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Preliminary Identification of a Prime Number Other Than 2 and 3, the Origin of Twin Prime Numbers, the Structure of the Chain of Prime Numbers and the Set of Prime Numbers Less Than a Given Integer

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Abstract

The application of the Euclidean division theorem for the positive integers allowed us to establish a set which contains all the prime numbers and this set we called it set of supposedly prime numbers and we noted it E_{sp}. We subsequently established from the previous set the set of non-prime numbers (the set of numbers belonging to this set and which are not prime) denoted E_{np}. We then extracted from the set of supposedly prime numbers the numbers which are not prime and the set of remaining number constitutes the set of prime numbers denoted E_p. We have deduced from the previous set, the set of prime numbers between two natural numbers. We have explained during our demonstrations the origin of the twin prime numbers and the structure of the chain of prime numbers.

Keywords

Supposedly Prime Numbers, Non-Prime Numbers, Prime Numbers, Prime Numbers Less Than a Given Integer, Prime Numbers between Two Given Integers

1. Introduction

A prime number is a number that has only two divisors: one and itself. The realm of prime numbers was considered an impenetrable realm. This article on prime numbers has removed a poisonous thorn from under the feet of scientists. Prime numbers play a very important role in securing information and therefore in the advancement of NTIC. There is a prize each year for the one or those who would have found the largest prime number; it is "the hunt for prime numbers". With the formulas and sets established in this article, we can determine all the largest prime numbers according to the measurement capabilities of our machines. This article has put an end to the mysteries of prime numbers, by putting light in the universe of prime numbers. With the formulas established in this article, one can perform a "primary" identification to know if a number is prime or not. We have shown in the article the nuance between prime number and other numbers that are not prime: this nuance depends on the parameters nij established in the article. We established the set of prime numbers and the set of prime numbers inferior to a given integer. The remainder of this article is organized as follows: In 1, set of supposedly prime numbers; In 2, preliminary identification of a prime number other than 2 and 3; In 3, the chain of prime numbers; In 4, set of non-prime numbers; In 5, set of prime numbers; In 6, set of prime numbers less than an integer; In 7, set of prime numbers between two integers; In 8, applications; In 9 conclusion followed by a bibliography, a biography and thank.

2. Set of Supposedly Prime Numbers

2.1. Observation

The first nine prime numbers are: 2; 3; 5; 7; 11; 13; 17; 19; 23.

Arrange these numbers by indicating the added value for each number to have the following number:

$2 \rightarrow +1$

$3 \rightarrow +2$

$5 \rightarrow +2$

$7 \rightarrow +4$

$11 \rightarrow +2$

$13 \rightarrow +4$

$17 \rightarrow +2$

$19 \rightarrow +4$

$23 \rightarrow +2$

Note: We note that from five it is enough to add alternately 2 to obtain the following prime number then 4 to obtain the prime number, which follows this following prime number and so on. When we continue to add alternately 2 and 4, we get the following numbers:

$25 \rightarrow +4$

$29 \rightarrow +2$

$31 \rightarrow +4$

$35 \rightarrow +2$

$37 \rightarrow +4$

$41 \rightarrow +2$

$43 \rightarrow +4$

$47 \rightarrow +2$

$49 \to +4$

$53 \to +2$

$55 \to +4$

$59 \to +2$

We can clearly see that from 23 we have a mixture of prime numbers and non-prime numbers. This inspires us with the idea of one (or more) formula(s) for prime numbers, hence the need to translate the previous numbers given by one or more formulas.

2.2. Formulas for the Numbers Obtained

Among these numbers, there are prime numbers and non-prime numbers, hence the name supposedly prime numbers. The numbers generated by this logic are said to be supposedly prime numbers.

Demonstration:

$5 \to 5$

$5+2 \to 7$

$5+2+4 \to 11$

$5+2+4+2 \to 13$

$5+2+4+2+4 \to 17$

$5+2+4+2+4+2 \to 19$

$5+2+4+2+4+2+4 \to 23$

Let n_1 and n_2 be two integer parameters such that:

n_1: the number of two added to 5 to have a new number.

n_2: the number of four added to 5 to have the same new number.

We can clearly see that a previous number is obtained by the relation:

$5+2n_1+4n_2$

Relations between n_1 and n_2:

<u>There are two possibilities:</u>

$n_1 = n_2$ or $n_1 = n_2 + 1$

- For $n_1 = n_2$, we have:
  $5+2n_1+4n_2 = 5+2n_2+4n_2 = 5+6n_2$ with $n_2 \geq 0$
- For $n_1 = n_2 + 1$,
  $5+2n_1+4n_2 = 5+2(n_2+1)+4n_2 = 7+6n_2$ with $n_2 \geq 0$

Note E_{sp}: the set of supposedly prime numbers.

$$E_{sp} = \{2;3;6n+5;6n+7 \text{ with } n \in \mathbb{N}\}$$

NB:

We note with calculations that this set seems to contain all the prime numbers but nothing proves it to us. The application of Euclidean division to positive integers gives us the same set which contains all the prime numbers.

2.3. Demonstrating That the Set: $\{2;3;6n+5;6n+7 \text{ with } n \in \mathbb{N}\}$ Contains All Prime Numbers

According to the Euclidean division theorem for positive integers, we have

$\forall(a,b)\in\mathbb{N}\times\mathbb{N}^*,\ \exists q,r\in\mathbb{N}\,/\,a=bq+r$ and $r<b$.

[1] Proz, Euclidean division-Definition and explanations, https://www.techno-science.net, May 04, 2022, 14h-44min.

$$N=\{a\in N\}=\{bq+r \text{ with } b\in\mathbb{N}^*;q,r\in\mathbb{N} \text{ and } r<b\}$$

We then write:

$$N=\{an+b \text{ with } a\in\mathbb{N}^* \text{ and } n,b\in\mathbb{N} \text{ such that } b<a \text{ and } b \text{ is between 0 and } a-1\}$$

We can write:

$$N=\bigcup_{b=0}^{a-1}\{an+b \text{ with } a\in N^*,n\in\mathbb{N}\}$$

Example:

- If $a=1$

$$N=\bigcup_{0}^{a-1}\{n,n\in N\}=\{n,n\in\mathbb{N}\}$$

- If $a=2$

$$N=\{2n,n\in\mathbb{N}\}\cup\{2n+1,n\in\mathbb{N}\}$$

- If $a=3$

$$N=\{3n,n\in\mathbb{N}\}\cup\{3n+1,n\in\mathbb{N}\}\cup\{3n+2,n\in\mathbb{N}\}$$

- If $a=4$

$$N=\{4n,n\in\mathbb{N}\}\cup\{4n+1,n\in\mathbb{N}\}\cup\{4n+2,n\in\mathbb{N}\}\cup\{4n+3,n\in\mathbb{N}\}$$

- If $a=5$

$$N=\{5n,n\in\mathbb{N}\}\cup\{5n+1,n\in\mathbb{N}\}\cup\{5n+2,n\in\mathbb{N}\}\cup\{5n+3,n\in\mathbb{N}\}\cup\{5n+4,n\in\mathbb{N}\}$$

- If $a=6$

$$N=\{6n,n\in\mathbb{N}\}\cup\{6n+1,n\in\mathbb{N}\}\cup\{6n+2,n\in\mathbb{N}\}\cup\{6n+3,n\in\mathbb{N}\}\cup\{6n+4,n\in N\}\cup\{6n+5,n\in\mathbb{N}\}$$

Consider the set

$$N=\{6n,n\in\mathbb{N}\}\cup\{6n+1,n\in\mathbb{N}\}\cup\{6n+2,n\in\mathbb{N}\}\cup\{6n+3,n\in\mathbb{N}\}\cup\{6n+4,n\in N\}\cup\{6n+5,n\in\mathbb{N}\}$$

Note 1:

The elements of $\{6n,n\in\mathbb{N}\}$ are even.

