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Determination of a Mathematical Model of Erosion Taking into Account the Intensity of Rainfall and Soil Slopes from the Global MNT Model

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Abstract

Our study is being carried out in the Wouri Estuary more precisely in the Nylon area, Douala. This area is influenced by abundant rainfall which promotes the phenomenon of rain erosion. This erosion contributes to the degradation of structures and soils. To better understand and predict this phenomenon of rainfall erosion, we set out to establish a mathematical model that takes into account precipitation and topography. To this end, the data collected in the field and in the laboratory made it possible. First, we graphically modeled the variation of the potential as a function of the intensity of rainfall and the slope of the ground. Next, we identified a mathematical model from cubic spline surface interpolation. Finally, we obtained the mathematical model which makes it possible to evaluate and predict the erosion potential. The results obtained allowed to have an erosion potential of 153.67 t/ha/year with field data and 153.94 t/ha/year with laboratory data. We compared the results obtained with those existing in the literature on the same study site. This comparison made it possible to validate the established mathematical model. This mathematical model is a decision support tool and can predict problems related to water, erosion and the environment.

Keywords

Erosion, Erosion Potential, Mathematical Model, Rainfall Intensity, Soil Slope

1. Introduction

Soil erosion is the process of detachment, transport and deposition of soil particles [1]. In general, erosion is responsible for the variation in relief and the degradation of structures. Several types of erosion are mentioned in the literature, in particular water erosion, a natural phenomenon which can worsen under the combined action of particular climatic and anthropogenic conditions [2]. The quantity of ejected particles varies as a function of the kinetic energy of precipitation [3] [4]. The interest of the study relates to the quantification of the displaced masses via models. To do this, the methods used in the assessment and mapping of erosion (current and/or risk) vary according to the objectives, means and scales of work. Quantification can be done by direct measurements and indirect evaluations (Mathematical Modeling). Direct measurements (topographic measurements, rainfall simulation) constitute the mainstay of the methods developed to quantify the phenomenon of erosion [5] [6]. Topographic measurements ensure the follow-up of the topographic evolution of the soil surface after each rainfall event while the rainfall simulation makes it possible to determine certain hydrodynamic characteristics of soils on a small scale and under various rain conditions and soils [7] [8]. Mathematical Modeling with Time has made it possible to develop a particular scientific language which translates into a set of équations which aim to reproduce the behavior of complex systems in space and time. Specialized computer models play an important role in the agro-environmental management of a watershed because they make it possible to simulate the impact of agricultural activities and conservation measures on water quality [9] [10]. These prediction tools, most often deterministic and spatialized, integrate and put into practice recent knowledge concerning the mechanisms of tearing, transport and sedimentation. Since the agri-environmental issue has attracted more and more attention from government agencies in recent years, research groups and consulting firms have adopted various procedures to assess pollution from diffuse sources. Many of these procedures involve the development and use of computer models to perform reliable and repetitive simulations. More recently, some of these models were coupled with geographic information systems (GIS) to facilitate the data management and speed up task processing. The global MNT model (Mbiakouo-Njeugna-Toko) [8] makes it possible to model the relief of soils and to quantify the masses of displaced sediments that influence structures, dwellings and watersheds. This model integrates topographic data and coordinates of land points. This model has the advantage of modeling the relief and estimating the amounts of sediment displaced without using GIS, remote sensing and other digital satellite models which are very expensive. To help fight against this phenomenon, following the work of [8], we proposed to establish a mathematical model capable of determining and predicting the erosion potential by taking into account the intensity parameters of rainfall, soil slope and erosion potential from the global MNT model. To do this, we will perform a graphic modeling to illustrate the variations in the erosion potential according to the intensity of rain and the slope of the ground, and determine the mathematical model.

2. Materials and Methods

2.1. Presentation of the Study Site

The site submitted to our study is the Nylon area. The Nylon zone is located in the district municipality of Douala III, on the south-eastern fringe of the agglomeration. The district covers an area of approximately 113 km^2 for an estimated population of 1,350,000 inhabitants and is head quarter in Logbaba. With its 380,000 inhabitants, the Nylon zone is one of the most populous sectors of the city of Douala. It is made up of around fifteen neighborhoods (Bilongue, Vie Tranquille, Oyack, Diboum, Ndogpassi, etc.) which are mostly very precarious, far from the city center and poorly served. The Nylon district, studied within the framework of this subject, is part of the zone bearing the same name. As part of our study, we are going to work at the level of Dakar block III of the said zone, just from the [8]. **Figure 1** below illustrates the ground plan of the study area.

