}_{ek}$ is applied for the analysis of statics systems [39] [40].

For decision making, we will use also $\mathcal{S}_{ek}$-framework. $\mathcal{S}_{ek}$ is described by function $\Gamma_{ek}:\{k_s(t)\}\to\{e(t)\}$, where $k_s(t)\in R$ is a coefficient of structural properties [39] systems (45) in space $\mathcal{P}_{ye}$

$$k_s(t)=\frac{e(t)}{y(t)}. \tag{52}$$

Next, we construct sector sets for system (45) in the space $\mathcal{P}_{ke}=(k,e)$ and will be decision-making on the class $\mathcal{F}_h$. The solution to this problem is based on the analysis of proposed frameworks.

5.5. About Properties $\mathrm{I}_{N,g}$

Consider the set $\mathrm{I}_{N,g}$ properties ensured the solution of hysteresis F1 structural identification. Let fulfill to conditions

(i) the set I_o ensures the solution of the model (49) parametric identification problem.

(ii) the input $u(t)$ ensures obtaining informative framework $\mathcal{S}_{ey}(\mathrm{I}_{N,g})$ or $\mathcal{S}_{ek}(\mathrm{I}_{N,g})$.

If $u(t)$ has properties (i), (ii), then input $u(t)$ is representative.

Let the framework $\mathcal{S}_{ey}$ is closed and its area is not zero. Designate altitude $\mathcal{S}_{ey}$ as $h(\mathcal{S}_{ey})$, where the altitude is the distance between two points of the opposite sides of framework $\mathcal{S}_{ey}$. Then the framework $\mathcal{S}_{ey}$ is identified on set $\mathrm{I}_{N,g}$.

Let $u\in\mathcal{PE}_\alpha$, where $\mathcal{PE}_\alpha$ is the constancy excitation property

$$\mathcal{PE}_\alpha : u^2(t)\geq\alpha \tag{53}$$

fair for $\exists\alpha>0$ and $\forall t\geq t_0$ on some interval $T>0$.

Statement 1 [37]. Let (i) the linear part of system (45) is stable and nonlinearity satisfies the condition (47); (ii) the input $u(t)$ is piecewise continuous, limited and constantly exciting; (iii) exists $\delta_S>0$ such that $h(\mathcal{S}_{ey})\geq\delta_S$. Then the framework $\mathcal{S}_{ey}$ is identified on set $\mathrm{I}_{N,g}$.

Proof of Statement 1. Consider input $u(t)$ satisfied to condition 1). $u(t)$ corresponds Fourier series containing a sinusoid with frequency ω_i. The output $y(t)\in\mathrm{I}_{N,g}$ contains components of this spectrum and has a phase shift. The variable x_1 is the result of the differentiation $y(t)$. Hence, x_1 contains components with this frequent spectrum. Therefore, the framework $\mathcal{S}$ (phase portrait) on a phase plane (y,x_1) has a closed form. The $\mathcal{S}_{ey}$-framework has the same form. Determine the distance $h(\mathcal{S}_{ey})$ between opposite points of the framework $\mathcal{S}_{ey}$. $y(t),x_1(t)$ satisfy the condition 2) statement 1. Therefore, for almost all $t\in J_{N,g}$ $h(\mathcal{S}_{ey})>\delta_S$. ■

The framework $\mathcal{S}_{ey}$ which has referred properties, we will name *h*-identified. Further, we believe that $\mathcal{S}_{ey}$ has the specified properties.

Features of the *h*-identifiability.

1) h-identifiability is a concept not parametric identification, and structural identification;

2) The demand for parametric identifiability is the base h-identifiability;

3) h-identifiability determines more stringent demands to the system input.

Feature 3 means that "the bad" input can satisfy a constancy excitation condition. Such input can give a so-called "insignificant" $\mathcal{S}_{ey}$-framework (framework $\mathcal{NS}_{ey}$) [37] which will have property h-identifiability.

5.6. Framework $\mathcal{NS}_{ey}$

Consider the framework $\mathcal{S}_{ey}$. Let $\mathcal{S}_{ey} = \mathcal{F}^l_{\mathcal{S}_{ey}} \cup \mathcal{F}^r_{\mathcal{S}_{ey}}$, where $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ are left and right fragments $\mathcal{S}_{ey}$. Determine for $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ secants

$$\gamma^l_S = a^l y, \quad \gamma^r_S = a^r y, \tag{54}$$

where a^l, a^r are the numbers determined using the method of least squares (LSM).

Theorem 5 [37]. Let (i) the framework $\mathcal{S}_{ey}$ is h-identifiable; (ii) the framework $\mathcal{S}_{ey}$ have the form $\mathcal{S}_{ey} = F^l_{\mathcal{S}_{ey}} \cup F^r_{\mathcal{S}_{ey}}$, where $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ are left and right fragments $\mathcal{S}_{ey}$; (iii) for $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ secants (54) are obtained. Then $\mathcal{S}_{ey}$ is $\mathcal{NS}_{ey}$-framework, if

$$\left| a^l - a^r \right| > \delta_h, \tag{55}$$

where $\delta_h > 0$ is some number.

Theorem 5 proves based on sets homothety.

Remark 5. $\mathcal{NS}_{ey}$-frameworks are characteristic for systems with many-valued nonlinearities. They are the input inadequate use result.

6. Structural Identification and Structural Identifiability BWH

We have noted (see introduction) that structural identifiability (SID) is the result of structural identification. Therefore, we will consider the SID basics guaranteed SI.

Apply the SI model (49) and represent the system (45) as (system $S_{y\varphi}$)

$$\begin{cases} S_y : \begin{cases} \dot{\tilde{X}} = A\tilde{X} + I\zeta, \\ \tilde{y} = C^{\mathrm{T}} \tilde{X}, \end{cases} \\ S_\varphi : e = f\left(y, x_1\right), \end{cases} \tag{56}$$

where $\tilde{X} \in R^n$ is a variable describing the general solution of the system (45), $\zeta \in R$ is a bounded perturbation appearing as the analysis result of the variable e.

