Scientific Research Publishing

Journal of Geoscience and Environment Protection, 2022, 10, 46-53
https://www.scirp.org/journal/gep
ISSN Online: 2327-4344
ISSN Print: 2327-4336

An Algorithm to Determine the Truncated Weibull Parameters for Distribution of Throats and Pores in Random Network Models

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How to cite this paper: Shi, F., Wu, J. D., Chang, T., Sun, C. J., & Wu, X. Z. (2022). An Algorithm to Determine the Truncated Weibull Parameters for Distribution of Throats and Pores in Random Network Models. *Journal of Geoscience and Environment Protection, 10,* 46-53.
https://doi.org/10.4236/gep.2022.1010004

Received: August 25, 2022
Accepted: October 22, 2022
Published: October 25, 2022

Abstract

In random network models, sizes for pores and throats are distributed according to a truncated Weibull distribution. As a result, parameters defining the shape of the distribution are critical for the characteristic of the network. In this paper, an algorithm to distribute pores and throats in random network was established to more representatively describe the topology of porous media. First, relations between Weibull parameters and the distribution of dimensionless throat sizes were studied and a series of standard curves were obtained. Then, by analyzing the capillary pressure curve of the core sample, frequency distribution histogram of throat sizes was obtained. All the sizes were transformed to dimensionless numbers ranged from 0 to 1. Curves of the core were compared to the standard curves, and truncated Weibull parameters could be determined according an inverse algorithm. Finally, aspect ratio and average length of throats were adjusted to simultaneously fit the porosity and the capillary pressure curves and the whole network was established. The predicted relative permeability curves were in good agreement with the experimental data of cores, indicating the validity of the algorithm.

Keywords

Random Network Models, Capillary Pressure Curve, Average Dimensionless Throat Sizes, Truncated Weibull Distribution

1. Introduction

Network models have been widely used to study fluid flow in porous media. There are two general kinds of networks: networks that representatively describe the topology of real core samples and random networks which only capture a few characteristic of porous media. The former ones can be used to accurately predict experimental data such as relative permeabilities and oil recovery (Øren et al., 1998), but constructing this kind of networks is costly. By contrast, random networks are founded with very low cost and mainly used to investigate non-Newtonian flow (Lopez et al. 2003; Taha, 2007) and microscopic mechanism of enhanced recovery of oil such as polymer flooding (Sun et al., 2011) or surfactant flooding (Bo et al., 2003, SPE84906). What's more, wetting condition plays a significant role in determining transport properties in porous media (Romero-Zeron, 2009; Rostami et al., 2010; Kasin, 2011) and random networks can qualitatively describe wettability effects (Zhao et al., 2010) if wetting conditions can be properly or quantitatively expressed (Øren et al., 2003; Feng et al., 2012) and random networks are representative. So, parameter optimization to construct valid networks is of great importance.

In network models, pores and throats are approximated as cylindrical ducts of arbitrary shapes and different sizes of cross-sections (Patzek, 2000). This is suitable for studying seepage characteristics under the influence of different parameters through stochastic network model. However, no method has been studied and proposed for assigning pore throat parameters of the network model so that the constructed model could represent the pore throat parameters and seepage characteristics of the actual core for specific oilfields.

In this paper, a new algorithm is established to calculate the truncated Weibull parameters which determine the sizes of throats and play critical roles in determining the shape of capillary pressure curves and relative permeability curves.

2. Methodology

2.1. Standard Curves of Dimensionless Average Throat Size vs. Weibull Parameters

Sizes of throats are distributed according to truncated Weibull function, which can be written as:

$$r=\left(r_{\max}-r_{\min}\right)\left(-\delta\ln\left[x\left(1-e^{-1/\delta}\right)+e^{-1/\delta}\right]\right)^{1/\gamma}+r_{\min} \tag{1}$$

In the formula, $r_{\max}$ and $r_{\min}$ are the inscribed radius of the biggest and smallest throats respectively. x is a random number ranged from 0 to 1. δ and γ are Weibull parameters.