The elements of $\{6n+2,n\in\mathbb{N}\}$ are even.

The elements of $\{6n+3,n\in\mathbb{N}\}$ are multiples of 3.

The elements of $\{6n+4,n\in\mathbb{N}\}$ are even.

When we eliminate these four previous sets in N we are left with the following two sets: $\{6n+1,n\in\mathbb{N}\}$ and $\{6n+5,n\in\mathbb{N}\}$.

Consequently the set $\{6n+1,n\in\mathbb{N}\}\cup\{6n+5,n\in\mathbb{N}\}$ contains all the prime numbers except 2 and 3.

Note 2:

$\{6n+1,n\in\mathbb{N}\}=\{1\}\cup\{6n+1,n\in\mathbb{N}^*\}$.

Demonstrate that $\{6n+1,n\in\mathbb{N}^*\}=\{6n+7,n\in N\}$.

Let $p=n-1$ with $n\in\mathbb{N}^*$ so $p\in\mathbb{N}$ and $n=p+1$,

$6n+1=6(p+1)+1=6p+6+1=6p+7, p\in\mathbb{N}$.

Then $\{6n+1, n\in\mathbb{N}\}=\{1\}\cup\{6n+1, n\in\mathbb{N}^*\}$.

So: $\{6n+1, n\in\mathbb{N}\}=\{1\}\cup\{6n+1, n\in\mathbb{N}^*\}=\{1\}\cup\{6n+7, n\in\mathbb{N}\}$,

$\{6n+1; 6n+5, n\in\mathbb{N}\}=\{1\}\cup\{6n+5; 6n+7, n\in\mathbb{N}\}$.

We note that the set $\{6n+5; 6n+7, n\in\mathbb{N}\}$ contains all prime numbers except 2 and 3.

Name E_{sp}: The Set of supposedly prime numbers,

$E_{sp}=\{2;3;6n+5;6n+7 \text{ with } n\in\mathbb{N}\}$ contains all prime numbers.

3. Preliminary Identification of a Prime Number Other Than 2 and 3

Let: $U_n=6n+5$ and $V_n=6n+7$ with $n\in\mathbb{N}$.

We have: $\frac{U_n-5}{6}=n$ and $\frac{V_n-7}{6}=n$ with $n\in\mathbb{N}$.

Consequence 1:

A number N is a supposed prime number other than 2 and 3 if and

Only if $\frac{N-5}{6}\in\mathbb{N}$ or $\frac{N-7}{6}\in\mathbb{N}$.

Consequence 2:

Since every prime number is a supposedly prime number, then if a number N different from 2 and 3 is prime then $\frac{N-5}{6}\in\mathbb{N}$ or $\frac{N-7}{6}\in\mathbb{N}$.

4. The Chain of Prime Numbers

4.1. Graphical Representation of Supposedly Prime Numbers in an Orthonormal Fram (Figure 1)

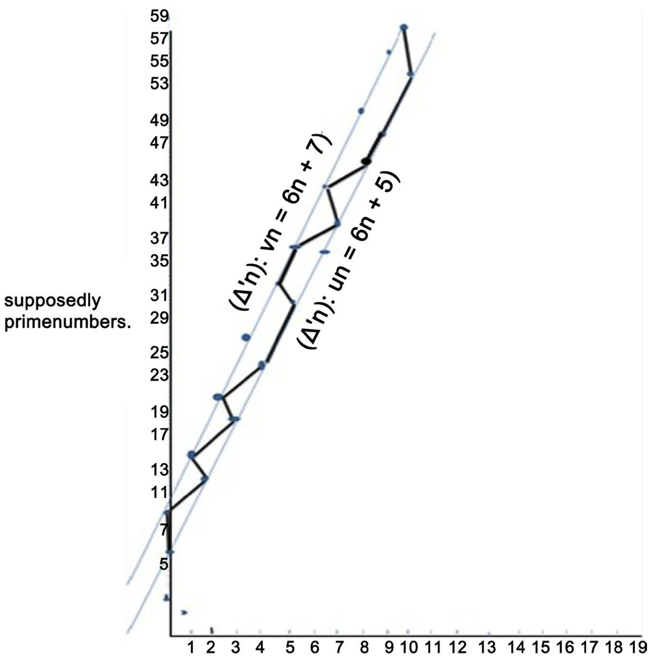

Figure 1. Representation of the prime numbers in an orthonormal fram.

4.2. Interpretations

The pairs ($U_n = 6n+5; V_n = 6n+7$ with $n \in \mathbb{N}$) are ordered. When we obtain two non-prime numbers for fixed n, we have a chain break. The first chain break is obtained with $n = 5\times 7 = 35$.

$6\times 35+5 = 215$ divisible by 5.

$6\times 35+7 = 217$ divisible by 7 According to what precedes, the chain of prime numbers is a **broken line presenting points of discontinuities**.

4.3. Twin Prime Numbers

4.3.1. Definition

The twin prime are two primes which only differ by two [2].
https://en.wikipedia.org/wiki/Twin_prime

4.3.2. State

What are called twin primes (*i.e.* two primes which only differ by two) are two prime numbers U_n and V_n such that:

$$U_n = 6n+5 \text{ and } V_n = 6n+7 \text{ with } n \in \mathbb{N} \ (n \text{ fixed}).$$

5. Set of Non-Prime Numbers

5.1. Definition

A non-prime number is a supposedly prime number that is not prime.

Remark: there are other non-prime numbers such as even numbers and those multiples by three but they are not taken into consideration in this article.

5.2. Class of Non-Prime Numbers

Let $U_n = 6n+5$ and $V_n = 6n+7$ with $n \in \mathbb{N}$.

The non-prime numbers are the products: $U_iU_j; V_iV_j$ and U_iV_j with $i; j \in \mathbb{N}^2$.