6.1. System S_φ

Consider the identifiability problem system S_φ. Let conditions hold.

B1. The input is constantly excited at the interval J.

B2. The analysis of $\mathcal{S}_{ey}$ gives the solution to the estimation problem the nonlinear properties of the system $S_{y\varphi}$.

We state the basic concepts, generalizing the results [22].

Definition 2. If $u(t)$ satisfies B1 and B2 conditions, then the input $u(t)$ is representative.

Let the framework $\mathcal{S}_{ey}$ closed, and the area $\mathcal{S}_{ey}$ is not zero. Denote height $\mathcal{S}_{ey}$ as $h(\mathcal{S}_{ey})$ where height is the distance between two points opposite sides of the framework $\mathcal{S}_{ey}$.

Theorem 6 [37]. Let (i) the linear part of the system (45) is stable; (ii) the nonlinearity $\varphi(\cdot)$ satisfies the condition (47); (iii) the input is bounded and constantly excited; 4) $h(\mathcal{S}_{ey}) \geq \delta_S$, where $\delta_S > 0$. Then the framework $\mathcal{S}_{ey}$ is identified on the set $\mathrm{I}_{N,g}$.

Definition 3. The framework $\mathcal{S}_{ey}$ is called *h*-identifiable if theorem 6 holds for $\mathcal{S}_{ey}$.

Let $\mathcal{S}_{ey}$ be *h*-identifiable. Introduce designations: $\mathcal{D}_y = \operatorname{dom}(\mathcal{S}_{ey})$ is definition range of the framework $\mathcal{S}_{ey}$, $D_y = D_y(\mathcal{D}_y) = \max_t y(t) - \min_t y(t)$ is diameter $\mathcal{D}_y$. Let $u(t) \in \mathrm{U}$, where U is an acceptable set of inputs for the system (45). The set U contains representative inputs.

Definition 4. If $\mathcal{D}_y$ of the framework $\mathcal{S}_{ey}$ has a maximum diameter D_y, the input S-synchronizes the system (45).

Consider a reference framework $\mathcal{S}_{ey}^{ref}$. $\mathcal{S}_{ey}^{ref}$ is the framework $\mathcal{S}_{ey}$ reflecting all properties of the function $\varphi(y)$. Designate by the diameter $D_y(\mathcal{S}_{ey}^{ref})$ as D_y^{ref}. D_y^{ref} exists if the input the system (45) is S-synchronizing.

Definitions 2, 3 show if $\mathcal{S}_{ey} \cong \mathcal{S}_{ey}^{ref}$, then $\left|D_y - D_y^{ref}\right| \leq \varepsilon_y$, where $\varepsilon_y \geq 0$, $\cong$ is the proximity sign. Elements of the subset U_S have property

$$\left|D_y\left(\mathcal{S}_{ey}\left(u(t)\big|_{u\in \mathrm{U}_S}\right)\right) - D_y^{ref}\right| \leq \varepsilon_y. \tag{57}$$

Synchronization $u(t) \in \mathrm{U}$ is the choice of such input $u_h(t) \in \mathrm{U}$ that reflects all features $\varphi(y)$ in $\mathcal{S}_{ey}$. It is true if $u(t)$ ensures $\max_{u_h} D_y$ and $\mathcal{S}_{ey} \neq \mathcal{NS}_{ey}$. We interpret the choice $u_h(t) \in \mathrm{U}$ as ensuring synchronization between structures of a model and the system. $\mathcal{d}_{h,y} = \max_{u_h} D_y$ is the condition of *h*-identifiability which can represent as

$$\left|D_y\left(\mathcal{S}_{ey}\left(u(t)\big|_{u\in \mathrm{U}_S}\right)\right) - \mathcal{d}_{h,y}\right| \leq \varepsilon_y. \tag{58}$$

The condition for $\mathcal{NS}_{ey}$

$$\left|D_y\left(\mathcal{S}_{ey}\left(u(t)\big|_{u\in \mathrm{U}\setminus\mathrm{U}_S}\right)\right) - \mathcal{d}_{h,y}\right| > \varepsilon_y. \tag{59}$$

(58) can be interpreted as proximity domain

$$\mathcal{Q}_D = \left|\mathcal{S}_{ey}\left(u(t)\big|_{u\in \mathrm{U}_S}\right) - \mathcal{S}_{ey}^{ref}\right|, \tag{60}$$

which is understood as $\left|\dot{y}(t) - \dot{y}^{ref}(t)\right| \leq \varepsilon_y$ for almost $\forall t \geq \tilde{t}$.

We will write $\delta\mathcal{Q}_D \leq \varepsilon_y$ if considered frameworks are close.

Domain $\mathcal{Q}_D$ is the S-synchronizability area on $\{u_h(t)\}$ or the structural identifiability domain on $\{\mathcal{S}_h(u_h(t))\}$, where $\mathcal{S}_h$ is the phase portrait of the system (45) if the condition $\delta Q_D \le \varepsilon_y$ is true for Q_D almost $\forall t \ge t^*$.

So, two criteria (55) and (59) presented for the existence of the insignificant framework $\mathcal{NS}_{ey}$. Structure of systems S_φ and (45) are structurally non-identifiable in this case.

Let the input $u_h(t)$ synchronize the system (45). If $u(t)$ is S-synchronizing, then we will write $u_h(t) \in \mathrm{S}$. Note that a finite set $\{u_h(t)\} \in \mathrm{S}$ exists for the system (45). The choice of optimal $u_h(t)$ depends on $d_{h,y}$ and (58). The hold of the condition (58) is one of the prerequisites for SI of the system (45).