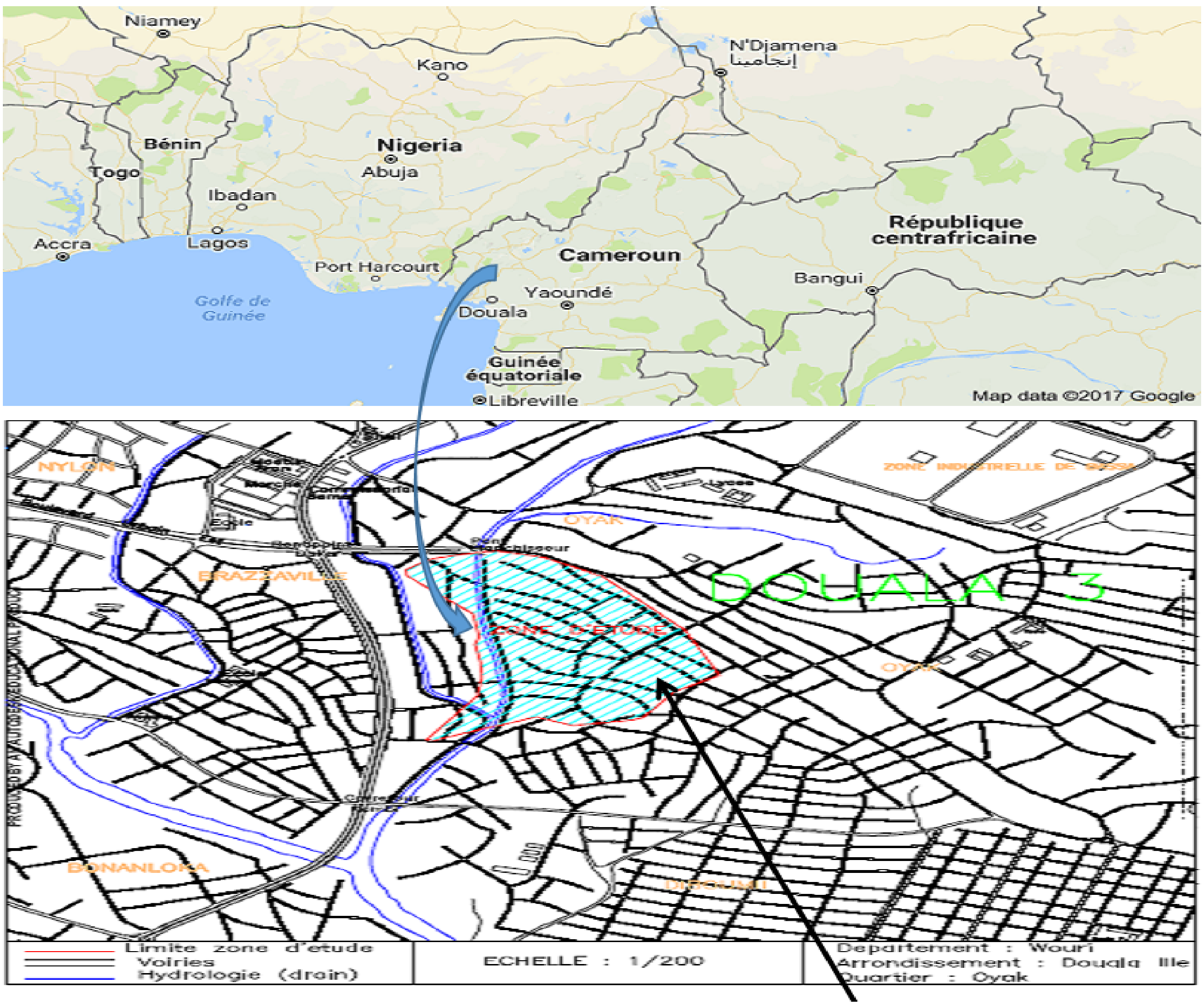

Figure 1. Ground plan showing the location of the study area (Mbiakouo, 2018).