Let us calculate U_iU_j:

$$U_iU_j = (6i+5)(6j+5) = 36ij+30i+30j+25 = 36ij+30(i+j)+18+7$$
$$= 6(6ij+5(i+j)+3)+7 = V_k$$

with $k = 6ij+5(i+j)+3$ then U_iU_j is class *V*:

Let us calculate V_iV_j:

$$V_iV_j = (6i+7)(6j+7) = 36ij+42i+42j+49 = 36ij+42i+42j+42+7$$
$$= 6(6ij+7i+7j+7)+7 = V_k$$

with $k = 6ij+7(i+j)+7$ then U_iU_j is class *V*:

Let us calculate U_iV_j:

$$U_iV_j = (6i+5)(6j+7) = 36ij+42i+30j+35 = 36ij+42i+30j+30+5$$
$$= 6(6ij+7i+5j+5)+5 = U_k$$

with $k = 6ij+7i+5j+5$ then U_iV_j is class *U*:

Consequences:

Non-prime numbers have the form:

$6n_3 + 5$ with $n_3 \in \{6n_1n_2 + 7n_1 + 5n_2 + 5 \text{ with } n_1; n_2 \in \mathbb{N}^2\}$

$6n_3 + 7$ with $n_3 \in \{6n_1n_2 + 7(n_1 + n_2) + 7; 6n_1n_2 + 5(n_1 + n_2) + 3 \text{ with } n_1; n_2 \in \mathbb{N}^2\}$

Conclusion:

The previous formulas reveal the famous secret of non-prime numbers (which differentiates them from prime numbers) and allow us to remove the nuance between prime numbers and non-prime numbers. The alternation between prime numbers and non-prime numbers is not a question of periodicity. This alternation depends on the integer parameters n_{ij} of the non-prime numbers.

$$n_{ij} \in \{6ij + 7i + 5j + 5 \text{ with } i, j \in \mathbb{N}^2\}$$
$$\cup \{6ij + 7(i + j) + 7; 6ij + 5(i + j) + 3 \text{ with } i, j \in \mathbb{N}^2\}$$

NB:

A number N is a supposed prime number other than 2 and 3 if and

$$\text{Only if } \frac{N-5}{6} \in \mathbb{N} \text{ or } \frac{N-7}{5} \in \mathbb{N}$$

Since every prime number is a supposedly prime number, then if a number N different from 2 and 3 is prime then

$$\frac{N-5}{6} \in \mathbb{N} \text{ or } \frac{N-7}{6} \in \mathbb{N}.$$

A natural number N different from 2 and 3 is prime if and only if

$$\frac{N-5}{6} \in \mathbb{N} \setminus \{6ij + 7i + 5j + 5 \text{ with } i, j \in \mathbb{N}^2\} \text{ or}$$

$$\frac{N-7}{6} \in \mathbb{N} \setminus \{6ij + 7(i + j) + 7; 6ij + 5(i + j) + 3 \text{ with } i, j \in \mathbb{N}^2\}.$$

5.3. Representation of the Set of Non-Prime Numbers

Name E_{np}: The Set of non-prime numbers.

5.3.1. First form of Representation

The first form of representation of E_{np} results from the formulas previously established.

$$E_{np} = \{6n_3 + 5 \text{ with } n_3 \in \{6n_1n_2 + 7n_1 + 5n_2 + 5 \text{ with } n_1, n_2 \in \mathbb{N}^2\}; 6n_3 + 7 \text{ with}$$
$$n_3 \in \{6n_1n_2 + 7(n_1 + n_2) + 7; 6n_1n_2 + 5(n_1 + n_2) + 3 \text{ with } n_1, n_2 \in \mathbb{N}^2\}\}$$

5.3.2. Second form of Representation

The non-prime numbers are the products: $U_iU_j; V_iV_j$ and U_iV_j with $i; j \in \mathbb{N}^2$

$$U_iU_j = [6i + 5][6j + 5]$$

$$V_iV_j = [6i + 7][6j + 7]$$

$$U_iV_j = [6i + 5][6j + 7]$$

$$E_{np} = \{[6i+5][6j+5];[6i+7][6j+7];[6i+5][6j+7] \text{ with } i; j \in \mathbb{N}^2\}.$$

6. Set of Prime Numbers

$$E_p = E_{sp} \setminus E_{np}$$

6.1. Storage of Supposedly Prime Numbers

The pairs ($6n+5; 6n+7$ with $n \in \mathbb{N}$) are ordered and increasing according to the increasing values of n. The pair $(2;3)$ is ordered. The pairs would be a convenient artifice to respect the order of the supposed prime numbers.

$E_{sp} = \{(2;3);(6n+5;6n+7) \text{ with } n \in \mathbb{N}\}$.

6.2. Storage of Non-Prime Numbers

$$E_{np} = \{[6i+5][6j+5];[6i+7][6j+7];[6i+5][6j+7] \text{ with } i; j \in \mathbb{N}^2\}.$$

Remark:

This set requires a rearrangement to respect the order of non-prime numbers

$E_p = E_{sp} \setminus E_{np}$ We deduce from the above that:

$$E_p = \{(2;3);(6n+5;6n+7) \text{ with } n \in \mathbb{N}\}$$
$$\setminus \{[6i+5][6j+5];[6i+7][6j+7];[6i+5][6j+7] \text{ with } i; j \in \mathbb{N}^2\}$$

NB:

Parentheses are only a convenient artifice for respecting the order of prime numbers.

7. Set of Prime Numbers Less Than an Integer

7.1. Set of Supposedly Prime Numbers Less Than an Integer

Let $E_{sp<M}$: the set of supposedly prime numbers less than M with $M \in \mathbb{N}$

$$E_{sp} = \{(2;3) < M;(6n+5;6n+7) < M \text{ with } n \in \mathbb{N}\}$$

Question: what is the Maximum value of n? There are two possibilities:

Either we determine n with respect to $6n + 5$ with $n \in \mathbb{N}$ or we determine n with respect to $6n + 7$ with $n \in \mathbb{N}$.

- First possibility

  Let be $n_{\max_1}$ the value of $n_{\max}$ determined with respect to $6n + 7$ with $n \in \mathbb{N}$

$$6n_{\max_1} + 7 < M \Leftrightarrow n_{\max_1} < \frac{M-7}{6}$$

$$n_{\max_1} = E\left(\frac{M-7}{6}\right)$$

- Second possibility:

  Let be $n_{\max_2}$ the value of $n_{\max}$ determined with respect to $6n + 5$ with $n \in \mathbb{N}$

$$6n_{\max_2} + 5 < M \Leftrightarrow n_{\max_2} < \frac{M-5}{6}$$

$$n_{\max_2} = E\left(\frac{M-5}{6}\right)$$

We have:

$n_{\max} = n_{\max_1}$ or $n_{\max_2}$

Let us say:

$$M_1 = 6n_{\max_1} + 5 = 6E\left(\frac{M-7}{6}\right) + 5$$

$$M_2 = 6n_{\max_1} + 7 = 6E\left(\frac{M-7}{6}\right) + 7$$

$$M_3 = 6n_{\max_2} + 5 = 6E\left(\frac{M-5}{6}\right) + 5$$

$$M_4 = 6n_{\max_2} + 7 = 6E\left(\frac{M-5}{6}\right) + 7$$

We choose the largest number that is less than M among these four numbers $(M_1; M_2; M_3; M_4)$.