Definition 5. If framework $\mathcal{S}_{ey}$ is *h*-identified and conditions $\left\| a^r \right| - \left| a^r \right\| \le \delta_h$, (8) are satisfied, then the framework $\mathcal{S}_{ey}$ or the system (56) (system (45)) is structurally identified or h_{δ_h}-identifiable.

Remark 6. Conditions specified in definition 5 are the conditions for the structural identification of systems (45), (56).

Definition 5 shows if the system (45) is h_{δ_h}-identified then the framework $\mathcal{S}_{ey}$ has the maximum diameter of area $\mathcal{D}_y$.

Definition 6. The model (49) is *SM*-identifying if the framework $\mathcal{S}_{ey}$ is h_{δ_h}-identifiable.

The framework $\mathcal{S}_{ey}$ is defined on $u_h(t) \in \mathrm{S}$ and $u_h(t)$ satisfies condition B1. Therefore, $\mathcal{S}_{ey}$ corresponds to the nonlinearity $\varphi(y)$ defined on the class

$$\varphi(y) \in \mathcal{F}_\varphi = \left\{ \varphi(y) \in R \middle| \varphi(y, \mathrm{A}), \mathrm{A} \in R^{n_\mathrm{A}}, \alpha_i \in \mathrm{A}, \alpha_i \in \left[\bar{\alpha}_i, \bar{\bar{\alpha}}_i \right] \right\}, \tag{61}$$

where $\bar{\alpha}_i, \bar{\bar{\alpha}}_i$ are some numbers.

Note that the term *SM*-identifying does not coincide with the concept proposed in [41].

Theorem 7 [42]. Let (i) the input $u(t) \in \mathrm{S}$ is constantly excited; (ii) the system (45) phase portrait have *m* features; (iii) $\mathcal{S}_{ey}$-framework is h_{δ_h}-identified and contains fragments corresponding to features of the system (45). Then the model (49) is *SM*-identifying.

The theorem 7 shows if the model (49) is not *SM*-identifying then model (49) structure or the informational set (48) need to change.

Let c_S is the center of the framework $\mathcal{S}_{ey}$ on the set $J_y = \{y(t)\}$, c_{D_y} is the center of the area $\mathcal{D}_y$.

Theorem 8. Let the set $\mathrm{U_S}$ given for the system $S_{y\varphi}$ and (i) exists $\varepsilon \ge 0$ such that $\left| c_S - c_{D_y} \right| \le \varepsilon$; (ii) $\left\| a^l \right| - \left| a^r \right\| \le \delta_h$, where a^l, a^r are coefficients of secants (54) for $\left(\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}} \right) \subset \mathcal{S}_{ey}$. Then the system (56) is h_{δ_h}-identifiable, the input $u_h(t) \in \mathrm{S}$, and the framework $\mathcal{S}_{ey}$ defines the class $\mathcal{F}_\varphi$.

Proof of Theorem 8. Consider the input $u_h(t) \in \mathrm{U_S}$. Since condition $\left| a^l - a^r \right| \le \delta_h$ is satisfied, the framework $\mathcal{S}_{ey}$ is symmetric concerning the point c_S plane (y, e). Consequently, definitional domains diameters of the fragments $\mathcal{F}^l_{\mathcal{S}_{ey}}, \mathcal{F}^r_{\mathcal{S}_{ey}}$ for the framework $\mathcal{S}_{ey}$ coincide up to a certain value $\varepsilon_\mathcal{F} \ge 0$ on the set $\{y(t)\}$, *i.e.*

$$\left| D_{\mathcal{F}_S^l}\left(\mathcal{D}_{\mathcal{F}_S^l}\right) - D_{\mathcal{F}_S^r}\left(\mathcal{D}_{\mathcal{F}_S^r}\right) \right| \leq \varepsilon_{\mathcal{F}}, \tag{62}$$

where $\mathcal{D}_{F_S^l}, \mathcal{D}_{F_S^r}$ are definitional domains $\mathcal{F}_{S_{ey}}^l, \mathcal{F}_{S_{ey}}^r$. Then the framework $\mathcal{S}_{ey}$ centre is equal to $c_{D_y} = 0.5\left(D_{\mathcal{F}_S^l} + D_{\mathcal{F}_S^r}\right)$. Since $D_{\mathcal{F}_S^l} + D_{\mathcal{F}_S^r} = D_y$, there exist $\varepsilon \geq 0$ such that $\left|c_S - c_{D_y}\right| \leq \varepsilon$. The fulfillment of conditions (i), (ii) guarantees $u(t) = u_h(t)$ and $\mathscr{d}_{h,y} = \max_{u_h} D_y$. Therefore, the framework $\mathcal{S}_{ey}$ contains all the features characteristic of the function $\varphi(y)$ at $u_h(t)$. So, $u_h(t) \in \mathrm{S}$, and system (45) is h_{δ_h}-identifiable. ■

As $\varphi(y) \in \mathcal{F}_\varphi$, then the area $\mathcal{D}_y$ have center $c_{D_y} \in J_{c_{D_y}}$, $J_{c_{D_y}}$ is some interval.

Len subset $\{u_{h,i}(t)\} \subset \mathrm{U_S} \subseteq \mathrm{U}$ ($i \geq 1$) which elements have the property of S-synchronizability exists. The framework $\mathcal{S}_{ey,i}(u_{h,i})$ has the diameter $D_{y,i}(\mathcal{D}_{y,i})$ and corresponds to every $u_{h,i}(t)$. As $u_{h,i}(t) \in \mathrm{S}$ the diameter $D_{y,i}$ has the property $\mathscr{d}_{h,\Sigma}$-optimality.

Let the hypothetical framework $\mathcal{S}_{ey}$ (a framework $\mathcal{S}_{ey}^{ref}$) of the system (45) have diameter $\mathscr{d}_{h,\Sigma}$.