Assume that there are $(n+1)$ throats in random network. If network is sufficiently big, x in Equation (1) are statistically evenly valued as $\frac{0}{n}$, $\frac{1}{n}$, $\frac{2}{n}$, and so on with the biggest value $\frac{n}{n}$. Sizes of the throats are transformed to dimensionless numbers between 0 and 1:

$$r_D=\frac{r}{r_{\max}} \tag{2}$$

As a result, $r_{D\max} = \frac{r_{\max}}{r_{\max}} = 1$ and $r_{D\min} = \frac{r_{\min}}{r_{\max}} \approx 0$. Number the throats in order of size. Using Equation (1) and Equation (2), the dimensionless radius of all throats are

$$\begin{cases} r_{D0} = (1 - r_{D\min})\left(-\delta \ln\left[\frac{0}{n}\left(1 - e^{-1/\delta}\right) + e^{-1/\delta}\right]\right)^{1/\gamma} + r_{D\min} \\ r_{D1} = (1 - r_{D\min})\left(-\delta \ln\left[\frac{1}{n}\left(1 - e^{-1/\delta}\right) + e^{-1/\delta}\right]\right)^{1/\gamma} + r_{D\min} \\ \vdots \\ r_{Dn} = (1 - r_{D\min})\left(-\delta \ln\left[\frac{n}{n}\left(1 - e^{-1/\delta}\right) + e^{-1/\delta}\right]\right)^{1/\gamma} + r_{D\min} \end{cases} \quad (3)$$

The dimensionless average throat size can be calculated by

$$\overline{R}_D = \frac{\sum_{i=0}^{n} r_{Di}}{n+1} = \frac{1 - r_{D\min}}{n+1} \sum_{i=0}^{n} \left(-\delta \ln\left[\frac{i}{n}\left(1 - e^{-1/\delta}\right) + e^{-1/\delta}\right]\right)^{1/\gamma} + r_{D\min} \quad (4)$$

Relationship of $\overline{R}_D$ versus Weibull parameters are depicted in semi logarithmic coordinates, as shown in Figure 1.

In Figure 1, when δ is fixed, $\overline{R}_D$ decreases linearly with increase of $\lg\gamma$. While γ is fixed, $\overline{R}_D$ also decrease as δ increases. The bigger δ is, the less it influences $\overline{R}_D$.

2.2. Standard Curves of Frequency Distribution of Dimensionless Throat Size under Different Weibull Parameters

Classify the range from 0 to 1 evenly into ten small intervals: 0 to 0.1, 0.1 to 0.2 and so on. Values between 0 and 0.1 are all considered as the value of 0.05, et al., for reasons of convenience. Calculate all dimensionless throat sizes with Equation (3) and distribution frequency are obtained by adding up those dimensionless throat sizes which are valued in the same small intervals.

The frequency distribution curves under different γ are shown in Figure 2

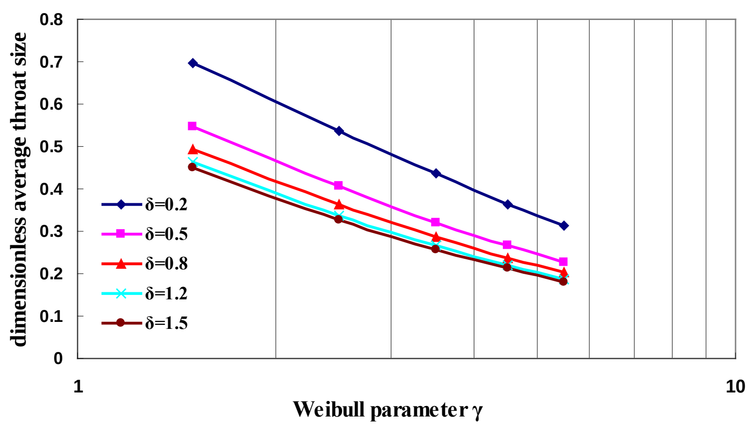

Figure 1. Relations between dimensionless average throat size and Weibull parameters.

with δ fixed as 0.2 and in **Figure 3** with δ fixed as 0.8.

Peak dimensionless throat sizes, $r_{D,peak}$, are the dimensionless throat sizes that correspond to the biggest frequency. From **Figure 2** and **Figure 3**, when δ is fixed, peak dimensionless throat sizes decrease monotonously as γ increases.

That is to say, more throats are assigned small values in these conditions.

2.3. Data Acquisition and Pretreatment of the Core Sample

Capillary pressure curves of the rock core sample are acquired by mercury intrusion method and porosity are calculated after measurement of void space and bone volume of the core in laboratory. According to the capillary pressure curves, frequency distribution histograms of throat sizes are obtained. $P_{S\min}$, the pressure corresponds to the minimum wetting phase saturation and P_T, the threshold capillary pressure can be read from the mercury intrusion curves (Qin & Li, 2006).

The maximum and minimum throat size, $r_{\max}$ and $r_{\min}$, can be calculated by replacing P_c in Equation (5) with P_T and $P_{S\min}$ respectively:

$$r = \frac{2\sigma\cos\theta}{P_c} \tag{5}$$

In which σ is the surface tension of the mercury-air system used in the core experiment and θ is mercury-rock contact angle.

2.4. Numerical Algorithm of Weibull Parameters for Throat Distribution

Transform the frequency distribution histogram of throat sizes into frequency distribution curve of dimensionless throat sizes with Equation (2) through which peak dimensionless throat size can be read. Dimensionless average throat size $\bar{R}_D$ can be obtained by analysis of frequency distribution histogram (Qin & Li, 2006) and calculation with Equation (2). The following steps are carried out to determine the parameters γ and δ,and the flowchart is shown in **Figure 4**.

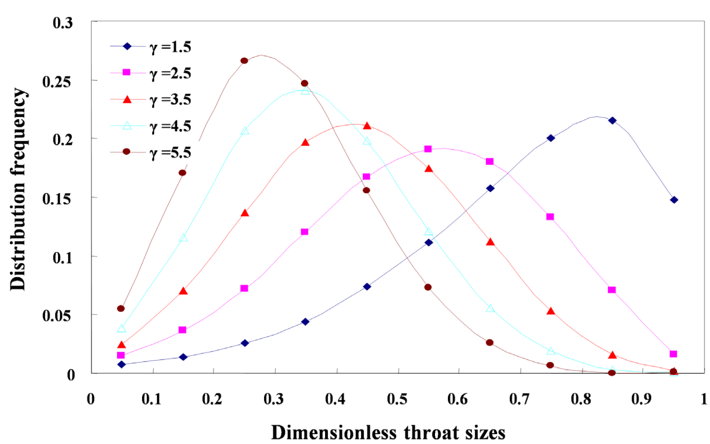

Figure 2. Frequency distribution curves of dimensionless throat sizes when $\delta = 0.2$.

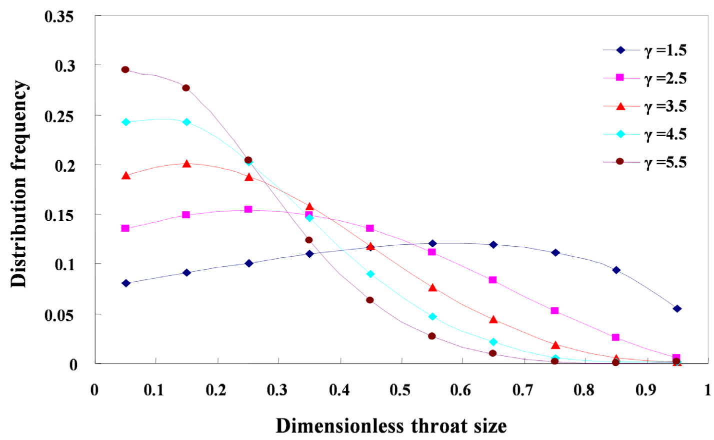

Figure 3. Frequency distribution curves of dimensionless throat sizes when $\delta = 0.8$.