2.2. Materials and Methods

In order to establish a mathematical model that better reflects the phenomenon of water erosion and makes it possible to quantify the masses of sediment displaced, we used two methods for data collection: the NEME-type mini rain simulator, field data developed in the global MNT model [8]. The data collected by these methods include rainfall intensities, soil slopes and erosion potential. Following the data collection, a graphical analysis which leads to a mathematical model is made.

2.3. Graphical Analysis and Mathematical Modeling

Graphical analysis is done with MATLAB R2017a software. It consists of generating the graphs which highlight the best interpolation between the data vectors (rainfall intensities, soil slopes and erosion potential) and gives as output the equivalent mathematical model.

After having generated the graph which makes it possible to obtain the best interpolation between the parameters of rainfall intensity, soil slopes and erosion potential, we look for the general form of the equivalent mathematical model. Note that the best interpolation is obtained by cubic spline. This phase is first done in mesh rectangles of the study area. Next it is crucial to determine the parameters of the model. Once this equation has been found, we extrapolate to the level of the watershed to bring out the desired mathematical model. Then, a comparison of the results is made with those obtained by [8] to validate the mathematical model obtained.

2.4. Determination of the Parameters of the Mathematical Model

The interpolation makes it possible to obtain the global model whose general shape of the surface adjustment by cubic spline is mentioned in equation (II.1) [11] and whose parameters $L_i(x)$, $L_j(y)$ and $M\_(i, j)$ are to be determined.

$$\boxed{S(x,y)=\sum_{i=0}^{N}\sum_{j=0}^{N}L_i(x)L_j(y)z_{i,j}} \tag{1}$$

With:

$$\begin{aligned}L_i(x)=\frac{1}{h}\Bigg[&\frac{(x_j-x)^3}{3!}M_{i,j-1}+\frac{(x-x_{j-1})^3}{3!}M_{i,j}+(x_j-x)\left(\delta_{i,j-1}-\frac{h^2}{3!}M_{i,j-1}\right)\\&+(x-x_{j-1})\left(\delta_{i,j}-\frac{h^2}{3!}M_{i,j}\right)\Bigg]\end{aligned} \tag{2}$$

$$\begin{aligned}L_j(y)=\frac{1}{h}\Bigg[&\frac{(y_i-y)^3}{3!}M_{j,i-1}+\frac{(y-y_{i-1})^3}{3!}M_{j,i}+(y_i-y)\left(\delta_{j,i-1}-\frac{h^2}{3!}M_{j,i-1}\right)\\&+(y-y_{i-1})\left(\delta_{j,i}-\frac{h^2}{3!}M_{j,i}\right)\Bigg]\end{aligned} \tag{3}$$

And

$$h\left(M_{i,j-1}+4M_{i,j}+M_{i,j+1}\right)=\frac{6}{h}\left(\delta_{i,j-1}-2\delta_{i,j}+\delta_{i,j+1}\right) \tag{4}$$

$$h=\frac{x_N-x_0}{N} \tag{5}$$

In the case of data collected in the field, $N = 9$. In the case of data collected in the laboratory $N = 8$. For reasons of simplicity we have chosen as boundary conditions "free ends" also called in this case "natural cubic splines". This condition adds two equations:

$$M_{i,0}=0 \text{ et } M_{i,N}=0 \tag{6}$$

Matrix expression established allowing to determine $M\_(i, j)$

$$h\begin{pmatrix} 4 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 1 & 4 & 1 & \ddots & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 & 4 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} M_{0,1} \\ M_{0,2} \\ \vdots \\ \vdots \\ M_{N,N-3} \\ M_{N,N-2} \\ M_{N,N-1} \end{pmatrix}=\frac{6}{h}\begin{pmatrix} \delta_{0,0}-2\delta_{0,1}+\delta_{0,2} \\ \delta_{0,1}-2\delta_{0,2}+\delta_{0,3} \\ \vdots \\ \vdots \\ \delta_{N,N-4}-2\delta_{N,N-3}+\delta_{N,N-2} \\ \delta_{N,N-3}-2\delta_{N,N-2}+\delta_{N,N-1} \\ \delta_{N,N-2}-2\delta_{N,N-1}+\delta_{N,N} \end{pmatrix}$$