This number will be the last number when we arrange the supposedly prime numbers in ascending order.

We write:

$$E_{sp<M} = \left\{(2;3) < M; (6n+5; 6n+7) < M \text{ with } n \in \left\{[0; n_{\max}] \cap \mathbb{N}\right\}\right\}$$

7.2. Set of Non-Prime Numbers Less Than an Integer *M*

Let $E_{np<M}$, the set of non-prime numbers less than an integer M

$$E_{np<M} = \left\{[6i+5][6j+5] < M; [6i+7][6j+7] < M; [6i+5][6j+7] < M \text{ with } M \in \mathbb{N} \text{ and } i; j \in \mathbb{N}^2\right\}.$$

Remark:

This set requires a rearrangement to respect the order of non-prime numbers less than *M*. When *M* is less than 25 all supposedly prime numbers less than *M* are prime so $E_{sp} = E_p$.

Question:

What are the maximum values of *i* and *j* for each of products?

- $[6i+5][6j+5] < M$

  $j_{\max}$ is obtained for $i = 0$

$$i = 0 \Leftrightarrow 5[6j_{\max} + 5] < M \Leftrightarrow j_{\max} < \frac{\frac{M}{5} - 5}{6} \Leftrightarrow j_{\max} < \frac{M-25}{30}$$

$$j_{\max} = E\left(\frac{M-25}{30}\right)$$

$$i_{\max} = j_{\max} = E\left(\frac{M-25}{30}\right)$$

$$[6i+5][6j+5] < M \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M-25}{30}\right)\right] \cap \mathbb{N}\right\}^2$$

- $[6i+7][6j+7]<M$

  $j_{\max}$ is obtained for $i=0$

$$i=0 \Leftrightarrow 5[6j_{\max}+7]<M \Leftrightarrow j_{\max}<\frac{\frac{M}{7}-7}{6} \Leftrightarrow j_{\max}<\frac{M-49}{42}$$

$$j_{\max}=E\left(\frac{M-49}{30}\right)$$

$$i_{\max}=j_{\max}=E\left(\frac{M-49}{42}\right)$$

$$[6i+7][6j+7]<M \Rightarrow i,j \in \left\{\left[0;E\left(\frac{M-49}{42}\right)\right]\cap\mathbb{N}\right\}^2$$

- $[6i+5][6j+7]<M$

$$i=0 \Leftrightarrow 5[6j_{\max}+7]<M \Leftrightarrow 6j_{\max}+7<M/5$$

$$\Leftrightarrow j_{\max}<\frac{M-35}{30} \Rightarrow j_{\max}=E\left(\frac{M-35}{30}\right)$$

$$i=0 \Leftrightarrow 7[6j_{\max}+5]<M \Leftrightarrow 6j_{\max}+5<M/7$$

$$\Leftrightarrow j_{\max}<\frac{M-35}{42} \Rightarrow j_{\max}=E\left(\frac{M-35}{42}\right)$$

$$[6i+5][6j+7] \Rightarrow i \in \left\{\left[0;E\left(\frac{M-35}{42}\right)\right]\cap\mathbb{N}\right\} \text{ and } j \in \left\{\left[0;E\left(\frac{M-35}{30}\right)\right]\cap\mathbb{N}\right\}$$

Let

$$M \in \mathbb{N}^* \setminus \{1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24\}$$

$$E_{np}=\left\{\begin{array}{l} [6i+5][6j+5]<M \Rightarrow i,j \in \left\{\left[0;E\left(\frac{M-25}{30}\right)\right]\cap\mathbb{N}\right\}^2 \\ [6i+7][6j+7]<M \Rightarrow i,j \in \left\{\left[0;E\left(\frac{M-49}{42}\right)\right]\cap\mathbb{N}\right\}^2 \\ [6i+5][6j+7]<M \Rightarrow i \in \left\{\left[0;E\left(\frac{M-35}{42}\right)\right]\cap\mathbb{N}\right\} \text{ and } \\ \qquad j \in \left\{\left[0;E\left(\frac{M-35}{30}\right)\right]\cap\mathbb{N}\right\} \end{array}\right\}$$

7.3. Calculation Method for Non-Prime Numbers

7.3.1. Calculation Method for $[6i+5]\times[6j+5]$ and $[6i+7]\times[6j+7]$

For the products $[6i+5]\times[6j+5]$.

For each *i* chosen, the calculation starts with the corresponding *j*.

We multiply the numbers $6i+5$ by the numbers $6j+5$ until we obtain a number greater than or equal to *M*. the product obtained is eliminated when it is greater than or equal to *M*. The same logic is used for the calculation of the products $[6i+7]\times[6j+7]$ (**Figure 2**, **Figure 3**).

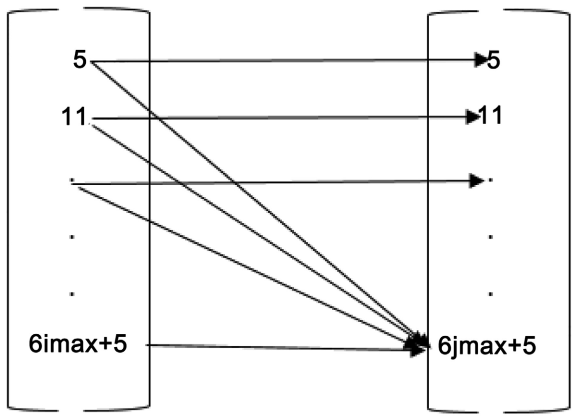

Figure 2. Calculation method for $[6i+5]\times[6j+5]$.

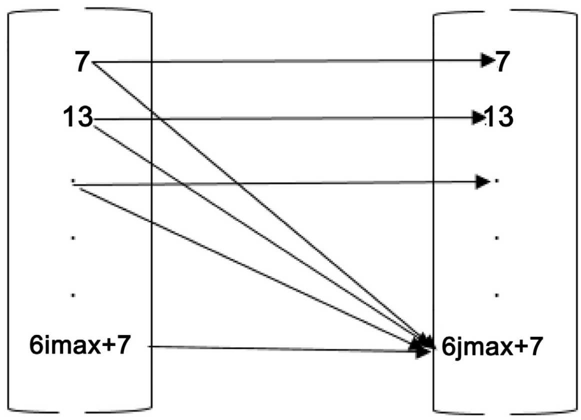

Figure 3. Calculation method for $[6i+7]\times[6j+7]$.

7.3.2. Calculation Method for $[6i+5]\times[6j+7]$

For each i chosen we multiply the $6i+5$ every $6j+7$.

The product obtained is eliminated when it is greater than or equal to M (**Figure 4**).

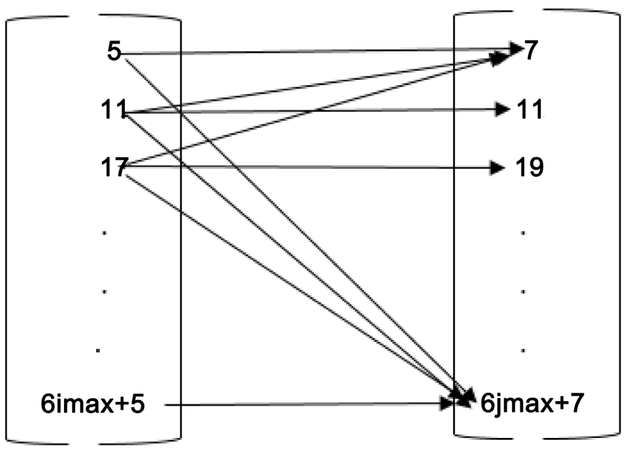

Figure 4. Calculation method for $[6i+5]\times[6j+7]$.

$$E_{p<M} = E_{sp<M} \setminus E_{np<M}$$

$$E_{p<M} = \left\{ \begin{array}{l} \left\{(2;3) < M; (6n+5; 6n+7) < M \text{ with } n \in \left\{[0; n_{\max}] \cap \mathbb{N}\right\}\right\} \\ \setminus \left\{ \begin{array}{l} [6i+5][6j+5] < M \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M-25}{30}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+7][6j+7] < M \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M-49}{42}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+5][6j+7] < M \Rightarrow i \in \left\{\left[0; E\left(\frac{M-35}{42}\right)\right] \cap \mathbb{N}\right\} \text{ and } \\ j \in \left\{\left[0; E\left(\frac{M-35}{30}\right)\right] \cap \mathbb{N}\right\} \end{array} \right\} \end{array} \right\}$$

8. Set of Prime Numbers between Two Integers

Let M_1 and M_2 be two integers such that $M_1 < M_2$.

Let M be a prime number between M_1 and M_2 therefore $M_1 < M < M_2$.

$E_{p<M_1}$: the set of prime numbers less than M_1;

$E_{p<M_2}$: the set of prime numbers less than M_2;

$E_{P<M_1<M<M_2}$: the set of prime numbers between M_1 and M_2.

We have:

$$E_{P<M_1<M<M_2} = E_{p<M_2} \setminus E_{p<M_1}$$

$$E_{P<M_1<M<M_2} = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left\{(2;3) < M_2; (6n+5; 6n+7) < M_2 \text{ with } n \in \left\{[0; n_{\max}] \cap \mathbb{N}\right\}\right\} \\ \setminus \left\{ \begin{array}{l} [6i+5][6j+5] < M_2 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M_2-25}{30}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+7][6j+7] < M_2 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M_2-49}{42}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+5][6j+7] < M_2 \Rightarrow i \in \left\{\left[0; E\left(\frac{M_2-35}{42}\right)\right] \cap \mathbb{N}\right\} \text{ and } \\ j \in \left\{\left[0; E\left(\frac{M_2-35}{30}\right)\right] \cap \mathbb{N}\right\} \end{array} \right\} \end{array} \right\} \\ \setminus \left\{ \begin{array}{l} \left\{(2;3) < M_1; (6n+5; 6n+7) < M_1 \text{ with } n \in \left\{[0; n_{\max}] \cap \mathbb{N}\right\}\right\} \\ \setminus \left\{ \begin{array}{l} [6i+5][6j+5] < M_1 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M_1-25}{30}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+7][6j+7] < M_1 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{M_1-49}{42}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+5][6j+7] < M_1 \Rightarrow i \in \left\{\left[0; E\left(\frac{M_1-35}{42}\right)\right] \cap \mathbb{N}\right\} \text{ and } \\ j \in \left\{\left[0; E\left(\frac{M_1-35}{30}\right)\right] \cap \mathbb{N}\right\} \end{array} \right\} \end{array} \right\} \end{array} \right\}$$

9. Applications

9.1. Determining the Prime Numbers Less Than 100

9.1.1. Determining the Supposed Prime Numbers Less Than 100

$$E_{p<100} = \{(2;3) < M_2; (6n+5; 6n+7) < 100 \text{ with } n \in \{[0; n_{\max}] \cap \mathbb{N}\}\}$$

$$E\left(\frac{100-5}{6}\right) = 15 \text{ and } E\left(\frac{100-7}{6}\right) = 15.$$

We have: $E\left(\frac{100-5}{6}\right) = 15$ and $E\left(\frac{100-7}{6}\right) = 15$

$$M_1 = 6 \times 15 + 5 = 95; M_2 = 6 \times 15 + 7 = 97.$$

Therefore, the largest supposedly prime less than 100 is 97.

Calculation method of $E_{sp<M}$.

$U_n = 6n+5$ and $V_n = 6n+7 \Rightarrow V_n = 6n+7 = 6n+5+2$ with $n \in \mathbb{N} \Rightarrow$ $V_n = U_n + 2$ and $U_{n+1} = 6(n+1)+5 = 6n+11 = 6n+7+4 = V_n + 4$

It is therefore sufficient to know the smallest supposed prime different from 2 and 3, that is to say 5, to construct the set of supposed prime numbers less than an integer *M*.

$$E_{sp<M} = \{(2;3), \overset{+2}{\longrightarrow} (5;7) \overset{+4}{\longrightarrow}, \overset{+2}{\longrightarrow} (11;13), \ldots\ldots\ldots\ldots\ldots, \overset{+2}{\longrightarrow} (U_{max}; V_{max})\}$$

$$E_{sp<100} = \{(2;3);(5;7);(11;13);(17;19);(23;25);(29;31);(35;37);(41;43);$$
$$(47;49);(53;55);(59;61);(65;67);(77;79);(83;85);(89;91);(95;97)\}$$

If we remove the parentheses, we get:

$$E_{sp<100} = \{2;3;5;7;11;13;17;19;23;25;29;31;35;37;41;43;47;49;$$
$$53;55;59;61;65;67;77;79;83;85;89;91;95;97\}.$$

9.1.2. Determining Non-Prime Numbers Less Than 100

$M = 100$

$$E_{np<100} = \left\{\begin{array}{l} [6i+5][6j+5] < 100 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{100-25}{30}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+7][6j+7] < 100 \Rightarrow i, j \in \left\{\left[0; E\left(\frac{100-49}{42}\right)\right] \cap \mathbb{N}\right\}^2 \\ [6i+5][6j+7] < 100 \Rightarrow i \in \left\{\left[0; E\left(\frac{100-35}{42}\right)\right] \cap \mathbb{N}\right\} \text{ and} \\ \quad j \in \left\{\left[0; E\left(\frac{100-35}{30}\right)\right] \cap \mathbb{N}\right\} \end{array}\right\}$$

$$E\left(\frac{100-25}{30}\right) = 2; E\left(\frac{100-49}{42}\right) = 1; E\left(\frac{100-35}{42}\right) = 1; E\left(\frac{100-30}{35}\right) = 2$$

$$[6i+5][6j+5] < 100 \Rightarrow i, j \in \{[0;2] \cap \mathbb{N}\}^2$$

$$[6i+7][6j+7] < 100 \Rightarrow i, j \in \{[0;1] \cap \mathbb{N}\}^2$$

$$[6i+5][6j+7]<100 \Rightarrow i \in \{[0;1] \cap \mathbb{N}\} \text{ and } j \in \{[0;2] \cap \mathbb{N}\}$$

- Calculation method of $[6i+5][6j+5]<100 \Rightarrow i, j \in \{[0;2] \cap \mathbb{N}\}^2$

$$i_{\max} = j_{\max} = 2 \Rightarrow U_{i_{\max}} = U_{j_{\max}} = 6 \times 2 + 5 = 17$$

$$[6i+5][6j+5]<100 = \begin{bmatrix} 5 \\ 11 \\ 17 \end{bmatrix} \times \begin{bmatrix} 5 \\ 11 \\ 17 \end{bmatrix} = \begin{Bmatrix} 25 \\ 55 \\ 35 \\ 85 \\ 55 \end{Bmatrix}$$

- Calculation method of $[6i+7][6j+7]<100 \Rightarrow i, j \in \{[0;1] \cap \mathbb{N}\}^2$

$$i_{\max} = j_{\max} = 2 \Rightarrow V_{i_{\max}} = V_{j_{\max}} = 6 \times 1 + 7 = 13$$

$$[6i+7][6j+7]<100 = \begin{bmatrix} 7 \\ 13 \end{bmatrix} \times \begin{bmatrix} 7 \\ 13 \end{bmatrix} = \begin{Bmatrix} 49 \\ 91 \end{Bmatrix}$$

- Calculation method of $[6i+5][6j+7]<100 \Rightarrow i \in \{[0;1] \cap \mathbb{N}\}$ and $j \in \{[0;2] \cap \mathbb{N}\}$

$$i_{\max} = 1 \Rightarrow U_{i_{\max}} = 6 \times 1 + 5 = 11 \text{ and } j_{\max} = 2 \Rightarrow V_{i_{\max}} = 6 \times 2 + 7 = 19$$

$$[6i+5][6j+7]<100 = \begin{bmatrix} 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 13 \\ 119 \end{bmatrix} = \begin{Bmatrix} 35 \\ 65 \\ 95 \\ 77 \end{Bmatrix}$$

We will arrange the products obtained in ascending order to obtain the order of non-prime numbers.

The non-prime numbers less than 100 are 25; 35; 49; 55; 65; 77; 85; 91.

If we extract the non-prime numbers less than 100 from the supposedly prime numbers less than 100, we will be left with the prime numbers less than 100.

$$E_{p<100} = \{2;3;5;7;11;13;17;19;23;29;31;37;41;$$
$$43;47;53;59;61;67;79;83;89;97\}$$

According to the above, the prime numbers less than 100 are the following numbers: 2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53; 59; 61; 67; 71; 73; 79; 83; 89; 97.

9.2. Determining the Set of Prime Numbers Less Than 1000

9.2.1. Determining the Set of Supposedly Prime Numbers Less Than 1000

$M = 1000$

$$E_{p<1000} = \{(2;3)<M;(6n+5;6n+7)<1000 \text{ with } n \in \{[0;n_{\max}] \cap \mathbb{N}\}\}$$

$$E\left(\frac{1000-5}{6}\right) = 165 \text{ and } E\left(\frac{1000-7}{6}\right) = 165$$

We have: $E\left(\frac{1000-5}{6}\right) = 165$ and $E\left(\frac{1000-7}{6}\right) = 165$

$$M_1 = 6 \times 165 + 5 = 995; M_2 = 6 \times 165 + 7 = 997$$

Therefore, the largest supposedly prime less than 1000 is 997.

$$E_{sp<1000} = \{(2;3);(5;7);(11;13);(17;19);(23;25);(29;31);(35;37);(41;43);(47;49); (53;55);(59;61);(65;67);(71;73);(77;79);(83;85);(89;91);(95;97); (101;103);(107;109);(113;115);(119;121);(125;127);(131;133);(137;139); (143;145);(149;151);(155;157);(161;163);(167;169);(173;175);(179;181); (185;187);(191;193);(197;199);(203;205);(209;211);(215;217);(221;223); (227;229);(233;235);(239;241);(245;247);(251;253);(257;259);(263;265); (269;271);(275;277);(281;283);(287;289);(293;295);(299;301);(305;307); (311;313);(317;319);(323;325);(329;331);(335;337);(341;343);(347;349); (353;355);(359;361);(365;367);(371;373);(377;379);(383;385);(389;391); (395;397);(401;403);(407;409);(413;415);(419;421);(425;427);(431;433); (437;439);(443;445);(449;451);(455;457);(461;463);(467;469);(473;475); (479;481);(485;487);(491;493);(497;499);(503;505);(509;511);(515;517); (521;523);(527;529);(533;535);(539;541);(545;547);(551;553);(557;559); (563;565);(569;571);(575;577);(581;583);(587;589);(593;595);(599;601); (605;607);(611;613);(617;619);(623;625);(629;631);(635;637);(641;643); (647;649);(653;655);(659;661);(665;667);(671;673);(677;679);(683;685); (689;691);(695;697);(701;703);(707;709);(713;715);(719;721);(725;727); (731;733);(737;739);(743;745);(749;751);(755;757);(761;763);(767;769); (773;775);(779;781);(785;787);(791;793);(797;799);(803;805);(809;811); (815;817);(821;823);(827;829);(833;835);(839;841);(845;847);(851;853); (857;859);(863;865);(869;871);(875;877);(881;883);(887;889);(893;895); (899;901);(905;907);(911;913);(917;919);(923;925);(929;931);(935;937); (941;943);(947;949);(953;955);(959;961);(965;967);(971;973);(977;979); (983;985);(989;991);(995;997)\}$$

If we remove the parentheses:

$$E_{sp<1000} = \{2;3;5;7;11;13;17;19;23;25;29;31;35;37;41;43;47;49;53;55;59;61;65;67; 71;73;77;79;83;85;89;91;95;97;101;103;107;109;113;115;119;121;125; 127;131;133;137;139;143;145;149;151;155;157;161;163;167;169;173;175; 179;181;185;187;191;193;197;199;203;205;209;211;215;217;221;223; 227;229;233;235;239;241;245;247;251;253;257;259;263;265;269;271; 275;277;281;283;287;289;293;295;299;301;305;307;311;313;317;319; 323;325;329;331;335;337;341;343;347;349;353;355;359;361;365;367; 371;373;377;379;383;385;389;391;395;397;401;403;407;409;413;415; 419;421;425;427;431;433;437;439;443;445;449;451;455;457;461;463; 467;469;473;475;479;481;485;487;491;493;497;499;503;505;509;511; 515;517;521;523;527;529;533;535;539;541;545;547;551;553;557;559; 563;565;569;571;575;577;581;583;587;589;593;595;599;601;605;607; 611;613;617;619;623;625;629;631;635;637;641;643;647;649;653;655; 659;661;665;667;671;673;677;679;683;685;689;691;695;697;701;703;$$

707; 709; 713; 715; 719; 721; 725; 727; 731; 733; 737; 739; 743; 745; 749; 751; 755; 757; 761; 763; 767; 769; 773; 775; 779; 781; 785; 787; 791; 793; 797; 799; 803; 805; 809; 811; 815; 817; 821; 823; 827; 829; 833; 835; 839; 841; 845; 847; 851; 853; 857; 859; 863; 865; 869; 871; 875; 877; 881; 883; 887; 889; 893; 895; 899; 901; 905; 907; 911; 913; 917; 919; 923; 925; 929; 931; 935; 937; 941; 943; 947; 949; 953; 955; 959; 961; 965; 967; 971; 973; 977; 979; 983; 985; 989; 991; 995; 997}

9.2.2. Determining the Set of Non-Prime Numbers Less Than 1000

$M = 1000$

$$E_{np<1000} = \left\{ \begin{array}{l} [6i+5][6j+5] < 1000 \Rightarrow i, j \in \left\{ \left[0; E\left(\frac{1000-25}{30}\right) \right] \cap \mathbb{N} \right\}^2 \\ [6i+7][6j+7] < 1000 \Rightarrow i, j \in \left\{ \left[0; E\left(\frac{1000-49}{42}\right) \right] \cap \mathbb{N} \right\}^2 \\ [6i+5][6j+7] < 1000 \Rightarrow i \in \left\{ \left[0; E\left(\frac{1000-35}{42}\right) \right] \cap \mathbb{N} \right\} \text{ and } \\ \qquad j \in \left\{ \left[0; E\left(\frac{1000-35}{30}\right) \right] \cap \mathbb{N} \right\} \end{array} \right\}$$

$$E\left(\frac{1000-25}{30}\right) = 32; E\left(\frac{1000-49}{42}\right) = 22; E\left(\frac{1000-35}{42}\right) = 22; E\left(\frac{1000-30}{35}\right) = 32$$

$$[6i+5][6j+5] < 1000 \Rightarrow i, j \in \{0,32\}^2$$

$$[6i+7][6j+7] < 1000 \Rightarrow i, j \in \{[0;22] \cap \mathbb{N}\}^2$$

$$[6i+5][6j+7] < 1000 \Rightarrow i \in \{0,22\} \quad \text{and} \quad j \in \{0,32\}$$

- Calculation of $[6i+5][6j+5] < 1000 \Rightarrow i, j \in \{0,32\}^2$ (**Figure 5**)

  $$i_{\max} = j_{\max} = 2 \Rightarrow U_{i_{\max}} = U_{j_{\max}} = 6 \times 32 + 5 = 197$$

- Calculation of $[6i+7][6j+7] < 1000 \Rightarrow i, j \in \{[0;32] \cap \mathbb{N}\}^2$ (**Figure 6**)

  $$i_{\max} = j_{\max} = 2 \Rightarrow V_{i_{\max}} = V_{j_{\max}} = 6 \times 22 + 7 = 139$$

- Calculation of $[6i+5][6j+7] < 1000 \Rightarrow i \in \{[0;22] \cap \mathbb{N}\}$ and $j \in \{[0;32] \cap \mathbb{N}\}$ (**Figure 7**)

  $$i_{\max} = 22 \Rightarrow U_{i_{\max}} = 6 \times 22 + 5 = 137 \quad \text{and} \quad j_{\max} = 32 \Rightarrow V_{i_{\max}} = 6 \times 32 + 7 = 199$$

According to the above, the non-prime numbers less than 1000 are the following numbers:

25; 35; 49; 55; 65; 77; 85; 91; 95; 115; 119; 121; 125; 133; 143; 145; 155; 161; 169; 175; 185; 187; 203; 205; 209; 215; 217; 221; 235; 245; 247; 253; 259; 265; 275; 277; 287; 289; 295; 299; 301; 305; 319; 323; 325; 329; 335; 341; 343; 355; 361; 365; 371; 377; 385; 493; 497; 505; 511; 515; 517; 527; 529; 533; 535; 539; 545; 551; 553; 559; 565; 575; 581; 583; 589; 595; 605; 611; 623; 625; 629; 635; 637; 643; 649; 665; 667; 671; 679; 685; 689; 695; 697; 703; 707; 713; 715; 721; 725; 731; 737; 745; 749; 755; 763; 767; 775; 779; 781; 785; 791; 793; 799; 803; 805; 815; 817; 833; 835; 841; 845; 847; 851; 865; 869; 871; 875; 889; 893; 893; 899; 901; 905; 913; 917; 923; 925; 931; 935; 943; 949; 955; 959; 961; 965; 973; 979; 985; 989; 995.

$$[6i+5][6j+5] < 1000 = \begin{bmatrix} 5 \\ 11 \\ 17 \\ 23 \\ 29 \\ 35 \\ 41 \\ 47 \\ 53 \\ 59 \\ 67 \\ 71 \\ 77 \\ 83 \\ 89 \\ 101 \\ 107 \\ 113 \\ 119 \\ 125 \\ 131 \\ 137 \\ 143 \\ 149 \\ 155 \\ 161 \\ 167 \\ 173 \\ 179 \\ 185 \\ 191 \\ 197 \end{bmatrix} \times \begin{bmatrix} 5 \\ 11 \\ 17 \\ 23 \\ 29 \\ 35 \\ 41 \\ 47 \\ 53 \\ 59 \\ 67 \\ 71 \\ 77 \\ 83 \\ 89 \\ 101 \\ 107 \\ 113 \\ 119 \\ 125 \\ 131 \\ 137 \\ 143 \\ 149 \\ 155 \\ 161 \\ 167 \\ 173 \\ 179 \\ 185 \\ 191 \\ 197 \end{bmatrix} = \left\{ \begin{matrix} 25;\ 55; \\ 85;\ 115; \\ 145;\ 175; \\ 205;\ 235; \\ 335;\ 385; \\ 415;\ 445; \\ 475;\ 505; \\ 535;\ 665; \\ 685;\ 715; \\ 745;\ 775; \\ 805;\ 835; \\ 865;\ 895; \\ 925;\ 955; \\ 985;\ 121; \\ 187;\ 253; \\ 319;\ 385; \\ 451;\ 517; \\ 583;\ 649; \\ 715;\ 781; \\ 847;\ 913; \\ 879;\ 289; \\ 391;\ 493; \\ 595;\ 697; \\ 799;\ 901; \\ 529;\ 667; \\ 805;\ 943; \\ 841. \end{matrix} \right\}$$

Figure 5. Non-prime numbers less than 1000 of the form $[6i+5][6j+5]$.

$$[6i+7][6j+7] < 1000 = \begin{bmatrix} 7 \\ 13 \\ 19 \\ 25 \\ 31 \\ 37 \\ 43 \\ 49 \\ 55 \\ 61 \\ 67 \\ 73 \\ 79 \\ 85 \\ 91 \\ 97 \\ 103 \\ 109 \\ 115 \\ 121 \\ 127 \\ 133 \\ 139 \end{bmatrix} \times \begin{bmatrix} 7 \\ 13 \\ 19 \\ 25 \\ 31 \\ 37 \\ 43 \\ 49 \\ 55 \\ 61 \\ 67 \\ 73 \\ 79 \\ 85 \\ 91 \\ 97 \\ 103 \\ 109 \\ 115 \\ 121 \\ 127 \\ 133 \\ 139 \end{bmatrix} = \left\{ \begin{matrix} 49;\ 91; \\ 133;\ 175; \\ 217;\ 259; \\ 301;\ 343; \\ 385;\ 427; \\ 469;\ 511; \\ 553;\ 595; \\ 637;\ 679; \\ 721;\ 763; \\ 805;\ 847; \\ 889;\ 931; \\ 973;\ 169; \\ 247;\ 325; \\ 403;\ 481; \\ 559;\ 637; \\ 715;\ 793; \\ 871;\ 949; \\ 361;\ 475; \\ 589;\ 703; \\ 817;\ 931; \\ 625;\ 775; \\ 925;\ 961. \end{matrix} \right\}$$

Figure 6. Non-prime numbers less than 1000 of the form $[6i+7][6j+7]$.

$$[6i+5][6j+7] < 1000 = \begin{bmatrix} 5 \\ 11 \\ 17 \\ 23 \\ 29 \\ 35 \\ 41 \\ 47 \\ 53 \\ 59 \\ 67 \\ 71 \\ 77 \\ 83 \\ 89 \\ 101 \\ 107 \\ 113 \\ 119 \\ 125 \\ 131 \\ 137 \end{bmatrix} \times \begin{bmatrix} 7 \\ 13 \\ 19 \\ 25 \\ 31 \\ 37 \\ 43 \\ 49 \\ 55 \\ 61 \\ 67 \\ 73 \\ 79 \\ 85 \\ 91 \\ 97 \\ 103 \\ 109 \\ 115 \\ 121 \\ 127 \\ 133 \\ 139 \end{bmatrix} = \begin{Bmatrix} 35; 65; 95; \\ 125; 155; 185; \\ 215; 245; 275; \\ 305; 335; 365; \\ 395, 425; 455; \\ 485; 515; 545; \\ 575; 605; 635; \\ 665; 695; 725; \\ 755; 785; 815; \\ 845; 875; 905; \\ 935; 965; 995; \\ 77; 143; 209; \\ 275; 341; 407; \\ 473; 539; 605; \\ 671; 737; 803; \\ 869; 935; 119; \\ 221; 323; 425; \\ 527; 629; 731; \\ 833; 935; 161; \\ 299; 437; 575; \\ 713; 851; 989; \\ 203; 377; 551; \\ 725; 899; 245; \\ 455; 665; 875; \\ 287; 533; 779; \\ 329; 611; 893; \\ 371; 689; 413; \\ 767; 455; 845 \\ 497; 923; 539 \\ 581; 623; 665; \\ 707; 749; 791; \\ 833; 875; 917; 959. \end{Bmatrix}$$

Figure 7. Non-prime numbers less than 1000 of the form $[6i+5][6j+7]$.

$$E_{p<1000} = E_{sp<1000} \setminus E_{np<1000}$$

It suffices to extract in $E_{sp<1000}$ all non-prime numbers.

$$\begin{aligned} E_{p<1000} = \{ & 2;3;5;7;11;13;17;19;23;29;31;37;41;43;47;53;59;61;67;71;73;79;83;89; \\ & 97;101;103;107;109;113;127;131;137;139;149;151;157;163;167;173;179; \\ & 181;191;193;197;199;211;223;227;229;233;239;241;251;257;263;269; \\ & 271;277;281;283;293;307;311;313;317;331;337;347;349;353;359;367; \\ & 373;379;383;389;397;401;409;419;421;431;433;439;443;449;453;457; \\ & 461;463;467;479;487;491;499;503;509;521;523;541;547;557;563;569; \\ & 571;577;587;593;599;601;607;613;617;619;631;641;647;653;659;661; \\ & 673;677;683;691;701;709;719;727;733;739;743;751;757;761;769;773; \\ & 787;797;809;811;821;823;827;829;839;853;857;859;863;877;881;883; \\ & 887;907;911;919;929;937;941;947;953;967;971;977;983;991;997 \} \end{aligned}$$

NB:

Verification:

[3] https://en.wikipedia.org/wiki/1000_(number).

9.3. Determining the Prime Numbers between 100 and 1000

The set of prime numbers between 100 and 1000 is deduced from the two previous set, it suffices to extract in $E_{p<1000}$ all the prime numbers less than 100.

$$E_{P<100<1000} = E_{p<1000} \setminus E_{p<100}$$

$$\begin{aligned} &E_{P<100<1000} \\ &= \{101;103;107;109;113;127;131;137;139;149;151;157;163;167;173;179;181; \\ &\quad 191;193;197;199;211;223;227;229;233;239;241;251;257;263;269;271; \\ &\quad 277;281;283;293;307;311;313;317;331;337;347;349;353;359;367;373; \\ &\quad 379;383;389;397;401;409;419;421;431;433;439;443;449;453;457;461; \\ &\quad 463;467;479;487;491;499;503;509;521;523;541;547;557;563;569;571; \\ &\quad 577;587;593;599;601;607;613;617;619;631;641;647;653;659;661;673; \\ &\quad 677;683;691;701;709;719;727;733;739;743;751;757;761;769;773;787; \\ &\quad 797;809;811;821;823;827;829;839;853;857;859;863;877;881;883;887; \\ &\quad 907;911;919;929;937;941;947;953;967;971;977;983;991;997\} \end{aligned}$$

10. Conclusion

The results obtained during our demonstration revealed the famous secret of prime numbers and showed that the alternation between prime numbers is not a question of periodicity but it depends on other parameters established previously. We hope that this article on prime numbers will put an end to the hunt for prime numbers and bring a boost in mathematics more specifically in the field of number theory by shedding light in the universe of prime numbers. We are envious to publish soon another article on the prime numbers dealing with the equations from the non-premier numbers that will be the subject of mathematical conjecture.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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