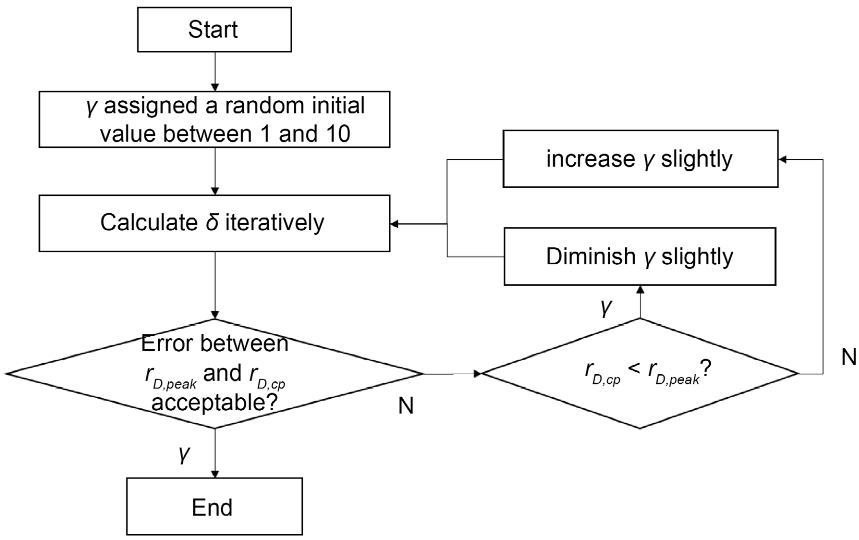

Figure 4. Flowchart of the proposed algorithm.

1) γ is assigned a random initial value between 1 and 10.

2) Calculate δ iteratively using Equation (4) or by Figure 1 with known $\bar{R}_D$ and γ.

3) With current δ and γ, the standard frequency distribution curve are determined (like those in Figure 2 or Figure 3) and its $r_{D,peak}$ is compared to the core's peak dimensionless throat sizes $r_{D,cp}$. If $r_{D,cp} < r_{D,peak}$, diminish γ slightly; If $r_{D,cp} > r_{D,peak}$, increase γ slightly.

4) Repeat (2) and (3) until the relative error between $r_{D,cp}$ and $r_{D,peak}$ are allowable.

5) δ and γ are determined and throat size distribution of the random network is obtained together with the upper and lower limits of the throat size.

3. Research Results and Discussion

In networks, aspect ratio, the ratio between the pore radius and connecting throat radius, is used to distribute pore sizes, which can be expressed as

$$\alpha = \frac{r_{pore}}{r_{throat}} \tag{6}$$

According to spatial correlation principle, size of pore is larger than that of all its connecting throats. Known the throat distribution and aspect ratio (Lu et al., 1997; Sun, 2002), pore size can be obtained by

$$r_p = \max\left(\alpha\frac{\sum_{i=1}^{n_C} r_i}{n_C}, \max\left(r_i\right)\right) \tag{7}$$

in which n_c is the numbers of connecting throats of the pore.

In random networks, pores are distributed in regular node. Throat length is the distance between its connecting pores minus the radius of the two pores. So, the grid of networks determines the length of throats and further the porosity. If the grid lines are denser, pores are closer to each other and the porosity of the network is larger while the grid lines are looser, pores are farther away from each other, resulting in long throats, more bone volume and low porosity.

Core sample is selected. Experimental and treated data are listed in **Table 1**. Its capillary curves and imbibitions relative permeability curves are depicted in solid lines in **Figure 5** and **Figure 6** respectively.

Table 1. Experimental data of core sample and fluid.

Porosity	0.387
Throat size	1.93 μm - 8.96 μm
Oil-water tension in waterflooding	30 mN/m
Average coordinate number	4.27

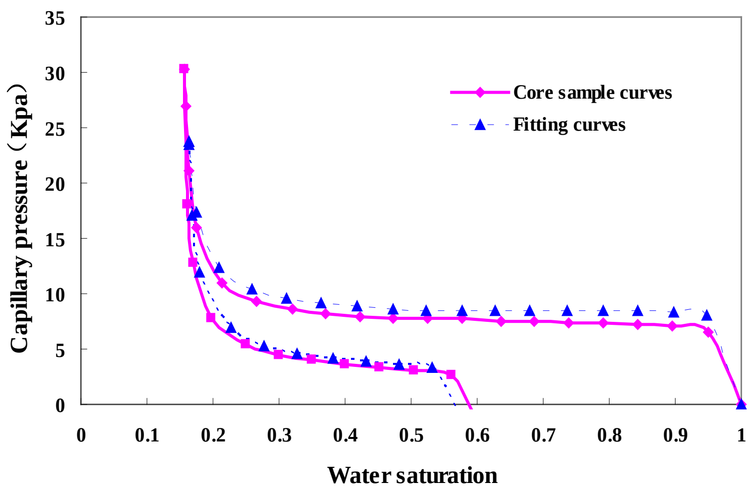

Figure 5. Capillary pressure curves from core sample and fitted curves by network model.

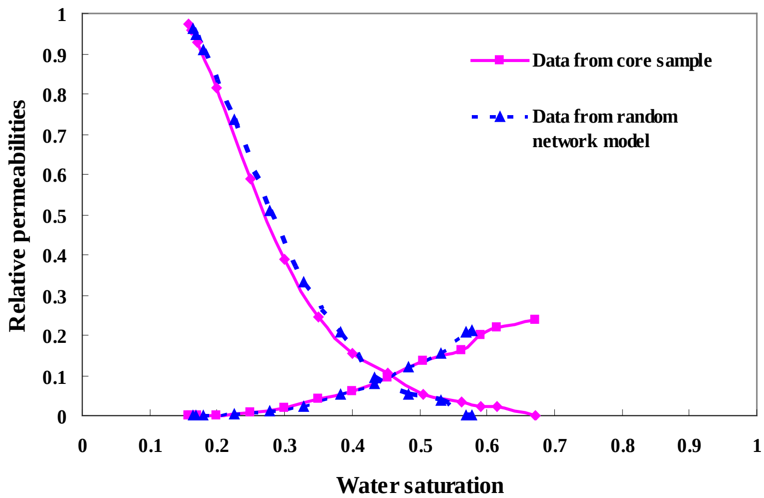

Figure 6. Relative permeability curves from core sample and from random network model.

According to the algorithm, $\delta = 0.18$ and $\gamma = 2.7$. Adjust the density of grid lines to fit the porosity with fine tuning coordinate numbers to simultaneously fit the capillary curves (dotted lines in Figure 4). Porosity of the network is 0.382 with an average coordinate number of 4.43.

Simulate water flooding with Valvatne's modeling code. Relative permeability curves with the random network are dotted in Figure 5. From the Figure, the fitted curves are in good agreement with the experimental data from core sample, which validate the method to determine the truncated Weibull parameters for distribution of throats and pores.

4. Conclusion

An algorithm is established to determine the truncated Weibull parameters for distribution of throats and pores in random network models using this method, founded networks can more accurately and representatively describe the topology of rock cores.

In random network models, the two Weibull parameters determine the sizes of throats and play critical roles in determining the shape of capillary pressure curves and relative permeability curves. Knowing the dimensionless peak throat size and average throat size, Weibull parameters can be determined with iterations. Density of grid lines in random network has great impact on porosity. The denser the grid lines are, the larger porosity of the network is.

In future, pore throat parameters assigning methods based on artificial intelligence algorithms might be an interesting direction for further research.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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