Definition 7. The framework $\mathcal{S}_{ey,i}$ has $\mathscr{d}_{h,\Sigma}$-optimality property on the set U_h if $\varepsilon_\Sigma > 0$ such that $\left|\mathscr{d}_{h,\Sigma} - D_{y,i}\right| \leq \varepsilon_\Sigma \quad \forall i = \overline{1, \#\mathrm{U}_h}$.

Definition 8. If $\left(\{u_{h,i}(t)\} = \mathrm{U}_h \subset \mathrm{U}\right) \& \left(u_{h,i}(t) \in \mathrm{S}\right)$, $i \geq 1$ and frameworks $\mathcal{S}_{ey,i}(u_{h,i})$ have $\mathscr{d}_{h,\Sigma}$-optimality property, then frameworks $\mathcal{S}_{ey,i}(u_{h,i})$ are structurally indiscernible on sets $\{u_{h,i}(t)\}$.

So, the h_{δ_h}-identifiability estimate can be obtained from any input, following definitions 6, 7.

Definition 9. If frameworks $\mathcal{S}_{ey,i}(u_{h,i})$ have $\mathscr{d}_{h,\Sigma}$-optimality property, then $\mathcal{S}_{ey,i}(u_{h,i})$ is locally structurally identifiable on the set U_h.

Let the framework $\mathcal{S}_{ey,i}(u_{h,i})$ having $\mathscr{d}_{h,\Sigma}$-optimality property is $\mathcal{S}_{ey,i}^{\Sigma}$, and the locally structurally identified framework $\mathcal{S}_{ey,i}(u_{h,i})$ is $\mathcal{S}_{ey,i}^{LSI}$.

The framework $\mathcal{S}_{ey}$ is locally structurally identifiable on the set $\mathrm{U}_h \subseteq \mathrm{U_S}$ if

$$\left(\exists u_h \in \mathrm{S}\right) \text{ what } \left(\mathcal{S}_{ey} \cong \mathcal{S}_{ey}^{\Sigma}\right) \rightarrow \mathcal{S}_{ey} \cong \mathcal{S}_{ey}^{LSI}. \tag{63}$$

Remark 7. We consider nonlinearities satisfying condition (47). Therefore, notes made above are valid.

Definition 10. The framework $\mathcal{S}_{ey,i}(u_i \notin \mathrm{U_S})$ that does not have the $\mathscr{d}_{h,\Sigma}$-optimality property is locally structurally non-identifiable on the set U_h.

The framework $\mathcal{S}_{ey,i}(u_i \notin \mathrm{U_S})$ that is structurally non-identifiable on the set U_h defines a class $\mathcal{F}_\varphi^N \not\subset \mathcal{F}_\varphi$.

Remark 8. The described approach applies to the nonlinear system with a dynamic law of nonlinearity change. In this case, the hierarchical immersion method [43] is used for the structure estimation of the nonlinearity.

The identifiability of system S_y considered in [44]. Let the phase portrait $\mathcal{S}$ constructed for the system.

6.2. Non-Identifiability Degree

Obtain the non-identifiability degree estimate of the system (56). Definitional domains of $\mathcal{S}$ and $\mathcal{S}_{ey}$ are coincident. Therefore, the diameter $D\left(\mathcal{S}_{ey}\right)$ of the framework $\mathcal{S}_{ey}$ is known. Consider the set $\left\{u_i\left(t\right)\right\}$ having the property $\mathcal{PE}_{\alpha}$. Determine the framework $\mathcal{S}_{ey,i}$ for each $u_i\left(t\right)$ and obtain $D_{y,i}\left(\mathcal{S}_{ey,i}\right)$. Suppose $\mathcal{d}_{h,y}=\max_{u_i}\left|D_y\left(\mathcal{D}\left(\mathcal{S}_{ey,i}\right)\right)\right|$ and denote the corresponding input as u_h. Determine diameters $d_{y,j}=\left|D_{y,j}\left(\mathcal{D}\left[\mathcal{S}_{ey,j}\left(u_j\in\mathcal{U}\right)\right]\right)\right|$ for all inputs $\mathcal{U}=\left\{u_i\left(t\right)\right\}\setminus\left\{u_h\right\}$. Since $u_h\in\mathrm{S}$, therefore $\mathcal{d}_{h,y}>D_{y,j}$ $\forall j\geq 1$. As $u_h\in\mathrm{S}$, therefore $\mathcal{d}_{h,y}>D_{y,j}$ $\forall j\geq 1$. Then evaluate the non-identifiability degree as

$$\mathcal{SI}_j=\mathcal{SI}\left(\mathcal{S}_{ey,j}\right)=\frac{\mathcal{d}_{h,y}-d_{y,j}}{\mathcal{d}_{h,y}} \tag{64}$$

$\mathcal{SI}$ shows that S_{BW}-system (1) is structurally identifiable if $\mathcal{SI}_j\to 0$.

If estimates for the fragments $\mathcal{F}_{\mathcal{S}}^{l},\mathcal{F}_{\mathcal{S}}^{r}$ of the phase portrait $\mathcal{S}$ are known, then the identifiability degree is defined as

$$SI=SI\left(\mathcal{S}\right)=\frac{d_y^l\left(\mathcal{F}_{\mathcal{S}}^{l}\right)}{d_y^r\left(\mathcal{F}_{\mathcal{S}}^{r}\right)}, \tag{65}$$

where $d_y^l\left(\mathcal{F}_{\mathcal{S}}^{l}\right),d_y^r\left(\mathcal{F}_{\mathcal{S}}^{r}\right)$ are diameters of fragments $\mathcal{F}_{\mathcal{S}}^{l},\mathcal{F}_{\mathcal{S}}^{r}$. The system S_{φ} is structurally identifiable if $SI\left(\mathcal{S}\right)\leq O\left(1\right)$ where $O\left(1\right)$ is neighborhood 1.