A code under MATLAB R2017a made it possible to determine all the values of $M(i, j)$. The Equations (1)-(6) made it possible to determine $S(x, y)$.

$$\begin{aligned} S(x,y)=&L_1(x)L_6(y)z_{1,6}+L_2(x)L_7(y)z_{2,7}+L_3(x)L_8(y)z_{3,8} \\ &+L_4(x)L_1(y)z_{4,1}+L_5(x)L_9(y)z_{5,9}+L_6(x)L_2(y)z_{6,2} \\ &+L_7(x)L_3(y)z_{7,3}+L_8(x)L_5(y)z_{8,5}+L_9(x)L_4(y)z_{9,4} \end{aligned} \tag{7}$$

We proceeded in the same way with the data obtained from the mini rain simulator and made an extrapolation in the whole of the watershed whose area is estimated at 24.223 ha for slopes of the order of 9.64% and a coefficient run off $C_r = 0.25$.

The relation (II.9) [8] made it possible to determine the different values of the rainfall intensities.

$$Q=\frac{8760\times C_r\times A\times I}{360} \tag{8}$$

The overall approach which makes it possible to obtain the desired mathematical model is presented in appendix (I).

3. Results and Discussions

In this part of the work, it is a question of presenting the results of the graphic analysis obtained from the data from the field (**Figure 2(a)**) and the mini rainfall

Table 1. Flow rate values l/s [8].

i	0	1	2	3	4	5	6	7	8
Q	0.13	0.575	0.69	0.72	0.725	0.98	1.4	2.07	2.55

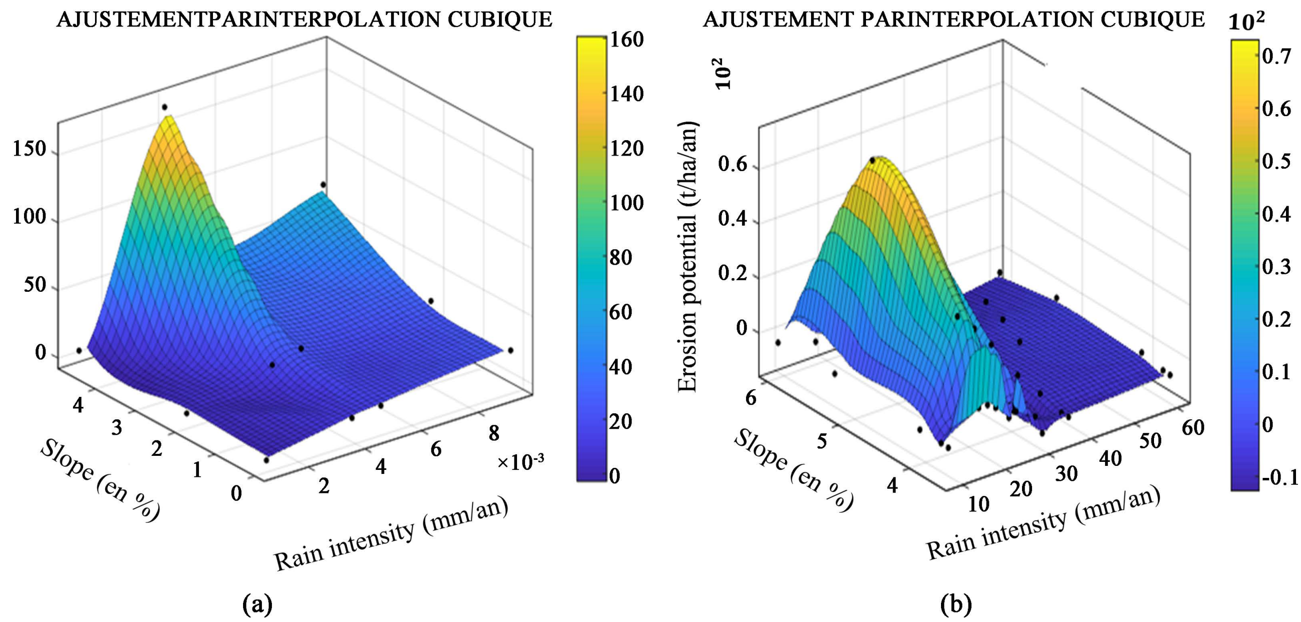

Figure 2. Surface obtained from data from the mini rainfall simulator (a) and the terrain (b).

simulator (**Figure 2(b)**). It also involves presenting the mathematical model obtained and comparing the results of the erosion potentials obtained from each of the methods used.