Example 1. Consider the BWH from Section 3.4. Consider four variant inputs

$$\begin{aligned}&u_0\left(t\right)=2-2\sin\left(0.15\pi t\right),u_1\left(t\right)=2-2\sin\left(0.35\pi t\right),\\&u_2\left(t\right)=2-2\sin\left(0.5\pi t\right),u_3\left(t\right)=2-2\sin\left(0.15\pi t\right)+0.2\sin\left(0.35\pi t\right).\end{aligned} \tag{66}$$

Calculate diameters for the phase portrait definitional domain

$$D_{y,0}\left(\mathcal{S}_0\right)=3.75,\quad D_{y,1}\left(\mathcal{S}_1\right)=1.728,\quad D_{y,2}\left(\mathcal{S}_2\right)=1.08,\quad D_{y,3}\left(\mathcal{S}_3\right)=3.967. \tag{67}$$

Results obtained for the system $\mathcal{S}_{BW}$ steady state. The analysis showed $u_0\left(t\right)\in\mathrm{S}$. We assume that the system $\mathcal{S}_{BW}$ with the phase portrait $\mathcal{S}_0$ is the standard and $\mathcal{d}_{h,y}=D_{y,0}\left(\mathcal{S}_0\right)$. The degree of non-identifiability of the system $\mathcal{S}_{BW}$ for various u_i

$$\mathcal{SI}_1=0.549,\quad \mathcal{SI}_2=0.718,\quad \mathcal{SI}_3=-0.035. \tag{68}$$

We see that the $\mathcal{S}_{BW}$-system is structurally non-identifiable with u_1,u_2, and the $\mathcal{S}_{BW}$-system with input u_3 is structurally indistinguishable from input u_0. So, frameworks $\mathcal{S}_{ey,1}\left(u_1\right),\mathcal{S}_{ey,2}\left(u_2\right)$ are frameworks of class $\mathcal{NS}_{ey}$, and the framework $\mathcal{S}_{ey,3}\left(u_3\right)$ belongs to class $\mathcal{S}_{ey}^{LSI}$.

6.3. Hierarchical Immersion Method

If nonlinear processes are complex, then the model (49) will be inadequate. Then the hierarchical immersion method (HIM) [44] is used to design the $\mathcal{S}_{ey}$-framework. HIM realizes the subsequent stages of synthesis $\mathcal{S}_{ey}$ if the model (47) is inadequate. The method is based on the application (49) in a new structural space and the synthesis for $\tilde{\mathcal{S}}_{ey}$ new framework. If the new model (49) is significant, HIM stops. Otherwise, a new iteration is implemented.

6.4. SI and SID Bouc-Wen Hysteresis

Consider the BWH from section 3.4. Introduce the framework $\mathcal{S}_{ey}$ to estimate the S_{BW}-system structural identifiability. $\mathcal{S}_{ey}$ is the hysteresis estimation in the structural space $\mathcal{P}_{ey}$. Apply the model

$$\dot{\hat{x}} = -0.199x + 0.471f \tag{69}$$

and calculate the error $e = \dot{x} - \dot{\hat{x}}$.

The framework $\mathcal{S}_{ey}$ described by the mapping $s_{ey}: y(t) \to e(t)$ and is showed in Figure 7.

Apply the approach proposed in [45]. Draw the straight line parallel to the ordinate axis through point c_S. Obtain two fragments $\left(\mathcal{F}_{\mathcal{S}_{ey}}^{l}, \mathcal{F}_{\mathcal{S}_{ey}}^{r}\right) \subset \mathcal{S}_{ey}$. Determine secants for the left $\mathcal{F}_{\mathcal{S}_{ey}}^{l}$ and right $\mathcal{F}_{\mathcal{S}_{ey}}^{r}$ fragment

$$\begin{aligned} \gamma_e^l &= 0.0313y - 0.146,\ r_{ye,l}^2 = 0.912, \\ \gamma_e^r &= 0.032y - 0{,}15,\ r_{ye,r}^2 = 0.926, \end{aligned} \tag{70}$$

Let $h(\mathcal{S}_{ey})$ be the distance between the opposite sides of the framework $\mathcal{S}_{ey}$. The framework $\mathcal{S}_{ey}$ satisfies conditions of theorem 6. The height $h(\mathcal{S}_{ey}) \geq 0.02$, and the input $f(t)$ is constantly excited and S-synchronized. Therefore, the $\mathcal{S}_{ey}$-framework (system S_{BW}) is h_{δ_h}-identifiable. Figure 8 confirms this conclusion. Models (69) is *SM*-identifying.

Consider the structural identification of BWH. Apply the hierarchical immersion method for estimating the BWH structure. Calculate the derivative for e applying numerical derivation. This procedure is sensitive to calculation errors. Therefore, perform smoothing $\dot{e}(t)$ applying the Fourier transform.

Denote the obtained variable as $\varepsilon = \dot{e}$. Further analysis has shown that ε did not depend on x (see Figure 8). Thus, ε depends on $\dot{x}$ or z.

Consider the framework $\mathcal{S}_{\varepsilon\tilde{x}}$ described by the mapping $\Gamma_{\varepsilon\hat{x}}: \hat{\dot{x}} \to \varepsilon$, where $\hat{\dot{x}}$ is the estimate of the derivative $\dot{x}$. Determine the secant γ_ε for $\mathcal{S}_{\varepsilon\tilde{x}}$:

$$\gamma_{\varepsilon\hat{x}} = 0.0324\hat{\dot{x}},\ r_{\varepsilon\hat{x}}^2 = 0.86. \tag{71}$$

The model (71) presents in Figure 8. Therefore, Figure 8 and the model (71) confirm effect $\dot{x}$ on hysteresis properties.

Estimate the relationship between variables z and ε. Use the variable e as the estimation z. Apply the denote $\hat{z} = e$. The analysis shows $\hat{z}$ and ε not relates by the linear dependence. Therefore, the correlation between ε and the combination $\hat{z}$ and $\hat{\dot{x}}$ exists. Eliminate the effect of the linear component $\hat{\dot{x}}$ on ε. Obtain the variable $\vartheta = \varepsilon - \gamma_{\varepsilon\hat{x}}$. Go to into the space $\mathcal{P}_{\vartheta\mu} = (\vartheta, \mu)$, $\mu = |\hat{z}|^h \hat{\dot{x}},\ h > 0$.

The example of the relation estimation is shown in Figure 9, where $h = 0.5$. The secant $\gamma_{\vartheta\mu}$ framework $\mathcal{S}_{\vartheta\mu}$ has the form $\gamma_{\vartheta\mu} = 0.354\mu$, coefficient of determination $r_{\vartheta\mu}^2 = 0.82$. The parameter h cannot correspond to the parameter n in (3). The cause of such discrepancy follows from the proposed approach. True of BWM parameters estimates based on the use of the parametric identification.

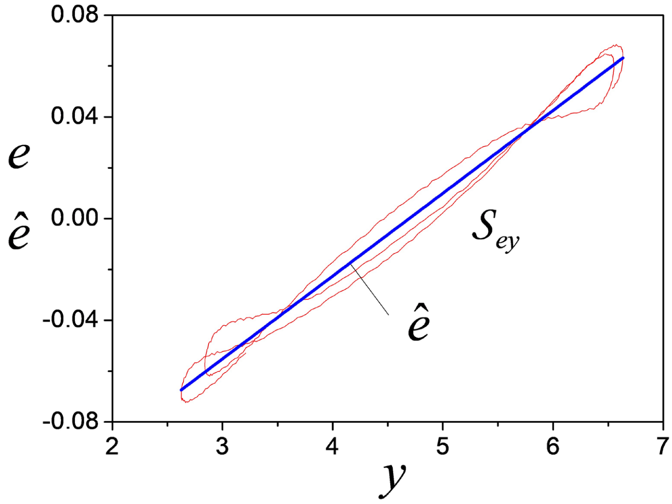

Figure 7. Framework S_{ey}.

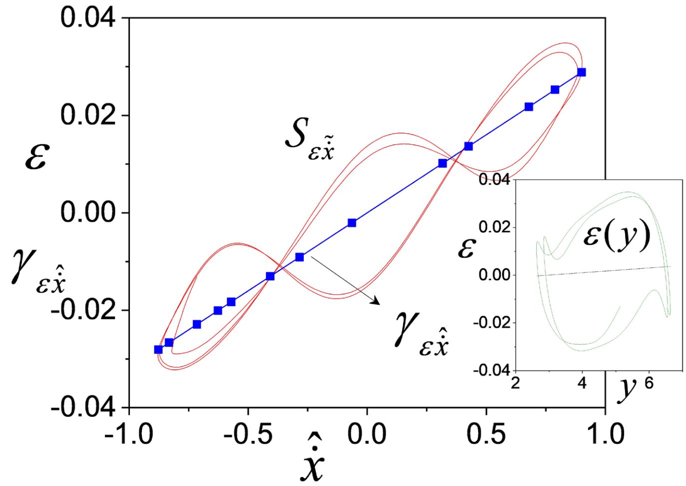

Figure 8. Framework for effect $\hat{x}$ estimation of S_{BW}-system.

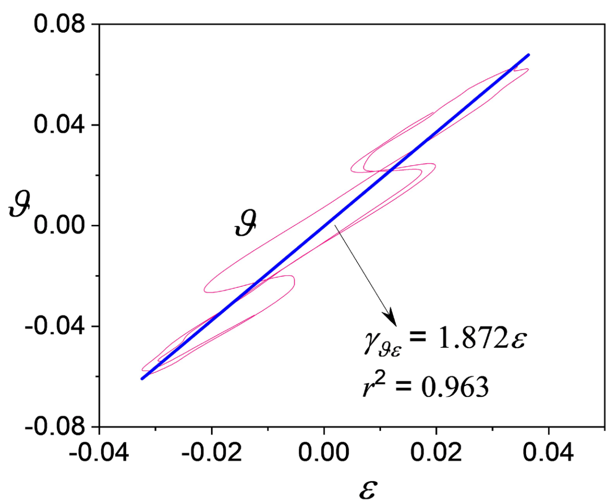

Figure 9. Estimation of correlation ε and ϑ.

The effect estimates of the variable μ can be obtained in space $\mathcal{P}_{\varepsilon\mu}$. This conclusion follows from **Figure 9**, where the dependency $\vartheta = \vartheta(\varepsilon)$ is presented.

Remark 9. Secant (71) can use as the output for estimating structural relationships in BWM.

Figure 10 confirms the validity of the proposed approach. The framework reflects the relationship between the reference and received hysteresis estimates. The secant $\gamma_{z\hat{z}}$ has the form

$$\gamma_{z\hat{z}} = 0.033z - 0.0068, \quad r_{z\hat{z}}^2 = 0.836. \tag{72}$$

So, the structure analysis has shown that the hysteresis dynamics depends on variables z and $\dot{x}$. The system (1), (2) output does not influence the change of the hysteresis. The structure analysis is based on the application of adequate mathematical methods and guaranteed decision-making on the structure of the system S_{BW}.

The HIM stop rule. Let $I_i(I_o)$ is an informational set on which the framework $\mathcal{S}_i$ is defined, where i is the hierarchical immersion level. Examples of sets $I_i(I_o)$ and frameworks are presented above. Let $\mathcal{NS}_{i+1}$ is the insignificant framework, and at the level i the system is structurally identifiable.

Let $\mathcal{NS}_{i+1}$ is the insignificant framework, and the system S_{BW} is structurally identifiable at the level i. $i+1$ is a sign of structural non-identifiability the system at the level $i+1$. $\mathcal{NS}_{i+1}$ is a sign of the system (1)-(3) structural non-identifiability at level $i+1$.

Theorem 9. The system S_{BW} is structurally identifiable on the set $I_i(I_o)$ if $\mathcal{S}_{i+1} = \mathcal{NS}_{i+1}$ at the level $i+1$.

The proof of theorem 9 follows from the analysis of secant for framework at each step i.

Figure 11 represents the framework $\mathcal{S}_{\tilde{\varepsilon}\tilde{\mu}} = \mathcal{NS}$ and the secant, where $\tilde{\mu} = |\hat{z}||\hat{\dot{x}}|$, $\tilde{\varepsilon} = \vartheta - \gamma_{\vartheta\tilde{\mu}}$. Obtain the model for the variable $\tilde{\varepsilon}$ on the set $\{\hat{\dot{x}}(t), \tilde{\mu}(t)\}$

$$\hat{\tilde{\varepsilon}} = \hat{a}_1\tilde{\mu} + \hat{a}_2\hat{\dot{x}}, \tag{73}$$

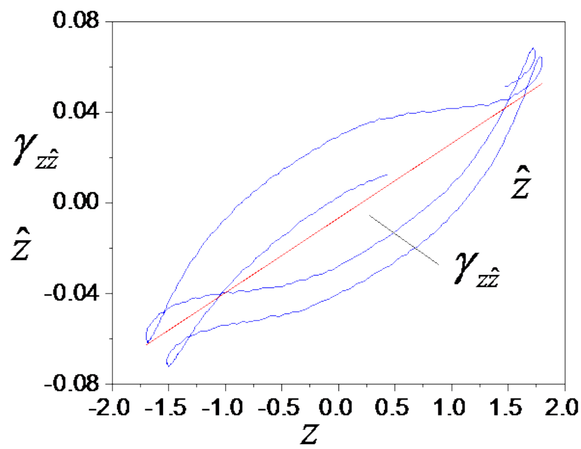

Figure 10. Estimation of proximity z and $\hat{z}$.

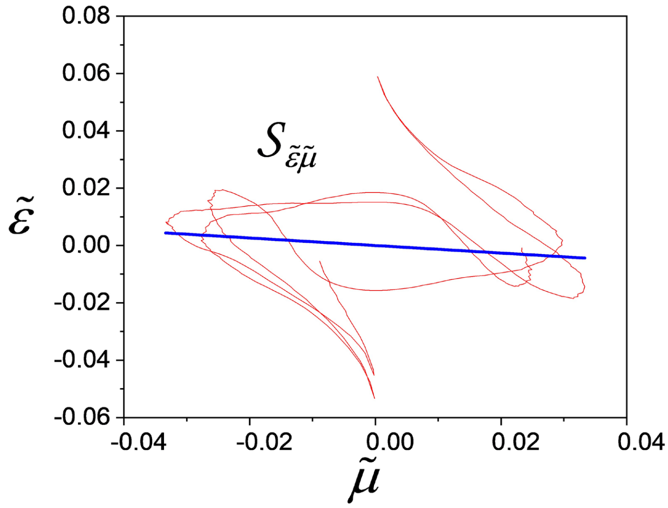

Figure 11. Insignificant framework $\mathcal{S}_{\tilde{\varepsilon}\tilde{\mu}}$.

and introduce the misalignment $\tilde{\kappa} = \tilde{\varepsilon} - \hat{\tilde{\varepsilon}}$. An approximation $\tilde{\kappa}$ by model $\hat{\tilde{\kappa}} = f(\tilde{\mu})$ shows that this relationship is insignificant. This conclusion confirms the presence of the third term in the right part of the Equation (3).

So, we propose the approach for structure estimating of the Bouc-Wen model based on the set I_o analysis. The approach is based on the hierarchical immersion method and the analysis of geometrical frameworks. Frameworks describe the state of the system nonlinear part at each SI stage.

7. Excitation Constancy Effect on System Identifiability

Let input $u(t)$ of the system (56) have the property

$$u(t) \in \mathcal{PEF}_{\alpha,\omega_k}, \tag{74}$$

where

$$u_h(t): (u_h \in \mathcal{PE}_\alpha) \& (u_h \in \mathcal{PF}_{\omega_h}) \& (u_h \in \mathrm{S}), \quad \mathcal{PF}_{\omega_h}: u_h(t) = \mathcal{RF}_h(\Omega_h), \tag{75}$$

$\mathcal{RF}(\Omega_k)$ is a model for $u_k(t)$ based on the Fourier series and given on the set of frequencies $\Omega_k = \{\omega_1, \omega_2, \cdots, \omega_k\}$.

Let $u_k \in \mathrm{U}_k$, $\mathrm{U}_k = \mathrm{U} \setminus \mathrm{U}_\mathrm{S}$. Consequently, $u_k \notin \mathrm{S}$. For $u_h \in \mathrm{S}$ is hold

$$u_h(t): (u_h \in \mathcal{PE}_\alpha) \& (u_h \in \mathcal{PF}_{\omega_h}) \& (u_h \in \mathrm{S}), \quad \mathcal{PF}_{\omega_h}: u_h(t) = \mathcal{RF}_h(\Omega_h), \tag{76}$$

where $\Omega_h \neq \Omega_k$.

Compare (75), (76) and obtain

$$\left(\mathcal{RF}_h(\Omega_h) \neq \mathcal{RF}_k(\Omega_k)\right) \Rightarrow \mathcal{S}_{ey}^h \neq \mathcal{S}_{ey}^k \Rightarrow \mathcal{S}_{ey}^k = \mathcal{NS}_{ey}. \tag{77}$$

From (77) have

$$\left(\mathcal{D}_y\left(\mathcal{S}_{ey}^h\right) \neq \mathcal{D}_y\left(\mathcal{S}_{ey}^k\right)\right) \Rightarrow \left[D_y\left(\mathcal{S}_{ey}^h\right) \geq D_y\left(\mathcal{S}_{ey}^k\right)\right]. \tag{78}$$

The definitional domain of frameworks $\mathcal{S}_{ey}^h, \mathcal{S}_{ey}^k$ do **not** coincide, and $\mathcal{S}_{ey}^h$ is $d_{h,\Sigma}$-optimal on the set U_h. Therefore, the fulfillment of condition (58) follows from inequality (78). Consequently, the structure of the system (45) nonlinear part with u_k has indicators that do not coincide with the structurally identifiable parameters of the system (45) with u_h.

So, the CE condition of the input affects the S_φ-system h_{δ_h}-identifiable and, consequently, the system (56).

The statement is true.

Theorem 10 [37]. Let (i) the input u_k to condition (75); (ii) the $\mathcal{S}_{ey}^k$-framework corresponds to the input u_k; (iii) there is the input $u_h \in \mathrm{S}$ such that the condition (76) satisfied; (iv) conditions (77), (78) holds. Then (a) the S_φ-system is structurally non-identifiable by the input u_k; (b) structural parameters of the S_φ-system do not correspond to the system $S_{y\varphi}$ with the identifiable framework $\mathcal{S}_{ey}^h$.

The input amplitude can influence on the SI of nonlinear systems. Modify conditions (75), (76)

$$u_k(t):(u \in \mathcal{PE}_\alpha)\&\left(u \in \mathcal{PF}_{\omega_k}\right)\&\left(\overline{u_h \in \mathrm{S}}\right),\quad \mathcal{PF}_{\omega_k}: u_k(t) = \mathcal{RF}_k(G_k, \Omega_k), \tag{79}$$

$$u_h(t):(u_h \in \mathcal{PE}_\alpha)\&\left(u_h \in \mathcal{PF}_{\omega_h}\right)\&\left(u_h \in \mathrm{S}\right),\quad \mathcal{PF}_{\omega_h}: u_h(t) = \mathcal{RF}_h(G_h, \Omega_h), \tag{80}$$

where G_k, G_h are model $\mathcal{RF}_k$, $\mathcal{RF}_h$ parameter vectors.

Present models $\mathcal{RF}_k$, $\mathcal{RF}_h$ as

$$\mathcal{RF}_h(G_h, \Omega_h) = g_h \widetilde{\mathcal{RF}_h}\left(\tilde{G}_h, \Omega_h\right),\quad \mathcal{RF}_k(G_k, \Omega_k) = g_k \widetilde{\mathcal{RF}_k}\left(\tilde{G}_k, \Omega_k\right), \tag{81}$$

$\widetilde{\mathcal{RF}_h}\left(\tilde{G}_h, \Omega_h\right)$, $\widetilde{\mathcal{RF}_k}\left(\tilde{G}_k, \Omega_k\right)$ are modifications of models (78), (76); $g_h = \max_i g_{h,i}$, $i = \overline{1, \#\Omega_h}$, $g_{h,i}$ is an element G_h; $g_k = \max_i g_{k,i}$, $i = \overline{1, \#\Omega_k}$. g_p ($p = k, h$) denotes the generalized amplitude of the input.

Condition (77) transformed into the form

$$g_h \widetilde{\mathcal{RF}_h}\left(\tilde{G}_h, \Omega_h\right) \neq g_k \widetilde{\mathcal{RF}_k}\left(\tilde{G}_k, \Omega_k\right). \tag{82}$$

Since $u_h \in \mathrm{S}$ then $g_h \geq g_k$. This conclusion follows from

$$D_h\left(\mathcal{S}(u_h)\right) \geq D_k\left(\mathcal{S}(u_k)\right) \Rightarrow \left|\widetilde{\mathcal{RF}_h}\left(\tilde{G}_h, \Omega_h\right)\right| \geq \left|\widetilde{\mathcal{RF}_k}\left(\tilde{G}_k, \Omega_k\right)\right|, \tag{83}$$

and the model $\widetilde{\mathcal{RF}_h}\left(\tilde{G}_h, \Omega_h\right)$ approximates the input ensuring S-synchronization of the system $S_{y\varphi}$.

Obtain $d_{h,\Sigma}$-optimality of the diameter $D_h\left(\mathcal{S}_{ey}^h\right)$ from $\mathcal{S}(u_h) \Rightarrow \mathcal{S}_{ey}^h$. The framework $\mathcal{S}_{ey}^k$ does not have this property (see (80)). Therefore, the input $u_k \notin \mathrm{S}$, which has a smaller generalized amplitude, gives the diameter $D_k\left(\mathcal{S}_{ey}^k\right)$.

Theorem 11 [43]. Let (i) the input u_k of the system (45) satisfies the condition (79); (ii) the framework $\mathcal{S}_{ey}^k$ corresponds to input u_k; (iii) there is the input $u_h \in \mathrm{S}$ such that the condition (80) holds; (iv) conditions (77), (78) are hold. Then (a) the S_φ-system is structurally non-identifiable by the input u_k; (b) structural parameters of the system S_φ do not correspond to the system (45) with the identifiable framework $\mathcal{S}_{ey}^h$ if $g_h \geq g_k$.

So, the properties influence of input on SI and the structural identifiability of the system with BWH show.

8. Conclusion

The estimate problem of Bouc-Wen hysteresis parameters is relevant under uncertainty. The existing approaches to the identification are based on the parametric paradigm and consider a priori information. Under uncertainty, the BWM synthesis requires time-consuming research. The parametric approach plays an approximative role for a given a priori model structure. It allows you to describe the behaviour of the system or set trends in its development. The structure is a hidden and non-formalized property of the system. Therefore, indirect and object methods should be used that reveal the features (structure) of the system. The paper proposes a structural-identification approach (CIA) for analyzing features of the Bouc-Wen hysteresis under uncertainty. Geometric frameworks are the basis of the CIA. The GF analysis allows for the evaluation of the BWH structure and identifiability. The proposed approaches demonstrate the possibilities of the stated paradigm.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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