3.1. Modeling of the Erosion Phenomenon

3.1.1. Results and Discussion of Graphical Analysis

Observation of these curves shows that the interpolation by cubic spline best reflects the graphical modeling of the data involved in the graphic analysis. These graphs also highlight the influence of slopes and abundant rainfall on the erosion potential. The greater the slope and the abundant precipitation, the greater the potential for erosion. As precipitation increases, the cohesion of soil particles (sandy loam-clay) becomes established and helps reduce the amount of sediment displaced. These low values of the erosion potential can also be explained by the fact of the screening effect due to the thickness of the layers of water above the surfaces. This water table (**Table 1**) with the high slope favors the run off which through the tangential force loosens the soil particles in a reduced way, hence the reduction of the erosion potential. The interpolation of the data from the mini simulator and those collected in the field and used in the global DEM model makes it possible, after graphical analysis, to obtain mathematical models which make it possible to predict the erosion potential in time and space.

After using the data from the field and from the laboratory, we obtained the following mathematical models at the plot scale:

- Model obtained with field data

$$S(x,y)=\frac{1}{h^2}\left[ax^3y^3-bx^3y+cxy^3-dxy\right] \quad (9)$$

- Model obtained with laboratory data:

$$S(x,y)=\frac{1}{h^2}\Big[A(0.017-x)^3(3.199-y)^3+(B+Cy)(0.017-x)^3 \\ +(D+Ex)(3.199-y)^3+Fxy+Gx+Hy+I\Big] \quad (10)$$

An extrapolation on the field of study makes it possible to obtain a global model on the watershed:

$$\boxed{P_{oT}=A_o\times S(x,y)} \quad (11)$$

3.1.2. Assessment of Erosion Potential

From the previous model, it is up to us to determine the erosion potential and compare the results with those obtained by [8]. To achieve this, we used data on the average annual rainfall (4190 mm) for the Douala estuary, slopes of around 3% on average. The results presented in **Table 2** below are those obtained by the mathematical model taking into account the data obtained by the mini rain simulator and those obtained in the field and developed in the global DEM model.

These values of the erosion potential obtained, being in agreement with those of [8] represent the quantities of displaced sediment which contributed to burying the lowlands of the watershed and silting up the various drains. If we suppose that these values increase in a very short time, we will observe the foundations of the houses entirely stripped, the structures and superstructures started then weakened, the houses which will be completely covered in the lowlands and the watershed sufficiently silted up.

Figure 3 illustrate the summary of the results obtained by the developed mathematical model, the results obtained by the mini simulator, the global DEM model and the USLE model.

- Erosion potentials obtained in the field

The results presented in **Figure 3**, make it possible to validate the mathematical model obtained from the field data. It emerges from **Figure 3** above that the results obtained by the mathematical model, the global MNT model and the USLE model present a relative error of the order of 3.39×10^{-2}.

- Erosion potentials obtained on the NEME type rain simulator

From the results presented in **Figure 3**, we note that the value found from the data of the NEME type rain simulator by our mathematical model (153.94 t/ha/year) is close to that found by [8] (153.07 t/ha/year) and that found using the relation of USLE (159.07 t/ha/year). These results allow to have a relative error of the order of 3.33×10^{-2} between those obtained by the relation of USLE and the mathematical model obtained. From these results presented in the field

Table 2. Evaluation of the erosion potential using the mathematical model.

Data types	Annual rainfall intensity	Ground slope	Average erosion potential
Field data	4190 mm	2% to 3%	153.67 t/ha/an.
NEME type mini rain simulator data			153.94 t/ha/an.

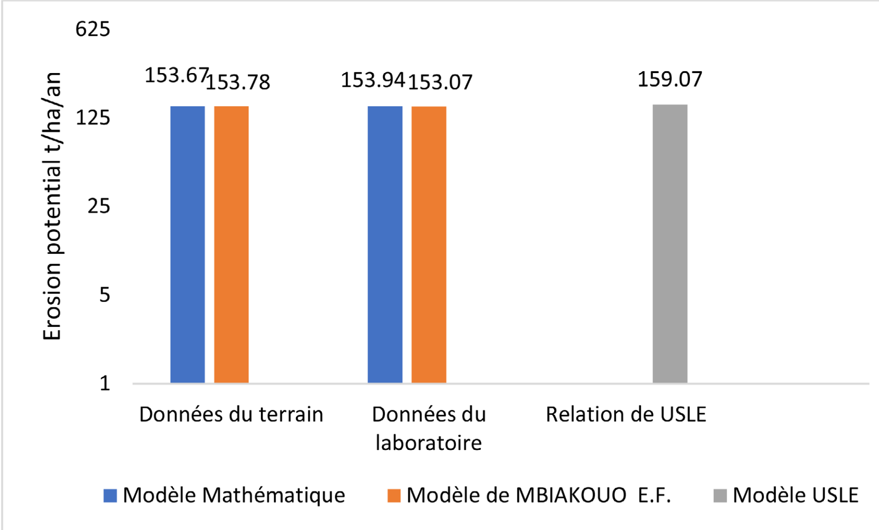

Figure 3. Comparison of the results of the erosion potential obtained by three methods.

and on the NEME type rain simulator, we can therefore validate our established mathematical model.

4. Conclusion

Water erosion, more particularly rain erosion, has very harmful consequences. Our study was carried out in the Nylon estuary area of Douala because in this area rainfall erosion is accelerated. This phenomenon leads to the burial of dwellings in lowlands, the formation of gullies. In order to help eliminate the consequences of this phenomenon by predicting the erosion potential, following the work of Mbiakouo *et al.* (2018), we proposed a mathematical model for predicting the erosion potential. The determination of the soil loss over the study area was carried out using data from the rain simulator and the global digital DEM model. The erosion potential obtained by the mathematical model established and by the USLE relationship is of the order of 150 t/ha/year for average slopes of 2% to 3%. This value highlights the impact of heavy rainfall on structures and homes in the Douala estuary. The results of the graphical analysis obtained by the data from the mini simulator designed and produced are in agreement with those obtained by the data from the global DEM model developed by [12]. We can therefore say from the results obtained that our established mathematical model is in agreement with the results obtained with the global DEM model and the empirical model of USLE. The mathematical model established to predict the erosion potential is easy to use in practice and takes into account the interaction between the various factors of erosion in particular the intensity of rainfall, the slope of the soil, the texture of the soil, the land cover rate. This established mathematical model makes it possible to bypass the use of the USLE model and all the others that are proposed. The investment cost of the mathematical model is very low compared to those involving satellites (GIS), remote sensing and isotope tracers.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Nomenclature

Small Letters

h: Order rain intensity;

$x = I$: represents the rainfall intensity m/year;

$y = P$: represents the slope of the ground %;

z: represents the erosion potential t/ha/year;

x: represents the rainfall intensity m/year;

y: represents the slope of the ground %;

z: represents the erosion potential t/ha/year.

Capital Letters

$L_i(x)$: parameter related to the intensity of rain;

$L_j(y)$: parameter linked to the slope of the ground;

$M(i, j)$: parameter linked to $L_i(x)$ (second derivative of the cubic spline);

Q: represents the flow in l/s;

A: represents the area of the study area in ha;

C_r: represents the run off coefficient without unit;

I: represents the rainfall intensity in m/year;

N: is the number of intervals between data points;

$S(x, y)$: erosion potential at the level of the meshed areas (in t/ha/year);

P_{oT}: erosion potential at the watershed level (in t/ha/year);

A_o: the surface of the soil exposed to erosion after each precipitation.

Greek Symbols

$\delta(i, j)$: Kronecker symbol.

Annex I

This approach is generally represented by the following organization chart: