Advances in Pure Mathematics, 2023, 13, 369-401
https://www.scirp.org/journal/apm
ISSN Online: 2160-0384
ISSN Print: 2160-0368

Mathematical Foundations of Sustainable Economy Development

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How to cite this paper: Gonchar, N.S. (2023) Mathematical Foundations of Sustainable Economy Development. *Advances in Pure Mathematics*, **13**, 369-401.
https://doi.org/10.4236/apm.2023.136024

Received: April 27, 2023
Accepted: June 25, 2023
Published: June 28, 2023

Abstract

The principles of sustainable economic development are formulated on the basis of the developed concept of description of economic systems. Mathematical algorithms for finding equilibrium states were built and the conditions for complete clearing of the markets were found. The definition of the phenomenon of recession is given and theorems are established that formulate the conditions for the existence of a recession in the economic system. The recession depth parameter is introduced. For the input-output production model, the necessary and sufficient conditions under which the economic system functions in a profitable mode have been established. The conditions for supply vectors of firms for special technological mappings and consumers demand under which complete clearing of markets takes place are found.

Keywords

Technological Mapping, Economic Equilibrium, Clearing Market, Recession Phenomenon, Sustainable Economic Development

1. Introduction

This article is a continuation of the previous article entitled "economy equilibrium and sustainable development" and it is the implementation of the ideas laid down in [1] for the formulation of the concept of sustainable economic development at the microeconomic level. In it, we clarify the issue of the influence of spontaneous output of goods by firms on the phenomenon of recession, formulate the concept of sustainable economic development.

Section 2 is devoted to the construction of algorithms for finding equilibrium states the existence of which was proved in previous paper "economy equilibrium and sustainable development". Lemma 2 provides sufficient conditions for the consistency of the supply structure with the demand structure, namely, that supply vectors belong to the interior of the cone formed by consumer demand vectors. Lemma 3 establishes the existence of a strictly positive solution of the system of equations for a square non-negative indecomposable matrix. Theorem 2 gives the necessary and sufficient conditions for the existence of an equilibrium price vector for which the market is cleared. Theorem 3 gives the necessary and sufficient conditions for the existence of a strictly positive equilibrium price vector under the condition that the structure of the supply is weakly consistent with the structure of the choice.

Lemma 4 characterizes the structure of the equilibrium state in the case of constant consumer incomes. Theorem 4 gives the necessary and sufficient conditions for the existence of an equilibrium state under which the markets are completely cleared in the case when consumer incomes are constant. Theorem 5 gives the necessary and sufficient conditions for the existence of economic equilibrium in the case of fixed consumer incomes. If the equilibrium vector of prices is not strictly positive, then in this case the important concept of the real vector of consumption is introduced. The system of equilibrium equations with the real vector of consumption is degenerate of a certain multiplicity, and the corresponding equilibrium vector of prices is called the generalized equilibrium price vector. The concept of the parameter of the recession level, which is a characteristic of the depth of the recession, is introduced.

Section 4 formulates the conditions for the sustainable development of the economy. Theorem 6 considers the case when production technologies are described by input-output technological mapping. The necessary and sufficient conditions for the existence of a strictly positive solution of the system of equations (63) have been found. These conditions are the conditions of belonging to the polyhedral cone of the gross product vector component by component multiplied by the vector built on the taxation vector. This cone is formed by the columns of the matrix, which is the product of the matrix of direct costs and the matrix of total costs. The probability of such a vector belonging into the specified cone is found. Provided that the structure of the supply is consistent with the structure of choice, Theorems 8 and 10 formulate the conditions for the structure of firms' output, the levels of taxation at which each of the firms can provide such an income for which it can ensure its functioning in the next production cycle. Theorems 11 and 12 formulate the conditions for the existence of an equilibrium price vector under which the markets are completely cleared if firms have chosen continuous strategies of behavior. Theorems 13 and 14 are reformulations of Theorems 8, 10 and, respectively, Theorems 11, 12 in the case of taxation.

By sustainable economic development, we understand the growth of the gross domestic product over a long period of time. During this period, the wages of employees increase, the majority of firms operate in a profitable mode, and the exchange rate of the national currency is stable. To formulate this definition mathematically, we introduce such concepts as the consistency of the structure of the supply to the structure of the choice. At the firm level, this will mean studying the structure of demand for produced goods so that the volumes of goods released are aligned with the structure of demand. The latter will ensure, in accordance with proven Theorems, the appropriate profit for firms.

Important for testing the economy of the state is a proven theorem about the necessary and sufficient conditions for the functioning of the economy in a profitable mode for the technological mapping of input-output. In reality, information about the economic system is presented in value indicators, and this description is aggregated. If we consider the vector of relative equilibrium of prices, then in this case it is possible to detect deviations from the conditions of the proven theorem that apply to the aggregated economic description. Depending on the economic system, these can be deformations in the taxation system, wage levels, pricing, and foreign economic relations. Information of society can be about such deformations.

The phenomenon of recession is the opposite of the concept of sustainable development. It is accompanied by a decline in production, the presence of unsold goods produced in the economic system. In society, there is saturation with a certain group of goods. It is impossible to influence this phenomenon with macroeconomic measures. Money emission will not lead to the consumption of these surplus goods and will lead to inflation of the remaining goods. The theory of economic equilibrium should explain this phenomenon. The proven theorems show that when there is incomplete agreement between the structure of demand and the structure of supply, there is a surplus of manufactured goods, the price of which is not determined by demand and supply. The proven Theorems show that when there is incomplete agreement between the structure of demand and the structure of supply, there is a surplus of manufactured goods, the price of which is not determined by demand and supply due to the saturation of a certain group of goods. To characterize this phenomenon, an important concept of the vector of real consumption in the economic system is introduced. Based on this concept, the concept of the depth of recession is introduced, which is defined as the relative value of unsold goods for a certain period.

The principles of sustainable economic development formulated in this work are a priority and have not been formulated anywhere before at the micro economic level. At the macroeconomic level, they are formulated in works [2]-[5]. As for the algorithms for calculating the entered values, you should refer to the work [6].

2. Algorithms of the Equilibrium States Finding

Here and further, $R_+^n \setminus \{0\}$ is a cone formed from the nonnegative orthant R_+^n by ejection of the null vector $\{0\} = \{0, \cdots, 0\}$. Further, the cone $R_+^n \setminus \{0\}$ is denoted by R_+^n.

Let us give a series of definitions useful for what follows.

Definition 1. *By a polyhedral non-negative cone created by a set of vectors*

$\{a_i, i=\overline{1,t}\}$ *of n-dimensional space* R^n *we understand the set of vectors of the form*

$$d = \sum_{i=1}^{t} \alpha_i a_i,$$

where $\alpha = \{\alpha_i\}_{i=1}^{t}$ *runs over the set* R_+^t.

Definition 2. *The dimension of a non-negative polyhedral cone created by a set of vectors* $\{a_i, i=\overline{1,t}\}$ *in n-dimensional space* R^n *is maximum number of linearly independent vectors from the set of vectors* $\{a_i, i=\overline{1,t}\}$.

Definition 3. *The vector b belongs to the interior of the non-negative polyhedral r-dimensional cone,* $r \le n$, *created by the set of vectors* $\{a_1, \cdots, a_t\}$ *in n-dimensional vector space* R^n *if a strictly positive vector* $\alpha = \{\alpha_i\}_{i=1}^{t} \in R_+^t$ *exists such that*

$$b = \sum_{s=1}^{t} a_s \alpha_s,$$

where $\alpha_s > 0, s = \overline{1,t}$.

Let us give the necessary and sufficient conditions under which a certain vector belongs to the interior of the polyhedral cone.

Lemma 1. *Let* $\{a_1, \cdots, a_m\}$, $1 \le m \le n$, *be the set of linearly independent vectors in* R_+^n. *The necessary and sufficient conditions for the vector b to belong to the interior of the non-negative cone* K_a^+ *created by vectors* $\{a_i, i=\overline{1,m}\}$ *are the conditions*

$$\langle f_i, b\rangle > 0, \quad i=\overline{1,m}, \quad \langle f_i, b\rangle = 0, \quad i=\overline{m+1,n}, \tag{1}$$

where $f_i, i=\overline{1,n}$, *is a set of vectors being biorthogonal to the set of linearly independent vectors* $\bar{a}_i, i=\overline{1,n}$, *and* $\bar{a}_i = a_i, i=\overline{1,m}$.

The proof of Lemma 1 see: [1] [6].

Describe now an algorithm of constructing strictly positive solutions to the set of equations

$$\psi = \sum_{i=1}^{l} C_i y_i, \quad y_i > 0, \quad i=\overline{1,l}, \tag{2}$$

with respect to the vector $y = \{y_i\}_{i=1}^{l}$ or the same set of equations in coordinate form

$$\sum_{i=1}^{l} c_{ki} y_i = \psi_k, \quad k=\overline{1,n}, \tag{3}$$

for the vector $\psi = \{\psi_1, \cdots, \psi_n\}$ belonging to the interior of the polyhedral cone created by vectors $\left\{C_i = \{c_{ki}\}_{k=1}^{n}, i=\overline{1,l}\right\}$.

Theorem 1. *If a certain vector* ψ *belonging to the interior of a non-negative r-dimensional polyhedral cone created by vectors* $\left\{C_i = \{c_{ki}\}_{k=1}^{n}, i=\overline{1,l}\right\}$, *is such that there exists a subset of r linearly independent vectors of the set of vectors* $\{C_i, i=\overline{1,l}\}$, *such that the vector* ψ *belongs to the interior of the cone created by this subset of vectors, then there exist* $l-r+1$ *linearly independent non-*

negative solutions z_i *to the set of* Equation (3) *such that the set of strictly positive solutions to the set of* Equation (3) *is given by the formula*

$$y = \sum_{i=r}^{l} \gamma_i z_i, \tag{4}$$

where

$$z_i = \left\{ \langle \psi, f_1 \rangle - \langle C_i, f_1 \rangle y_i^*, \cdots, \langle \psi, f_r \rangle - \langle C_i, f_r \rangle y_i^*, 0, \cdots, y_i^*, 0, \cdots, 0 \right\}, \quad i = \overline{r+1, l},$$

$$z_r = \left\{ \langle \psi, f_1 \rangle, \cdots, \langle \psi, f_r \rangle, 0, \cdots, 0 \right\},$$

$$y_i^* = \begin{cases} \min\limits_{s \in K_i} \dfrac{\langle \psi, f_s \rangle}{\langle C_i, f_s \rangle}, & K_i = \left\{ s, \langle C_i, f_s \rangle > 0 \right\}, \\ 1, & \langle C_i, f_s \rangle \le 0, \forall\, s = \overline{1, r}, \end{cases}$$

and the components of the vector $\{\gamma_i\}_{i=r}^{l}$ *satisfy the set of inequalities*

$$\sum_{i=r}^{l} \gamma_i = 1, \quad \gamma_i > 0, \quad i = \overline{r+1, l},$$

$$\sum_{i=r+1}^{l} \langle C_i, f_k \rangle y_i^* \gamma_i < \langle \psi, f_k \rangle, \quad k = \overline{1, r}. \tag{5}$$

The proof of Theorem 1 see: [1] [6].

Definition 4. *Let* $C_i = \{c_{si}\}_{s=1}^{n} \in R_+^n, i = \overline{1, l}$, *be a set of demand vectors and let* $b_i = \{b_{si}\}_{s=1}^{n} \in R_+^n, i = \overline{1, l}$, *be a set of supply vectors. We say that the structure of supply is agreed with the structure of demand in the strict sense if for the matrix B the representation* $B = CB_1$ *is true, where the matrix B consists of the vectors* $b_i \in R_+^n, i = \overline{1, l}$, *as columns, and the matrix C is composed from the vectors* $C_i \in R_+^n, i = \overline{1, l}$, *as columns and* B_1 *is a square nonnegative indecomposable matrix.*

Definition 5. *Let* $C_i = \{c_{si}\}_{s=1}^{n} \in R_+^n, i = \overline{1, l}$, *be a set of demand vectors and let* $b_i = \{b_{si}\}_{s=1}^{n} \in R_+^n, i = \overline{1, l}$, *be a set of supply vectors. We say that the structure of supply is agreed with the structure of demand in the strict sense of the rank* $|I|$ *if there exists a subset* $I \subseteq N$ *such that for the matrix* B^I *the representation* $B^I = C^I B_1^I$ *is true, where the matrix* B^I *consists of the vectors* $b_i^I \in R_+^{|I|}, i = \overline{1, l}$, *as columns, and the matrix* C^I *is composed from the vectors* $C_i^I \in R_+^n, i = \overline{1, l}$, *as columns and* $B_1^I = \left| b_{is}^{1,I} \right|_{i,s=1}^{l}$ *is a square nonnegative indecomposable matrix, where* $b_i^I = \{b_{ki}\}_{k \in I}$, $C_i^I = \{c_{ki}\}_{k \in I}$ *and, moreover, the inequalities*

$$\sum_{i=1}^{l} c_{ki} y_i^I < \sum_{i=1}^{l} b_{ki}, \quad k \in N \setminus I, \quad y_i^I = \sum_{s=1}^{l} b_{is}^{1,I},$$

are valid.

Lemma 2. *Suppose that the set of supply vectors* $b_i = \{b_{si}\}_{s=1}^{n} \in R_+^n, i = \overline{1, l}$, *belongs to the polyhedral cone created by the set of demand vectors* $\left\{ C_i = \{c_{ki}\}_{k=1}^{n} \in R_+^n, i = \overline{1, l} \right\}$. *Then for the matrix* $B = \left| b_{ki} \right|_{k=1,i=1}^{n,l}$ *created by the columns of vectors* $b_i = \{b_{ki}\}_{k=1}^{n} \in R_+^n, i = \overline{1, l}$, *the representation*

$$B = CB_1 \tag{6}$$

is true, where the matrix $C=\left|c_{ki}\right|_{k=1,i=1}^{n,l}$ *is created by the columns of vectors* $C_i=\left\{c_{ki}\right\}_{k=1}^{n}\in R_{+}^{n}, i=\overline{1,l}$, *and the matrix* $B_1=\left|b_{mi}^1\right|_{m=1,i=1}^{l}$ *is nonnegative. If, in addition, the set of supply vectors* $b_i\in R_{+}^{n}, i=\overline{1,l}$, *belongs to the interior of the polyhedral cone created by the demand vectors* $\left\{C_i=\left\{c_{ki}\right\}_{k=1}^{n}\in R_{+}^{n}, i=\overline{1,l}\right\}$, *the matrix* B_1 *can be chosen by strictly positive.*

Proof. The first part of Lemma 2 follows from the second. If every vector $b_i, i=\overline{1,l}$, belongs to the interior of the polyhedral cone created by vectors $C_i, i=\overline{1,l}$, then due to Theorem 1 there exists a strictly positive vector $y_i=\left\{y_{ki}\right\}_{k=1}^{l}$ such that

$$b_{ki}=\sum_{s=1}^{l}c_{ks}y_{si},\quad k=\overline{1,n}.$$

Let us denote $y_{si}=b_{si}^1$, then we obtain

$$b_{ki}=\sum_{s=1}^{l}c_{ks}b_{si}^1,\quad k=\overline{1,n},\quad i=\overline{1,l}.$$

This proves Lemma 2. □

Lemma 3. *Let* $B_1=\left\|b_{ki}^1\right\|_{k,i=1}^{l}$ *be a square nonnegative indecomposable matrix. Then the problem*

$$\sum_{k=1}^{l}b_{ki}^1 d_k=\sum_{s=1}^{l}b_{is}^1 d_i,\quad i=\overline{1,l},\tag{7}$$

has the strictly positive solution relative to the vector $d=\left\{d_k\right\}_{k=1}^{l}$.

Proof. Since the matrix B_1 is indecomposable, then $\sum_{s=1}^{l}b_{is}^1>0, i=\overline{1,l}$. Let us consider the problem

$$\sum_{k=1}^{l}e_{ki}d_k^1=d_i^1,\quad i=\overline{1,l},\tag{8}$$

and prove that it has the strictly positive solution relative to the vector $d^1=\left\{d_k^1\right\}_{k=1}^{l}$, where we introduced the denotation

$$e_{ki}=\frac{b_{ki}^1}{\sum_{s=1}^{l}b_{ks}^1}.$$

To prove this let us consider the problem

$$\frac{d_i^1+\sum_{k=1}^{l}e_{ki}d_k^1}{2}=d_i^1,\quad i=\overline{1,l},\tag{9}$$

on the set $P=\left\{d^1=\left\{d_k^1\right\}_{k=1}^{l}, d_k^1\ge 0, k=\overline{1,l}, \sum_{k=1}^{l}d_k^1=1\right\}$. Due to the equalities $\sum_{i=1}^{l}e_{ki}=1, k=\overline{1,l}$, the function $H\left(d^1\right)=\left\{H_i\left(d^1\right)\right\}_{i=1}^{l}$, maps P into itself and it is continuous on it, where

$$H_i\left(d^1\right)=\frac{d_i^1+\sum_{k=1}^{l} e_{ki} d_k^1}{2}, \quad i=\overline{1,l},$$

Due to Brouwer theorem, there exists a fixed point of the mapping $H\left(d^1\right)$, that is,

$$\frac{d_i^1+\sum_{k=1}^{l} e_{ki} d_k^1}{2}=d_i^1, \quad i=\overline{1,l},$$

The same fixed point satisfies the problem (8). Since the matrix $E=\left\|e_{ki}\right\|_{k,i=1}^{l}$ is non negative and indecomposable we have $E^{l-1}>0$. From the fact that $Ed^1=d^1$ it follows $E^{l-1}d^1=d^1$. This proves that the vector d^1 is strictly positive. Lemma 3 is proved. □

Theorem 2. *Let the structure of supply agree with structure of demand in the strict sense with the supply vectors* $b_i=\{b_{ki}\}_{k=1}^{n}\in R_+^n, i=\overline{1,l}$, *and the demand vectors* $\left\{C_i=\{c_{ki}\}_{k=1}^{n}\in R_+^n, i=\overline{1,l}\right\}$, *and let* $\sum_{s=1}^{n} c_{si}>0, i=\overline{1,l}$, $\sum_{i=1}^{l} c_{si}>0, s=\overline{1,n}$. *The necessary and sufficient conditions of the solution existence of the set of equations*

$$\sum_{i=1}^{l} c_{ki}\frac{\langle b_i,p\rangle}{\langle C_i,p\rangle}=\sum_{i=1}^{l} b_{ki}, \quad k=\overline{1,n}, \tag{10}$$

relative to the vector p is belonging of the vector $D=\{d_i\}_{i=1}^{l}$ *to the polyhedral cone created by vectors* $C_k^{\mathrm{T}}=\{c_{ki}\}_{i=1}^{l}, k=\overline{1,n}$, *where* $B=CB_1$, $B_1=\left|b_{ki}^1\right|_{k,i=1}^{l}$ *is a nonnegative indecomposable matrix, the vector* $D=\{d_i\}_{i=1}^{l}$ *is a strictly positive solution to the set of equations*

$$\sum_{k=1}^{l} b_{ki}^1 d_k=y_i d_i, \quad y_i=\sum_{k=1}^{l} b_{ik}^1, \quad i=\overline{1,l}. \tag{11}$$

Proof. Two cases are possible: 1) $n\geq l$ and 2) $n<l$. Consider the first case.

The necessity. First, we assume that the vectors $C_i, i=\overline{1,l}$, are linear independent. Let there exist strictly positive solution $p_0\in R_+^n$ to the set of Equation (10). Since for the matrix B the representation $B=CB_1$ is true, where the matrix B_1 is strictly positive, from the equalities

$$\sum_{i=1}^{l} c_{ki}\frac{\langle b_i,p_0\rangle}{\langle C_i,p_0\rangle}=\sum_{i=1}^{l} b_{ki}, \quad k=\overline{1,n}, \tag{12}$$

we have the equalities

$$\sum_{i=1}^{l} c_{ki}\left[\frac{\langle b_i,p_0\rangle}{\langle C_i,p_0\rangle}-\sum_{k=1}^{l} b_{ik}^1\right], \quad k=\overline{1,n}. \tag{13}$$

Due to the vectors $C_i, i=\overline{1,l}$, are linear independent we obtain

$$\frac{\langle b_i,p_0\rangle}{\langle C_i,p_0\rangle}-\sum_{k=1}^{l} b_{ik}^1=0, \quad i=\overline{1,l}, \tag{14}$$

or

$$\langle b_i, p_0\rangle - y_i\langle C_i, p_0\rangle = 0, \quad i=\overline{1,l}. \tag{15}$$

But

$$\langle b_i, p_0\rangle = \sum_{s=1}^{l}\langle C_s, p_0\rangle b_{si}^1. \tag{16}$$

Substituting (16) into (15) we obtain

$$\sum_{s=1}^{l}\langle C_s, p_0\rangle b_{si}^1 = y_i\langle C_i, p_0\rangle, \quad i=\overline{1,l}. \tag{17}$$

Let us put $D=\{\langle C_s, p_0\rangle\}_{s=1}^{l}$ then $d_s=\langle C_s, p_0\rangle > 0, s=\overline{1,l}$. From this it follows that the vector D belongs to the interior of the polyhedral cone created by vectors $C_k^{\mathrm{T}}=\{c_{ki}\}_{i=1}^{l}, k=\overline{1,n}$.

Now, suppose that the vectors $C_i \in R_+^n, i=\overline{1,l}$ are linear dependent.

In this case we come to the case 2) $n<l$, at the beginning of the proof.

Introduce into consideration $l-n$ fictitious goods and let us consider the demand matrix constructed by the vectors-column $C_i^{\varepsilon}=\{c_{ki}(\varepsilon)\}_{k=1}^{l}\in R_+^l$, where $c_{ki}(\varepsilon)=c_{ki}, k=\overline{1,n}, i\le n$, $c_{ki}(\varepsilon)=0, k=\overline{n+1,l}, i\le n$, and $C_i^{\varepsilon}=\{c_{ki}(\varepsilon)\}_{k=1}^{l}$ where $c_{ki}(\varepsilon)=c_{ki}, k=\overline{1,n}, i>n$, $c_{ki}(\varepsilon)=\delta_{ki}\varepsilon, k=\overline{n+1,l}, l\ge i>n$. Denote the matrix $C^{\varepsilon}=|c_{ki}(\varepsilon)|_{k=1,i=1}^{l,l}$ for the sufficiently small positive $\varepsilon>0$. Then, the rank of the matrix C^{ε} is equal l for every sufficiently small positive $\varepsilon>0$. Let us to put $B^{\varepsilon}=C^{\varepsilon}B_1$. Suppose that the vector $p_0=\{p_i^0\}_{i=1}^{n}\in R_+^n$ is a solution to the problem

$$\sum_{i=1}^{l}c_{ki}\frac{\langle b_i, p_0\rangle}{\langle C_i, p_0\rangle} = \sum_{i=1}^{l}b_{ki}, \quad k=\overline{1,n}. \tag{18}$$

Then the vector $p_0^{\varepsilon}\in R_+^l$ is a solution to the problem

$$\sum_{i=1}^{l}c_{ki}(\varepsilon)\frac{\langle b_i^{\varepsilon}, p_0^{\varepsilon}\rangle}{\langle C_i^{\varepsilon}, p_0^{\varepsilon}\rangle} = \sum_{i=1}^{l}b_{ki}^{\varepsilon}, \quad k=\overline{1,l}, \tag{19}$$

for every sufficiently small $\varepsilon>0$, where we put $p_0^{\varepsilon}=\{p_i^{0,\varepsilon}\}_{i=1}^{l}$, $p_i^{0,\varepsilon}=p_i^0, i=\overline{1,n}$, $p_i^{0,\varepsilon}=0, i=\overline{n+1,l}$. The Equalities (19) can be written in the form

$$\sum_{i=1}^{l}c_{ki}\frac{\langle b_i^{\varepsilon}, p_0^{\varepsilon}\rangle}{\langle C_i^{\varepsilon}, p_0^{\varepsilon}\rangle} = \sum_{i=1}^{l}b_{ki}, \quad k=\overline{1,n}, \tag{20}$$

$$\varepsilon\frac{\langle b_i^{\varepsilon}, p_0^{\varepsilon}\rangle}{\langle C_i^{\varepsilon}, p_0^{\varepsilon}\rangle} = \varepsilon\sum_{s=1}^{l}b_{is}^1, \quad i=\overline{n+1,l}. \tag{21}$$

On such a vector p_0^{ε} we have

$$\frac{\langle b_i^{\varepsilon}, p_0^{\varepsilon}\rangle}{\langle C_i^{\varepsilon}, p_0^{\varepsilon}\rangle} = \frac{\langle b_i, p_0\rangle}{\langle C_i, p_0\rangle}, \quad i=\overline{1,l}. \tag{22}$$

Therefore, the Equalities (20), (21) for $\varepsilon>0$ is written in the form

$$\sum_{i=1}^{l}c_{ki}\frac{\langle b_i, p_0\rangle}{\langle C_i, p_0\rangle} = \sum_{i=1}^{l}b_{ki}, \quad k=\overline{1,n}, \tag{23}$$

$$\frac{\langle b_i, p_0 \rangle}{\langle C_i, p_0 \rangle} = \sum_{s=1}^{l} b_{is}^1, \quad i = \overline{n+1, l}. \tag{24}$$

If to take into account the equalities

$$\sum_{i=1}^{l} c_{ki} \left[\frac{\langle b_i, p_0 \rangle}{\langle C_i, p_0 \rangle} - \sum_{s=1}^{l} b_{is}^1 \right] = 0, \quad k = \overline{1, n}, \tag{25}$$

and Equalities (24) we obtain

$$\sum_{i=1}^{n} c_{ki} \left[\frac{\langle b_i, p_0 \rangle}{\langle C_i, p_0 \rangle} - \sum_{s=1}^{l} b_{is}^1 \right] = 0, \quad k = \overline{1, n}. \tag{26}$$

Since the vectors $C_i, i = \overline{1, n}$, are linear independent we obtain

$$\frac{\langle b_i, p_0 \rangle}{\langle C_i, p_0 \rangle} - \sum_{s=1}^{l} b_{is}^1 = 0, \quad i = \overline{1, n}. \tag{27}$$

Due to the equalities $\langle b_i, p_0 \rangle = \sum_{s=1}^{l} \langle C_i, p_0 \rangle b_{si}^1, i = \overline{1, l}$, the equalities are true

$$\sum_{s=1}^{l} \langle C_s, p_0 \rangle b_{si}^1 = y_i \langle C_i, p_0 \rangle, \quad i = \overline{1, l}, \tag{28}$$

where we put $\sum_{s=1}^{l} b_{is}^1 = y_i, i = \overline{1, l}$, $\langle C_i, p_0 \rangle = \sum_{s=1}^{n} c_{si} p_s^0$. The last means the needed.

Sufficiency. From the fact that the strictly positive vector $D = \{d_i\}_{i=1}^{l}$ solves the Problem (11) and it belongs to the polyhedral cone created by vectors $C_k^{\mathrm{T}} = \{c_{ki}\}_{i=1}^{l}, k = \overline{1, n}$, then there exists nonnegative vector $p^0 = \{p_s^0\}_{s=1}^{n}$ such that

$$\sum_{s=1}^{n} c_{si} p_s^0 = d_i, \quad i = \overline{1, l}. \tag{29}$$

Substituting $d_i, i = \overline{1, l}$, into (11) we obtain

$$\sum_{i=1}^{l} b_{ik}^1 \sum_{s=1}^{n} c_{si} p_s^0 = y_k \sum_{s=1}^{n} c_{sk} p_s^0, \quad i = \overline{1, l}, \tag{30}$$

or

$$\sum_{s=1}^{n} b_{sk} p_s^0 = y_k \sum_{s=1}^{n} c_{sk} p_s^0, \quad i = \overline{1, l}, \tag{31}$$

where $y_k = \sum_{i=1}^{l} b_{ki}^1$. But

$$\sum_{i=1}^{l} c_{ki} y_i = \sum_{i=1}^{l} b_{ki}, \quad k = \overline{1, n}. \tag{32}$$

The Equalities (31) (32) gives

$$\sum_{i=1}^{l} c_{ki} \frac{\langle b_i, p_0 \rangle}{\langle C_i, p_0 \rangle} = \sum_{i=1}^{l} b_{ki}, \quad k = \overline{1, n}, \tag{33}$$

where we put $\langle b_i, p_0 \rangle = \sum_{s=1}^{n} b_{si} p_s^0, \langle C_i, p_0 \rangle = \sum_{s=1}^{n} c_{si} p_s^0, i = \overline{1, l}$.

Theorem 2 is proved. □

Definition 6. *Let* $C_i \in R_+^n, i=\overline{1,l}$, *be a set of demand vectors and let* $b_i \in R_+^n, i=\overline{1,l}$, *be a set of supply vectors. We say that the structure of supply is agreed with the structure of demand in the weak sense if for the matrix B the representation* $B = CB_1$ *is true, where the matrix B consists of the vectors* $b_i \in R_+^n, i=\overline{1,l}$, *as columns, and the matrix C is composed from the vectors* $C_i \in R_+^n, i=\overline{1,l}$, *as columns and* B_1 *is a square matrix satisfying the conditions*

$$\sum_{s=1}^{l} b_{is}^1 \geq 0, \quad i=\overline{1,l}, \quad B_1 = \left| b_{is}^1 \right|_{i,s=1}^{l}. \tag{34}$$

Definition 7. *Let* $C_i \in R_+^n, i=\overline{1,l}$, *be a set of demand vectors and let* $b_i \in R_+^n, i=\overline{1,l}$, *be a set of supply vectors. We say that the structure of supply is agreed with the structure of demand in the weak sense of the rank* $|I|$ *if there exists a subset* $I \subseteq N$ *such that for the matrix* B^I *the representation* $B^I = C^I B_1^I$ *is true, where the matrix* B^I *consists of the vectors* $b_i^I \in R_+^{|I|}, i=\overline{1,l}$, *as columns, and the matrix* C^I *is composed from the vectors* $C_i^I \in R_+^n, i=\overline{1,l}$, *as columns and* B_1^I *is a square matrix, satisfying the conditions*

$$\sum_{s=1}^{l} b_{is}^{1,I} \geq 0, \quad i=\overline{1,l}, \quad B_1^I = \left| b_{is}^{1,I} \right|_{i,s=1}^{l}, \tag{35}$$

where $b_i^I = \{b_{ki}\}_{k \in I}$, $C_i^I = \{c_{ki}\}_{k \in I}$ *and, moreover, the inequalities*

$$\sum_{i=1}^{l} c_{ki} y_i^I < \sum_{i=1}^{l} b_{ki}, \quad k \in N \setminus I, \quad y_i^I = \sum_{s=1}^{l} b_{is}^{1,I},$$

are valid.

Theorem 3. *Let the structure of supply be agreed with structure of demand in the weak sense with the supply vectors* $b_i = \{b_{ki}\}_{k=1}^n \in R_+^n, i=\overline{1,l}$, *and the demand vectors* $\left\{C_i = \{c_{ki}\}_{k=1}^n \in R_+^n, i=\overline{1,l}\right\}$, *and let* $\sum_{s=1}^{n} c_{si} > 0, i=\overline{1,l}$, $\sum_{i=1}^{l} c_{si} > 0, s=\overline{1,n}$. *The necessary and sufficient conditions of the solution existence of the set of equations*

$$\sum_{i=1}^{l} c_{ki} \frac{\langle b_i, p \rangle}{\langle C_i, p \rangle} = \sum_{i=1}^{l} b_{ki}, \quad k=\overline{1,n}, \tag{36}$$

is belonging of the vector $D = \{d_i\}_{i=1}^l$ *to the polyhedral cone created by vectors* $C_k^{\mathrm{T}} = \{c_{ki}\}_{i=1}^l, k=\overline{1,n}$, *where* $B = CB_1$, $B_1 = \left| b_{ki}^1 \right|_{k,i=1}^{l}$ *is a square matrix, the vector* $D = \{d_i\}_{i=1}^l$ *is a strictly positive solution to the set of equations*

$$\sum_{k=1}^{l} b_{ki}^1 d_k = y_i d_i, \quad y_i = \sum_{k=1}^{l} b_{ik}^1 \geq 0, \quad i=\overline{1,l}. \tag{37}$$

Proof. The proof of Theorem 3 is the same as Theorem 2. □

3. Recession Phenomenon Description

In this section we present the description of recession phenomenon proposed by the author in the papers [6] [7] [8] [9]. How to explain the phenomenon of recession from the point of view of the theory of economic equilibrium? It is obvious that firms produce goods to meet the needs of consumers. This means that the produced goods find their buyer. Speaking in the language of economics, there is a non-zero demand for produced goods. When the demand for produced goods is saturated, a certain part of these goods does not find its buyers. Under these conditions, prices for these goods become undefined. The owners of these goods will try to sell them at lower prices. Therefore, the recession phenomenon is characterized by an excess of manufactured goods, the equilibrium state must allow degeneration, that is, there must be many states of economic equilibrium.

Below, we consider the case as the *i*-th consumer has fixed gain $D_i > 0, i = \overline{1,l}$. Here, we consider the partial case as the *i*-th consumer is characterized by demand vector $C_i = \{C_{ki}\}_{k=1}^n, i = \overline{i,l}$.

Lemma 4. *If the vector* $p^0 = \left(p_1^0, \cdots, p_n^0\right) \in B$ *solves the set of inequalities*

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s} = \psi_k, \quad k \in I,$$

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s} < \psi_k, \quad k \in J, \tag{38}$$

then $p_k^0 = 0, k \in J$, *where* $I \cup J = \{1, \cdots, n\}$, $I \cap J = \varnothing$, $D_i > 0, i = \overline{1,l}$, $\psi_i > 0, i = \overline{1,n}$,

$$B = \left\{ p = \{p_i\}_{i=1}^n \in R_+^n, \sum_{i=1}^{n} p_i \psi_i = \sum_{i=1}^{l} D_i \right\}.$$

Proof. Suppose that $p^0 = \left(p_1^0, \cdots, p_n^0\right) \in B$ solves the set of inequalities. Multiplying on p_k^0 the left and right hand sides of Inequalities (38) we obtain the inequality $\sum_{i=1}^{n} p_i^0 \psi_i \geq \sum_{i=1}^{l} D_i$. If we assume that $p_k^0 > 0, k \in J$, then we obtain the strict inequality $\sum_{i=1}^{n} p_i^0 \psi_i > \sum_{i=1}^{l} D_i$. The last means the contradiction. Lemma 4 is proved. □

Theorem 4. *The necessary and sufficient conditions of strictly positive solution existence in the set B of the set of equations*

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s} = \psi_k, \quad k = \overline{1,n}, \tag{39}$$

is belonging of the vector $\psi = \{\psi_k\}_{k=1}^n$ *to the interior of the cone created by the vectors* $C_i = \{C_{ki}\}_{k=1}^n, i = \overline{1,l}$, *and belonging of the vector* $\left\{\frac{D_i}{y_i}\right\}_{i=1}^l$ *to the interior of the cone, created by vectors* $C_k^{\mathrm{T}} = \left\{C_{kj}\right\}_{j=1}^l, k = \overline{1,n}$, *for a certain strictly positive vector* $y = \{y_k\}_{k=1}^l$, *satisfying to the set of equations*

$$\sum_{i=1}^{l} C_{ki} y_i = \psi_k, \quad k = \overline{1,n}. \tag{40}$$

Proof. Necessity. Let there exists strictly positive solution $p_0 \in B$ to the set of Equation (39). Denote

$$y_i = \frac{D_i}{\sum_{s=1}^{n} C_{si} p_s^0}, \quad i = \overline{1,l},$$

then we obtain the equalities

$$\sum_{i=1}^{l} C_{ki} y_i = \psi_k, \quad k = \overline{1,n},$$

$$\sum_{s=1}^{n} C_{si} p_s^0 = \frac{D_i}{y_i}, \quad i = \overline{1,l},$$

proving the needed.

The sufficiency. If the vector ψ belongs to the cone created by the vectors $C_i = \{C_{ki}\}_{k=1}^{n}, i = \overline{1,l}$, and between the strictly positive vector $y = \{y_k\}_{k=1}^{l}$, satisfying the set of equations

$$\sum_{i=1}^{l} C_{ki} y_i = \psi_k, \quad k = \overline{1,n}, \tag{41}$$

there exists a vector $y = \{y_k\}_{k=1}^{l}$, such that

$$\sum_{s=1}^{n} C_{si} p_s^0 = \frac{D_i}{y_i}, \quad i = \overline{1,l},$$

then the strictly positive vector $p_0 = \{p_s^0\}_{s=1}^{n}$ solves the set of Equation (39). It is evident that $p_0 \in B$. Theorem 4 is proved. □

Theorem 5. *The necessary and sufficient conditions of the existence of equilibrium price vector* $p_0 = \{p_i^0\}_{i=1}^{n}$ *in the set B such that*

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s^0} \le \psi_k, \quad k = \overline{1,n}, \tag{42}$$

is the existence of strictly positive vector $y = \{y_k\}_{k=1}^{l}$ *and vector* $\bar{\psi} = \sum_{i=1}^{l} C_i y_i \le \psi$ *such that a vector* $\left\{\frac{D_i}{y_i}\right\}_{i=1}^{l}$ *belongs to the cone created by vectors* $C_k^T, k = \overline{1,n}$.

Proof. The necessity. If $p_0 \in B$ and solves the set of Inequalities (42), then there exists non empty set $I \subseteq \{1, 2, \cdots, n\}$ such that

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s^0} = \psi_k, \quad k \in I, \tag{43}$$

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s^0} < \psi_k, \quad k \in J. \tag{44}$$

Really, the strict inequalities

$$\sum_{i=1}^{l} \frac{C_{ki} D_i}{\sum_{s=1}^{n} C_{si} p_s^0} < \psi_k, \quad k = \overline{1,n}, \tag{45}$$

is impossible, since the assumption of validity of the set Inequalities (45) leads to the inequality $\sum_{i=1}^{n} p_i^0 \psi_i > \sum_{i=1}^{l} D_i$. So, the set I is a nonempty one. Let us denote

$$y_i = \frac{D_i}{\sum_{s=1}^{n} C_{si} p_s^0}, \quad i = \overline{1,l},$$

$$\bar{\psi} = \sum_{i=1}^{l} C_i y_i,$$

then

$$\sum_{s=1}^{n} C_{si} p_s^0 = \frac{D_i}{y_i}, \quad i = \overline{1,l}.$$

The last means belonging of vector $\left\{ \frac{D_i}{y_i} \right\}_{i=1}^{l}$ to the cone created by vectors $C_k^{\mathrm{T}}, k = \overline{1,n}$.

The sufficiency. If there exists vector $y = \{ y_k \}_{k=1}^{l}$ and vector $\bar{\psi} = \sum_{i=1}^{l} C_i y_i \le \psi$ such that $\left\{ \frac{D_i}{y_i} \right\}_{i=1}^{l}$ belongs to the cone created by vectors $C_k^{\mathrm{T}}, k = \overline{1,n}$, then there exists vector $p_0 \ge 0$ such that

$$\sum_{s=1}^{n} C_{si} p_s^0 = \frac{D_i}{y_i}, \quad i = \overline{1,l}.$$

From this it follows that p_0 is an equilibrium price vector. Theorem 5 is proved. □

If the conditions of Theorem 5 are true, then we assume that the set $I \subset N$. The last means that the partial clearing of markets takes place. The vector $\bar{\psi}$ we call the vector of real consumption. The vector $\bar{\psi}$ does not coincide with the vector ψ. Let us consider the set of equations

$$\sum_{i=1}^{l} c_{ki} \frac{D_i + y_i \sum_{s \in J} c_{si} p_s}{\langle C_i, p \rangle} = \bar{\psi}_k, \quad k = \overline{1,n}, \tag{46}$$

$$\bar{\psi}_k = \sum_{i=1}^{l} y_i c_{ki}, \quad k = \overline{1,n}, \tag{47}$$

where the strictly positive vector $y = \{ y_i \}_{i=1}^{l}$ solves the set of inequalities

$$\sum_{i=1}^{l} c_{ki} y_i = \psi_k, \quad k \in I, \tag{48}$$

$$\sum_{i=1}^{l} c_{ki} y_i < \psi_k, \quad k \in N \setminus I. \tag{49}$$

Not only the equilibrium price vector p_0 solves the set of Equation (46). The price vector p solving the set of Equation (46) has the following structure $p=\{p_i\}_{i=1}^{n}$ $p_i=p_i^0, i\in I$ $p_i=p_i^1, i\in N\setminus I$, where $p_i^1, i\in N\setminus I$ are arbitrary nonnegative real numbers. So, any vector p having the above structure clears the market with the demand vectors $C_i\in R_+^n, i=\overline{1,l}$, and supply vector $\bar{\psi}$. But the set of Equation (46) does not determine uniquely the prices of goods that belongs to the set $N\setminus I$ in spite of that the demand for these goods is non zero. The cause is that the needs of consumers are completely satisfied on these goods. To determine the prices for goods from the set $N\setminus I$ it needs to remove the degeneracy that is in the set of Equation (46). For this purpose it is need to add the infinitely small term removing the degeneracy. This term should take into account the technologies of production of these goods and fiscal policy. For example, if the map $T_i(p)=\{t_{ki}(p)\}_{k=1}^{n}, t_{ki}(p)=0, k\in I, i=\overline{1,l}$ takes into account the technologies of production of goods from the set $N\setminus I$ and fiscal policy, then the set of equations

For this purpose it is need to add the infinitely small term removing the degeneracy. This term should take into account the technologies of production of these goods and fiscal policy. For example, if the map $T_i(p)=\{t_{ki}(p)\}_{k=1}^{n}, t_{ki}(p)=0, k\in I, i=\overline{1,l}$ takes into account the technologies of production of goods from the set $N\setminus I$ and fiscal policy, then the set of equations

$$\sum_{i=1}^{l} c_{ki}\frac{D_i+y_i\sum_{s\in J}c_{si}p_s}{\langle C_i+\varepsilon T_i(p),p\rangle}=\bar{\psi}_k,\quad k=\overline{1,n}, \tag{50}$$

$$\bar{\psi}_k=\sum_{i=1}^{l} y_i c_{ki},\quad k=\overline{1,n}, \tag{51}$$

determines the prices for goods from the set $N\setminus I$ under condition that set of equations

$$\langle T_i(p),p\rangle=0,\quad i=\overline{1,l}, \tag{52}$$

determines the vector $p_1=\{p_i^1\}_{i\in N\setminus I}$, solving the set of Equation (52). Here $\varepsilon>0$ and it is very small. Tending $\varepsilon>0$ to zero we obtain needed solution. It may happen that the specified procedure is not applicable. In this case, the prices for these goods will be determined by agreements. The vector $\bar{\psi}=\{\bar{\psi}_k\}_{k=1}^{n}$ we will call the vector of real consumption.

In the case, as $I\subset N$, the price vector p solving the set of Equation (46) and taking into account the procedure for determining the ambiguous part of the vector components, stated above, will be called the generalized equilibrium price vector.

So, real degeneracy of solutions has the set of Equation (46) and the generalized equilibrium price vector solves the set of Equation (46). The quantity of goods $\psi_k-\bar{\psi}_k, k\in N\setminus I$ does not find a consumer. To characterize this we introduce the parameter of recession level

$$R = \frac{\langle \psi - \bar{\psi}, p \rangle}{\langle \psi, p \rangle},$$

where p is a generalized equilibrium price vector solving the set of Equation (46). Note that the i-th consumer can purchase a set of goods $y_i C_i^0$ on credit, where $C_i^0 = \{c_{ki}^0\}_{k=1}^n$, $c_{ki}^0 = 0, k \in I, c_{ki}^0 = c_{ki}, i = \overline{1,l}$.

4. Conditions of the Sustainable Economy Development

A description of sustainable economic development at the macroeconomic level is proposed in [2] [3] [4] [5]. In this section, we formulate the principles of sustainable development at the microeconomic level. Each business project that starts the production of a certain group of goods plans to receive added value. In the production process, there are direct costs for production materials and labor costs. If the business project is such that it has sales markets with positive added value, then we say that this business project has concluded contracts with suppliers of materials and raw materials and a certain number of employees who sell their labour power. The set of concluded contracts will be characterized by a technological mapping. Let's consider a technological mapping that describes the production of one type of product by spending a vector of goods $\{a_{ki}\}_{k=1}^n$ on one unit of the manufactured i-th product. Such a technological mapping will be characterized by input-output matrix $\|a_{ki}\|_{k,i=1}^n$. Our goal is to ensure the sustainable development of the economic system. Suppose that x_i of units of the i-th product are produced in the economic system. The input vector will be equal $X_i = \{x_i a_{ki}\}_{k=1}^n$. Then the output vector will be equal $Y_i = \{x_i \delta_{ki}\}_{k=1}^n$. Each i-th consumer in the economic system we characterize by two vectors $b_i = \{b_{ki}\}_{k=1}^n, i = \overline{1,l}$ and $C_i = \{c_{ki}\}_{k=1}^n, i = \overline{1,l}$. The vectors $b_i = \{b_{ki}\}_{k=1}^n, i = \overline{1,l}$ we call the property vector. The i-th consumer wants to exchange the vector $b_i = \{b_{ki}\}_{k=1}^n$ on the vector $C_i = \{c_{ki}\}_{k=1}^n$ which we call demand vector. We assume that in production process all produced goods are distributed in accordance with the rule

$$\sum_{i=1}^n (Y_i - X_i) = \sum_{i=1}^l b_i. \tag{53}$$

Is there such a market mechanism that would provide such distribution of the product in the society by certain prices vector?

Definition 8. *The distribution of a product in society is called economically expedient if, in the process of production and distribution, in accordance with the concluded agreement, the i-th consumer owns a set of goods* $b_i = \{b_{ki}\}_{k=1}^n, i = \overline{1,l}$ *such that the vector* $b = \sum_{i=1}^l b_i$ *belongs to the interior of the cone formed by the vectors* $C_i = \{c_{ki}\}_{k=1}^n, i = \overline{1,l}$.

Let us introduce two matrices $C = |c_{ki}|_{k=1,i=1}^{n,l}$ and $B = |b_{ki}|_{k=1,i=1}^{n,l}$.

Definition 9. *If the representation* $B = CB_1$ *is valid for the matrix B such that there is a solution to the problem*

$$\sum_{k=1}^{l} b_{ki}^{1} d_k = \sum_{s=1}^{l} b_{is}^{1} d_i \tag{54}$$

in relation to the vector $d = \{d_k\}_{k=1}^{l}$, *which belongs to the cone formed by the vectors* $C_i^{\mathrm{T}} = \{c_{ki}\}_{k=1}^{n}$, *we will call the distribution of the product rational.*

In this work, we follow the concept of describing economic systems developed in [1]. The essence of this description is the supply of firms is primary and the choice of consumers is secondary. But at the same time, the fact that the structure of the supply consistent with the structure of the choice is important. Axioms of this description are presented in [1], where random fields of consumer choice and decision-making by firms are built on the basis of these axioms. We describe firms by technological mappings $y_i = F_i(x_i), x_i \in X_i$ from the CTM class, and the choice of the i-th consumer by a vector of goods $C_i = \{c_{ki}\}_{k=1}^{n}$ that he wants to consume in a certain period of the economy's functioning. Let us assume that m firms function in the economic system, which are described by technological mappings $y_i = F_i(x_i), x_i \in X_i, i = \overline{1,m}$, from CTM class. If the firms chose the production processes $x_i \in X_i, y_i \in F(x_i), i = \overline{1,m}$, then the final product $\sum_{i=1}^{m}(y_i - x_i)$ is produced in the economic system which in the process of distribution of products in accordance with the concluded contracts the i-th consumer received a vector of goods $b_i, i = \overline{1,l}$.

The condition of economic equilibrium is the fulfillment of inequalities

$$\sum_{i=1}^{l} c_{ki} \frac{\langle b_i, p\rangle}{\langle C_i, p\rangle} \le \sum_{i=1}^{l} b_{ki}, \quad k = \overline{1,n}. \tag{55}$$

The main idea that we lay down is the following: firms should produce such a quantity of goods, the implementation of which will ensure their functioning in the next production cycle, and for this they should have sufficient income. In addition, the necessary balances for the state must be ensured: state spending on defense, renewal of fixed assets, financing of education, etc. Is such a distribution of products the result of market exchange based on the existence of an equilibrium price vector. Among all possible distributions of products produced by firms, the important one is when firms will be profitable so that they can carry out the next production cycle. First, consider the case when the only consumers are the firms themselves. In this case, it is convenient to consider all those employed in production as firms that produce labor and sell it to other firms. The general formulation of the problem is as follows: firms implemented production processes $(x_i, y_i), x_i \in X_i, y_i \in F_i(x_i), i = \overline{1,m}$ as a result of which the final product $\sum_{i=1}^{m}(y_i - x_i)$ is produced in the economic system, which must be distributed so that the process of production and distribution of products is continuous. A condition for this is a system of equalities

$$\sum_{i=1}^{m} x_{ki} \frac{(1-\pi_i)\langle y_i, p\rangle}{\langle x_i, p\rangle} + \sum_{i=m+1}^{l} \frac{C_{ki} D_i(p)}{\langle C_i, p\rangle} = \sum_{i=1}^{m} y_{ki}, \quad k = \overline{1,n}, \tag{56}$$

$$\sum_{i=m+1}^{l} y_i^0 C_{ki} = \sum_{i=1}^{m} \pi_i y_{ki}, \quad k = \overline{1,n}. \tag{57}$$

Equalities (57) are the equations of material balances to ensure public procurement, defense orders, construction of educational institutions, renewal of fixed assets, etc. If to put $D_i(p) = y_i^0 \langle C_i, p \rangle$ and take into Consideration (57), then we obtain

$$\sum_{i=1}^{m} x_{ki} \frac{(1-\pi_i)\langle y_i, p \rangle}{\langle x_i, p \rangle} = \sum_{i=1}^{m} (1-\pi_i) y_{ki}, \quad k = \overline{1,n}, \tag{58}$$

where we denoted

$$\langle y_i, p \rangle = \sum_{k=1}^{n} y_{ki} p_k, \quad \langle x_i, p \rangle = \sum_{k=1}^{n} x_{ki} p_k,$$

and $\pi = \{\pi_i\}_{i=1}^{m}, 0 \le \pi_i \le 1, i = \overline{1,m}$ is the so-called generalized vector of taxation, the economic meaning of which will be clarified in the applied articles.

In order for the process of functioning of the economic system to be continuous, it is necessary that there should be an equilibrium price vector p_0 such that the Equalities (56) and the inequalities $\langle y_i - x_i, p_0 \rangle > 0, i = \overline{1,m}$, were valid.

Suppose that firms implemented the continuous strategies of firm behaviour $(x_i(p), y_i(p)), x_i(p) \in X_i, y_i(p) \in F_i(x_i(p)), i = \overline{1,m}, p \in P$, where $P = \left\{ p \in R_+^n, \sum_{i=1}^{n} p_i = 1 \right\}$ as a result of which the final product $\sum_{i=1}^{m} (y_i(p) - x_i(p))$ is produced in the economic system, which must be distributed so that the process of production and distribution of products is continuous. A condition for this is a system of equalities

$$\sum_{i=1}^{m} x_{ki}(p) \frac{(1-\pi_i)\langle y_i(p), p \rangle}{\langle x_i(p), p \rangle} + \sum_{i=m+1}^{l} \frac{C_{ki}(p) D_i(p)}{\langle C_i(p), p \rangle} = \sum_{i=1}^{m} y_{ki}(p), \quad k = \overline{1,n}, \tag{59}$$

$$\sum_{i=m+1}^{l} y_i^0(p) C_{ki}(p) = \sum_{i=1}^{m} \pi_i y_{ki}(p), \quad k = \overline{1,n}. \tag{60}$$

If to put $D_i(p) = y_i^0(p) \langle C_i(p), p \rangle$ and take into Consideration (60), then we obtain

$$\sum_{i=1}^{m} x_{ki}(p) \frac{(1-\pi_i)\langle y_i(p), p \rangle}{\langle x_i(p), p \rangle} = \sum_{i=1}^{m} (1-\pi_i) y_{ki}(p), \quad k = \overline{1,n}, \tag{61}$$

where we denoted

$$\langle y_i(p), p \rangle = \sum_{k=1}^{n} y_{ki}(p) p_k, \quad \langle x_i(p), p \rangle = \sum_{k=1}^{n} x_{ki}(p) p_k,$$

and $\pi = \{\pi_i\}_{i=1}^{m}, 0 \le \pi_i \le 1, i = \overline{1,m}$ is the so-called generalized vector of taxation, the economic meaning of which will be clarified in the applied articles.

In order for the process of functioning of the economic system to be continuous, it is necessary that there should be an equilibrium price vector p_0 such that the Equalities (61) and the inequalities $\langle y_i(p_0) - x_i(p_0), p_0 \rangle > 0, i = \overline{1,m}$, were valid.

Suppose that x_i^0 units of the i-th product are produced in the economic system. At the same time, input vector will be equal $X_i = \{x_i^0 a_{ki}\}_{k=1}^n$. The output vector will be equal $Y_i = \{x_i^0 \delta_{ki}\}_{k=1}^n$. Then the problem (58) is written in the form

$$\sum_{i=1}^n a_{ki} \frac{(1-\pi_i) x_i^0 p_i}{\sum_{s=1}^n a_{si} p_s} = (1-\pi_k) x_k^0, \quad k = \overline{1,n}. \tag{62}$$

Let us denote $(1-\pi_i) x_i^0 = x_i, i = \overline{1,n}$, then the problem (62) we can write in the form

$$\sum_{i=1}^n a_{ki} \frac{x_i p_i}{\sum_{s=1}^n a_{si} p_s} = x_k, \quad k = \overline{1,n}, \tag{63}$$

$$p_i - \sum_{s=1}^n a_{si} p_s > 0, \quad i = \overline{1,n}. \tag{64}$$

Theorem 6. *Let* $A = \|a_{ki}\|_{k,i=1}^n$ *be a non negative productive indecomposable matrix. The necessary and sufficient condition for the continuous functioning of the economic system is that the vector* $x = \{x_i\}_{i=1}^n$ *belongs to the interior of the cone formed by the vectors of the columns of the matrix* $A(E-A)^{-1}$.

Proof. Necessity. Suppose that there exists an equilibrium price vector p_0 satisfying the set of Equation (63) and the Inequalities (64). Substituting p_i^0 from the equalities

$$p_i^0 - \sum_{s=1}^n a_{si} p_s^0 = \delta_i^0, \quad \delta_i^0 > 0, \quad i = \overline{1,n}. \tag{65}$$

into Equalities (63) we obtain

$$x_k = \sum_{i=1}^n a_{ki} \frac{x_i \left(\sum_{s=1}^n a_{si} p_s^0 + \delta_i^0 \right)}{\sum_{s=1}^n a_{si} p_s} = \sum_{i=1}^n a_{ki} x_i + \sum_{i=1}^n a_{ki} \frac{x_i \delta_i^0}{\sum_{s=1}^n a_{si} p_s^0}, \quad k = \overline{1,n}. \tag{66}$$

If to introduce the vector $\alpha = \{\alpha_k\}_{k=1}^n$, where $\alpha_k = \frac{x_k \delta_k^0}{\sum_{s=1}^n a_{sk} p_s^0} > 0, k = \overline{1,n}$, then we obtain from the Equalities (66) the equality $x = A(E-A)^{-1} \alpha$. The last proves the necessity.

Sufficiency. From the beginning, we assume that A^{-1} exists. Then for the diagonal matrix

$$X = \|\delta_{ij} x_j\|_{i,j=1}^n \tag{67}$$

the representation $X = AB_1$ is true, where $B_1 = A^{-1} X = \|a_{ki}^{-1} x_i\|_{k,i=1}^n$. From the assumption Theorem 6 we have

$$b_k^1 = \sum_{i=1}^n a_{ki}^{-1} x_i = \left[(E-A)^{-1} \alpha \right]_k > 0, k = \overline{1,n}. \tag{68}$$

Let us prove that the set of equations

$$\sum_{k=1}^{n} a_{ki}^{-1} x_i d_k = b_i^1 d_i, \quad i = \overline{1,n}, \tag{69}$$

has strictly positive solution belonging to the cone created by vectors $\{a_{ki}\}_{k=1}^{n}, i = \overline{1,n}$. From (68) we obtain

$$x_k = \sum_{i=1}^{n} a_{ki} b_i^1. \tag{70}$$

The problem (69) is equivalent to the problem

$$d_k = \sum_{i=1}^{n} a_{ik} \frac{b_i^1}{x_i} d_i = \sum_{i=1}^{n} a_{ik} \frac{b_i^1}{\sum_{k=1}^{n} a_{ik} b_k^1} d_i, \quad i = \overline{1,n}. \tag{71}$$

Let us introduce the denotation

$$u_{ik} = a_{ik} \frac{b_i^1}{\sum_{k=1}^{n} a_{ik} b_k^1}, \quad i,k = \overline{1,n}, \tag{72}$$

and introduce the matrix $U = \left| u_{ik} \right|_{i,k=1}^{n}$.

Let us consider the non linear set of equations

$$d_k = \frac{d_k + \sum_{i=1}^{n} u_{ik} d_i}{1 + \sum_{k=1}^{n} \sum_{i=1}^{n} u_{ik} d_i}, \quad k = \overline{1,n}, \tag{73}$$

on the set $D = \left\{ d = \{d_i\}_{i=1}^{n}, d_i \geq 0, i = \overline{1,n}, \sum_{i=1}^{n} d_i = 1 \right\}$.

Due to Schauder Theorem [10] there exists a solution to the set of Equation (73) in the set D. The set of Equation (73) can be written in the form

$$\sum_{i=1}^{n} u_{ik} d_i = \lambda d_k, \tag{74}$$

where $\lambda = \sum_{k=1}^{n} \sum_{i=1}^{n} u_{ik} d_i$. Let us prove that $\lambda > 0$ and the solution to the set of equations $d = \{d_i\}_{i=1}^{n}$ is a strictly positive one due to the matrix A is indecomposable. Really, the vector $d = \{d_i\}_{i=1}^{n}$ satisfies the set of Equation (74) which can be written in operator form

$$U^{\mathrm{T}} d = \lambda d, \tag{75}$$

or

$$\left[U^{\mathrm{T}} \right]^{n-1} d = \lambda^{n-1} d. \tag{76}$$

Since the vector d belongs to the set D and the matrix U is indecomposable, then the vector $\left[U^{\mathrm{T}} \right]^{n-1} d$ is strictly positive. From this it follows that $\lambda > 0$ and the vector d is strictly positive. Let us prove that $\lambda = 1$. The problem (74) is equivalent to the problem

$$\lambda \sum_{k=1}^{n} a_{ki}^{-1} x_i d_k = b_i^1 d_i, \quad i = \overline{1,n}. \tag{77}$$

Summing over index i the left and right hand sides of the Equality (77) we have $\lambda\sum_{k=1}^{n} b_k^1 d_k = \sum_{i=1}^{n} b_i^1 d_i$. The last proves the needed. Therefore, there exists a solution to the problem (71). From Theorem 3 and (71) if to put

$$p_i = \frac{b_i^1 d_i}{\sum_{s=1}^{n} a_{is} b_s^1} \tag{78}$$

and introduce the price vector $p = \{p_i\}_{i=1}^{n}$, then $d_k = \sum_{s=1}^{n} a_{sk} p_s, k = \overline{1,n}$.

Or, relative to the equilibrium price vector we obtain the set of equations

$$p_i = \frac{b_i^1}{\sum_{s=1}^{n} a_{is} b_s^1} \sum_{s=1}^{n} a_{si} p_s, \quad i = \overline{1,n}. \tag{79}$$

Since for the vector x the representation $x = A(E-A)^{-1}\alpha$ is true, where α is strictly positive vector, then from (70) we obtain $b^1 = (E-A)^{-1}\alpha$, where $b^1 = \{b_k^1\}_{k=1}^{n}$. From the last we have

$$\frac{b_i^1}{\sum_{s=1}^{n} a_{is} b_s^1} = 1 + \frac{\alpha_i}{\sum_{k=1}^{\infty}\sum_{s=1}^{n} a_{is}^k \alpha_s} > 1, \quad i = \overline{1,n}, \tag{80}$$

where we denoted by a_{is}^k the matrix elements of the matrix A^k. So, the constructed solution to the problem (71) is such that the inequalities

$$p_i - \sum_{s=1}^{n} a_{si} p_s > 0, \quad i = \overline{1,n}, \tag{81}$$

are true if the conditions of the Theorem 6 are valid.

Suppose that the matrix A is degenerate. The proof of necessity is the same as in previous case. To prove sufficiency we consider the non degenerate matrix $A + \varepsilon E$, where $\varepsilon > 0$ and sufficiently small such that $(A+\varepsilon E)^{-1}$ exists for all $\varepsilon > 0$. Such is possible due to the fact that $\varepsilon = 0$ is a route of equation $\det(A+\varepsilon E) = 0$ and this equation has the finite number of routs. We assume that $x(\varepsilon) = (A+\varepsilon E)(E - A - \varepsilon E)^{-1}\alpha$, where the vector α is a strictly positive one. As before, repeating the all above arguments we come to the fact that the vector $d(\varepsilon) = \{d_k(\varepsilon)\}_{k=1}^{n}$ satisfies the set of equations

$$d_k(\varepsilon) = \sum_{i=1}^{n} (a_{ik} + \delta_{ik}\varepsilon) \frac{b_i^1(\varepsilon)}{\sum_{k=1}^{n} (a_{ik} + \delta_{ik}\varepsilon) b_k^1(\varepsilon)} d_i(\varepsilon), \quad i = \overline{1,n}, \tag{82}$$

and it is strictly positive. Since $b^1(\varepsilon) = \{b_i^1(\varepsilon)\}_{i=1}^{n} = (E - A - \varepsilon E)^{-1}\alpha$, $(A+\varepsilon E) b^1(\varepsilon) = (A+\varepsilon E)(E - A - \varepsilon E)^{-1}\alpha$ we obtain that there exists

$$\lim_{\varepsilon \to 0} b^1(\varepsilon) = (E-A)^{-1}\alpha = b^1 = \{b_k^1\}_{k=1}^{n}.$$

Since $d(\varepsilon)$ is bounded therefore there exists subsequence ε_n tending to zero such that there exists limit

$$\lim_{\varepsilon_n \to 0} d(\varepsilon_n) = d = \{d_k\}_{k=1}^n$$

satisfying the set of Equation (71). Following the above arguments we come to that the introduced vector $p = \{p_i\}_{i=1}^n$ satisfies to the set of Equation (79). Let us prove that the vector p satisfies the set of Equation (63). Really, since

$$\frac{x_i p_i}{\sum_{s=1}^n a_{si} p_s} = \frac{b_i^1 x_i}{\sum_{s=1}^n a_{is} b_s^1} = b_i^1 \tag{83}$$

Multiplying the left and right hand sides on a_{ki} and summing over i we have

$$\sum_{i=1}^n a_{ki} \frac{x_i p_i}{\sum_{s=1}^n a_{si} p_s} = \sum_{i=1}^n a_{ki} b_i^1 = x_k, \quad k = \overline{1,n}, \tag{84}$$

The rest statements of the Theorem 6 follow as before. Theorem 6 is proved.

□

Let us put $X_k^0 = x_k^0 p_k$, $\bar{A} = |\bar{a}_{ki}|_{k,i=1}^n, \bar{a}_{ki} = \frac{p_k a_{ki}}{p_i}$ Then the Equality (9) and Inequality (64) is written in the form

$$\sum_{i=1}^n \bar{a}_{ki} \frac{(1-\pi_i) X_i^0}{\sum_{s=1}^n \bar{a}_{si}} = (1-\pi_k) X_k^0, \quad k = \overline{1,n}, \tag{85}$$

$$1 - \sum_{s=1}^n \bar{a}_{si} > 0, \quad i = \overline{1,n}. \tag{86}$$

Theorem 7. *If there exists an equilibrium price vector* $p_0 = \{p_i^0\}_{i=1}^n$ *satisfying the inequalities*

$$\sum_{i=1}^n a_{ki} \frac{(1-\pi_i) x_i^0 p_i^0}{\sum_{s=1}^n a_{si} p_s^0} = (1-\pi_k) x_k^0, \quad k \in I,$$

$$\sum_{i=1}^n a_{ki} \frac{(1-\pi_i) x_i^0 p_i^0}{\sum_{s=1}^n a_{si} p_s^0} < (1-\pi_k) x_k^0, \quad k \in J, \tag{87}$$

then those industries indexes of which belong to the set J are unprofitable.

Proof. The proof of Theorem 7 follows from the fact that those components of equilibrium price vector $p_0 = \{p_i^0\}_{i=1}^n$ whose components belong to the set J are equal zero. □

Now let's calculate the probability of the economy functioning in continuous mode. For this purpose we consider the mapping $z = I(\pi) A (E - A)^{-1} x$ of n-dimensional cone R_+^n into itself, where $x \in R_+^n$. The image of cone R_+^n under the transformation $D = I(\pi) A (E - A)^{-1}$, where the matrix $I(\pi) = \left\| \delta_{ij} \frac{1}{1-\pi_i} \right\|_{i,j=1}^n$, is a polyhedral cone K_+^n, belonging to the cone R_+^n. It is not difficult to prove that the needed probability is

$$P=\lim_{h\to\infty}\frac{\int\limits_{\sum_{i=1}^{n}z_i\le h}\mathrm{d}z_1\cdots\mathrm{d}z_n}{\int\limits_{\sum_{i=1}^{n}x_i\le h}\mathrm{d}x_1\cdots\mathrm{d}x_n}=\frac{|\det D|}{\prod_{k=1}^{n}\sum_{i=1}^{n}D_{ik}}$$
$$=\frac{\left|\det A(E-A)^{-1}\right|}{\prod_{j=1}^{n}(1-\pi_j)\prod_{k=1}^{n}\sum_{i=1}^{n}\left[A(E-A)^{-1}\right]_{ik}\frac{1}{1-\pi_i}}. \tag{88}$$

Further, we consider the case when the firms produce more than one type of goods and are described by technological mappings from the CTM class. So, let the economic system consist of m firms described by technological mappings $x_i\in X_i,\ y_i\in F_i(x_i)$. So, we assume that the matrix B has the representation $B=CB_1$, where

$$C=\|C_{ks}\|_{k,s=1}^{n,l},\quad B_1=\|b_{ks}^1\|_{k,s=1}^{l},\quad b_{ks}^1=\sum_{i=1}^{n}\frac{\tau_{ki}C_{is}b_s^1}{\sum_{s=1}^{l}C_{is}b_s^1},\quad k,s=\overline{1,l}. \tag{89}$$

Theorem 8. *Let the matrix* $\left\|\sum_{k=1}^{l}C_{ik}\tau_{kj}\right\|_{i,j=1}^{n}$ *be a non negative and indecomposable one. Suppose that the matrix* $\tau=\|\tau_{kj}\|_{k=1,j=1}^{l,n}$ *is such that*

$$\sum_{i=1}^{n}\tau_{ki}=b_k^1>0,\quad k=\overline{1,l}. \tag{90}$$

Then the problem

$$\sum_{k=1}^{l}b_{ks}^1d_k=b_s^1d_s,\quad s=\overline{1,l}, \tag{91}$$

has a strictly positive solution belonging to the cone created by the column of matrix C^{T}, *under the conditions that* $\sum_{k=1}^{n}C_{ki}>0,\ i=\overline{1,l}$, *where* C^{T} *is a transposed matrix to the matrix C.*

Proof. Let us consider the non linear map

$$H(p)=\{H_i(p)\}_{i=1}^{n}, \tag{92}$$

$$H_i(p)=\frac{p_i+\dfrac{\sum_{j=1}^{n}p_j\sum_{k=1}^{l}C_{jk}\tau_{ki}}{\sum_{s=1}^{l}C_{is}b_s^1}}{1+\sum_{i=1}^{n}\dfrac{\sum_{j=1}^{n}p_j\sum_{k=1}^{l}C_{jk}\tau_{ki}}{\sum_{s=1}^{l}C_{is}b_s^1}},\quad i=\overline{1,n}, \tag{93}$$

on the set $P=\left\{p=\{p_i\}_{i=1}^{n},\ p_i\ge 0,i=\overline{1,n},\ \sum_{i=1}^{n}p_i=1\right\}$. It is continuous on P and maps it into itself. Therefore, there exists a fixed point p_0 of the map $H(p)$. The last leads to the set of equations

$$\frac{\sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk}\tau_{ki}}{\sum_{s=1}^{l} C_{is} b_s^1} = \lambda p_i^0, \quad i = \overline{1,n}, \tag{94}$$

where

$$\lambda = \sum_{i=1}^{n} \frac{\sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk}\tau_{ki}}{\sum_{s=1}^{l} C_{is} b_s^1}.$$

Let us prove that $\lambda > 0$ and p_0 is a strictly positive vector. Let us introduce the matrix

$$F = \left\| f_{ji} \right\|_{j,i=1}^{n}, \tag{95}$$

where

$$f_{ji} = \frac{\sum_{k=1}^{l} C_{jk}\tau_{ki}}{\sum_{s=1}^{l} C_{is} b_s^1}, \quad i,j = \overline{1,n}.$$

The set of Equation (94) can be written in the operator form

$$F^{\mathrm{T}} p_0 = \lambda p_0.$$

Since the matrix F^{T} is a nonnegative and indecomposable one and the fact that the vector p_0 satisfies also the set of equations

$$\left[F^{\mathrm{T}}\right]^{n-1} p_0 = \lambda^{n-1} p_0,$$

we obtain that $\left[F^{\mathrm{T}}\right]^{n-1} p_0$ is strictly positive vector. From here it follows that p_0 is a strictly positive vector and $\lambda > 0$. Here, we denoted by F^{T} the transposed matrix to the matrix *F*. Let us prove that $\lambda = 1$. Let us denote

$$d_k = \frac{1}{\lambda} \sum_{u=1}^{n} p_u^0 C_{uk}, \quad k = \overline{1,l}. \tag{96}$$

Then, from (94) we obtain

$$p_i^0 = \frac{\sum_{k=1}^{l} d_k \tau_{ki}}{\sum_{s=1}^{l} C_{is} b_s^1}. \tag{97}$$

Substituting (97) into (96), we obtain

$$d_j = \frac{1}{\lambda} \sum_{u=1}^{n} \frac{\sum_{k=1}^{l} d_k \tau_{ku} C_{uj}}{\sum_{s=1}^{l} C_{us} b_s^1}, \quad k = \overline{1,l}. \tag{98}$$

Let us introduce the denotation $d_j^1 = d_j b_j^1$, then

$$d_j^1 = \frac{1}{\lambda}\sum_{k=1}^{l}\sum_{u=1}^{n}\frac{\frac{d_k^1}{b_k^1}\tau_{ku}C_{uj}b_j^1}{\sum_{s=1}^{l}C_{us}b_s^1}, \quad j=\overline{1,l}. \tag{99}$$

Summing up over index j the left and right hand sides of (99), we obtain

$$\sum_{j=1}^{l} d_j^1 = \frac{1}{\lambda}\sum_{j=1}^{l} d_j^1. \tag{100}$$

Since $\sum_{j=1}^{l} d_j^1 \neq 0$, we have $\lambda = 1$. Therefore, the Equalities (98) is written in the form

$$d_i = \sum_{u=1}^{n}\frac{\sum_{k=1}^{l} d_k\tau_{ku}C_{ui}}{\sum_{s=1}^{l}C_{us}b_s^1}, \quad i=\overline{1,l}. \tag{101}$$

Or, multiplying the left and right sides of Equalities (101) on b_i^1, we obtain the equalities

$$\sum_{k=1}^{l} b_{ki}^1 d_k = b_i^1 d_i, \quad i=\overline{1,l}. \tag{102}$$

Theorem 8 is proved.

The direct consequence of Theorem 8 is the following:

Theorem 9. *Suppose that the conditions of Theorem* 8 *are valid, then the strictly positive solution* p_0, *constructed in Theorem* 8, *is a solution to the problem*

$$\sum_{i=1}^{l} C_{ki}\frac{\langle b_i, p_0\rangle}{\langle C_i, p_0\rangle} = \sum_{i=1}^{l} b_{ki}, \quad k=\overline{1,n}. \tag{103}$$

Proof. If to substitute

$$d_k = \sum_{u=1}^{n} p_u^0 C_{uk}, \quad k=\overline{1,l}, \tag{104}$$

into (102), we obtain

$$\sum_{m=1}^{n} p_m^0 C_{mi} b_i^1 = \sum_{k=1}^{l} b_{ki}^1 \sum_{j=1}^{n} p_j^0 C_{jk} = \sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk} b_{ki}^1 = \sum_{j=1}^{n} p_j^0 b_{ji}, \quad i=\overline{1,l}. \tag{105}$$

From the last equalities, we have

$$\frac{\sum_{j=1}^{n} p_j^0 b_{ji}}{\sum_{m=1}^{n} p_m^0 C_{mi}} = b_i^1, \quad i=\overline{1,l}. \tag{106}$$

Or,

$$\frac{\langle b_i, p_0\rangle}{\langle C_i, p_0\rangle} = b_i^1, \quad i=\overline{1,l}. \tag{107}$$

Taking into account the representation

$$\sum_{m=1}^{l} C_{im} b_{mj}^{1} = b_{ij}, \quad i = \overline{1,n}, \quad j = \overline{1,l}, \tag{108}$$

and summing up over the index *j*, we have

$$\sum_{m=1}^{l} C_{im} b_{m}^{1} = \sum_{j=1}^{l} b_{ij}. \tag{109}$$

From the last and (107), we have

$$\sum_{m=1}^{l} C_{im} \frac{\langle b_m, p_0 \rangle}{\langle C_m, p_0 \rangle} = \sum_{j=1}^{l} b_{ij}, \quad i = \overline{1,n}. \tag{110}$$

Theorem 9 is proved.

Theorem 10. *Suppose that the demand vectors* $C_i = \{C_{ki}\}_{k=1}^{n}, i = \overline{1,l}$ *belong to the interior of the nonnegative cone created by the supply vectors* $b_i = \{b_{ki}\}_{k=1}^{n}, i = \overline{1,l}$, *and the inequalities* $\sum_{j=1}^{n} C_{jk} > 0, k = \overline{1,l}$, *are true. Then, for the matrix C the representation* $C = BD$ *is valid, where the square matrix D is nonnegative one. Let us assume that the matrix D has an inverse matrix* $D^{-1} = B_1 = \| b_{ij}^{1} \|_{ij=1}^{l}$ *such that* $\sum_{j=1}^{l} b_{ij}^{1} = b_i^{1} > 0, i = \overline{1,l}$, *and for the matrix D the representation* $D = \tau C$ *is true. If the matrix* $\left\| \sum_{k=1}^{l} C_{jk} \tau_{ki} b_k^{1} \right\|_{j,i=1}^{l}$ *is nonnegative and indecomposable, then the problem*

$$\sum_{m=1}^{l} C_{im} \frac{\langle b_m, p \rangle}{\langle C_m, p \rangle} = \sum_{j=1}^{l} b_{ij}, \quad i = \overline{1,n}, \tag{111}$$

has a strictly positive solution p_0 *relative to the vector p, where*

$$\tau = \| \tau_{ki} \|_{k=1,i=1}^{l,n}, \quad C = \| C_{im} \|_{i=1,m=1}^{n,l}.$$

Proof. Let us consider the nonlinear map

$$H(p) = \{ H_i(p) \}_{i=1}^{n}, \tag{112}$$

$$H_i(p) = \frac{p_i + \sum_{j=1}^{n} p_j \left(\sum_{k=1}^{l} C_{jk} \tau_{ki} b_k^{1} \right)}{1 + \sum_{i=1}^{n} \sum_{j=1}^{n} p_j \left(\sum_{k=1}^{l} C_{jk} \tau_{ki} b_k^{1} \right)}, \quad i = \overline{1,n}, \tag{113}$$

on the set $P = \left\{ p = \{ p_i \}_{i=1}^{n}, p_i \ge 0, i = \overline{1,n}, \sum_{i=1}^{n} p_i = 1 \right\}$. It is continuous on the set P and maps it into itself. It means that the problem (113) has fixed point $p_0 \in P$ such that it solves the problem

$$\sum_{j=1}^{n} p_j^{0} \left(\sum_{k=1}^{l} C_{jk} \tau_{ki} b_k^{1} \right) = \lambda p_i^{0}, \quad i = \overline{1,n}, \tag{114}$$

where $\lambda = \sum_{i=1}^{n} \sum_{j=1}^{n} p_j^{0} \left(\sum_{k=1}^{l} C_{jk} \tau_{ki} b_k^{1} \right)$. As in the previous Theorem 8, it is proved that $\lambda > 0$ and the vector p_0 is a strictly positive one due to the Theorem 10 conditions. Let us introduce the vector $d=\{d_k\}_{k=1}^{l}$, where $d_k=\sum_{j=1}^{n}C_{jk}p_j^0, k=\overline{1,l}$. Then, from the Equalities (114) we obtain

$$p_i^0=\frac{1}{\lambda}\sum_{k=1}^{l}d_k\tau_{ki}b_k^1. \tag{115}$$

Or, multiplying on C_{im} the left and right hand sides of the Equalities (115) and summing up relative to the index *i* from 1 to *n*, we obtain

$$\begin{aligned} d_m&=\sum_{i=1}^{n}C_{im}p_i^0=\frac{1}{\lambda}\sum_{i=1}^{n}C_{im}\sum_{k=1}^{l}d_k\tau_{ki}b_k^1\\ &=\frac{1}{\lambda}\sum_{k=1}^{l}d_kb_k^1\sum_{i=1}^{n}\tau_{ki}C_{im}=\frac{1}{\lambda}\sum_{k=1}^{l}d_kb_k^1D_{km},\quad m=\overline{1,l}, \end{aligned} \tag{116}$$

$$\sum_{m=1}^{l}d_mD_{mk}^{-1}=\sum_{m=1}^{l}d_mB_{mk}^1=\frac{1}{\lambda}b_k^1d_k,\quad k=\overline{1,l}. \tag{117}$$

So, we obtain that there exists a strictly positive solution to the problem

$$\sum_{m=1}^{l}d_mB_{mk}^1=\frac{1}{\lambda}d_kb_k^1,\quad k=\overline{1,l}. \tag{118}$$

Summing up the left and right hand sides of Equalities (118) over the index k we have

$$\sum_{m=1}^{l}d_mb_m^1=\frac{1}{\lambda}\sum_{k=1}^{l}d_kb_k^1. \tag{119}$$

So, $\lambda=1$. From here we obtain that the problem

$$\sum_{m=1}^{l}d_mB_{mk}^1=d_kb_k^1,\quad k=\overline{1,l}, \tag{120}$$

has strictly positive solution belonging to the cone created by the columns of the matrix C^{T}, due to the fact that the representations for $d_m=\sum_{i=1}^{n}C_{im}p_i^0, m=\overline{1,l}$, is true. This proves Theorem 10, since the rest statement is proved as above in previous Theorems. □

Now, we consider the case as the matrices $C(p),B(p),B_1(p)$ depend on the price vector $p\in P$, where $P=\left\{p=\{p_k\}_{k=1}^{n}, p_k\ge 0, \sum_{k=1}^{n}p_k=1\right\}$. Suppose that for the matrix $B(p)$ the representation $B(p)=C(p)B_1(p)$ is true, where

$$C(p)=\left\|C_{ks}(p)\right\|_{k,s=1}^{n,l},\quad B_1(p)=\left\|b_{ks}^1(p)\right\|_{k,s=1}^{l},$$

$$b_{ks}^1(p)=\sum_{i=1}^{n}\frac{\tau_{ki}(p)C_{is}(p)b_s^1(p)}{\sum_{s=1}^{l}C_{is}(p)b_s^1(p)},\quad k,s=\overline{1,l}. \tag{121}$$

Theorem 11. *Let the matrix* $\left\|\sum_{k=1}^{l}C_{ik}(p)\tau_{kj}(p)\right\|_{i,j=1}^{n}$ *be a non negative and indecomposable one for every* $p\in P$. *Suppose that the matrix* $\tau(p)=\left\|\tau_{kj}(p)\right\|_{k=1,j=1}^{l,n}$ *is such that*

$$\sum_{i=1}^{n} \tau_{ki}(p) = b_k^1(p) > 0, \quad p \in P, \quad k = \overline{1,l}. \tag{122}$$

Then, the problem

$$\sum_{i=1}^{l} C_{ki}(p) \frac{\langle b_i(p), p \rangle}{\langle C_i(p), p \rangle} = \sum_{i=1}^{l} b_{ki}(p), \quad k = \overline{1,n}, \tag{123}$$

has a strictly positive solution p_0 *under the conditions that* $\sum_{k=1}^{n} C_{ki}(p) > 0,\ p \in P, i = \overline{1,l}$.

Proof. Let us introduce the non linear map

$$H(p) = \{H_i(p)\}_{i=1}^{n}, \tag{124}$$

$$H_i(p) = \frac{p_i + \dfrac{\sum_{j=1}^{n} p_j \sum_{k=1}^{l} C_{jk}(p) \tau_{ki}(p)}{\sum_{s=1}^{l} C_{is}(p) b_s^1(p)}}{1 + \sum_{i=1}^{n} \dfrac{\sum_{j=1}^{n} p_j \sum_{k=1}^{l} C_{jk}(p) \tau_{ki}(p)}{\sum_{s=1}^{l} C_{is}(p) b_s^1(p)}}, \quad i = \overline{1,n}, \tag{125}$$

on the set $P = \left\{ p = \{p_i\}_{i=1}^{n}, p_i \geq 0, i = \overline{1,n}, \sum_{i=1}^{n} p_i = 1 \right\}$. It is continuous on P and maps it into itself. Therefore, there exists a fixed point p_0 of the map $H(p)$. The last leads to the set of equations

$$\frac{\sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk}(p_0) \tau_{ki}(p_0)}{\sum_{s=1}^{l} C_{is}(p_0) b_s^1(p_0)} = \lambda p_i^0, \quad i = \overline{1,n}, \tag{126}$$

where

$$\lambda = \sum_{i=1}^{n} \frac{\sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk}(p_0) \tau_{ki}(p_0)}{\sum_{s=1}^{l} C_{is}(p_0) b_s^1(p_0)}.$$

Let us prove that $\lambda > 0$ and p_0 is a strictly positive vector. Let us consider the matrix

$$F = \left\| f_{ji}(p_0) \right\|_{j,i=1}^{n}, \tag{127}$$

where

$$f_{ji}(p_0) = \frac{\sum_{k=1}^{l} C_{jk}(p_0) \tau_{ki}(p_0)}{\sum_{s=1}^{l} C_{is}(p_0) b_s^1(p_0)}, \quad i, j = \overline{1,n}.$$

The set of Equation (126) can be written in operator form

$$F^{\mathrm{T}} p_0 = \lambda p_0.$$

Since the matrix F^{T} is a nonnegative and indecomposable one and the fact that the vector p_0 satisfies also the set of equations

$$\left[F^{\mathrm{T}}\right]^{n-1} p_0 = \lambda^{n-1} p_0,$$

we obtain that $\left[F^{\mathrm{T}}\right]^{n-1} p_0$ is strictly positive vector. From here it follows that p_0 is strictly positive vector and $\lambda > 0$. Here, we denoted by F^{T} the transposed matrix to the matrix *F*. Let us prove that $\lambda = 1$.

Let us denote

$$d_k = \frac{1}{\lambda} \sum_{u=1}^{n} p_u^0 C_{uk}\left(p_0\right), \quad k = \overline{1,l}. \tag{128}$$

Then from (126) we obtain

$$p_i^0 = \frac{\sum_{k=1}^{l} d_k \tau_{ki}\left(p_0\right)}{\sum_{s=1}^{l} C_{is}\left(p_0\right) b_s^1\left(p_0\right)}. \tag{129}$$

Substituting (129) into (128), we obtain

$$d_j = \frac{1}{\lambda} \sum_{u=1}^{n} \frac{\sum_{k=1}^{l} d_k \tau_{ku}\left(p_0\right) C_{uj}\left(p_0\right)}{\sum_{s=1}^{l} C_{us}\left(p_0\right) b_s^1\left(p_0\right)}, \quad k = \overline{1,l}. \tag{130}$$

Let us introduce the denotation $d_j^1 = d_j b_j^1$, then

$$d_j^1 = \frac{1}{\lambda} \sum_{k=1}^{l} \sum_{u=1}^{n} \frac{\frac{d_k^1}{b_k^1\left(p_0\right)} \tau_{ku}\left(p_0\right) C_{uj}\left(p_0\right) b_j^1\left(p_0\right)}{\sum_{s=1}^{l} C_{us}\left(p_0\right) b_s^1\left(p_0\right)}, \quad k = \overline{1,l}. \tag{131}$$

Summing up over index *j* the left and right hand sides of (131), we obtain

$$\sum_{j=1}^{l} d_j^1 = \frac{1}{\lambda} \sum_{j=1}^{l} d_j^1. \tag{132}$$

Since $\sum_{j=1}^{l} d_j^1 \neq 0$ we have $\lambda = 1$. Therefore, the Equalities (130) is written in the form

$$d_i = \sum_{u=1}^{n} \frac{\sum_{k=1}^{l} d_k \tau_{ku}\left(p_0\right) C_{ui}\left(p_0\right)}{\sum_{s=1}^{l} C_{us}\left(p_0\right) b_s^1\left(p_0\right)}, \quad i = \overline{1,l}. \tag{133}$$

Or, multiplying the left and right hand sides of Equalities (133) on $b_i^1\left(p_0\right)$ we obtain the equalities

$$\sum_{k=1}^{l} b_{ki}^1\left(p_0\right) d_k = b_i^1\left(p_0\right) d_i, \quad i = \overline{1,l}. \tag{134}$$

If to substitute

$$d_k = \sum_{u=1}^{n} p_u^0 C_{uk}(p_0), \quad k = \overline{1,l}, \tag{135}$$

into (134) we obtain

$$\begin{aligned}\sum_{m=1}^{n} p_m^0 C_{mi}(p_0) b_i^1(p_0) &= \sum_{k=1}^{l} b_{ki}^1(p_0) \sum_{j=1}^{n} p_j^0 C_{jk}(p_0) \\ &= \sum_{j=1}^{n} p_j^0 \sum_{k=1}^{l} C_{jk}(p_0) b_{ki}^1(p_0) \\ &= \sum_{j=1}^{n} p_j^0 b_{ji}(p_0), \quad i = \overline{1,l}.\end{aligned} \tag{136}$$

From the last equalities we have

$$\frac{\sum_{j=1}^{n} p_j^0 b_{ji}(p_0)}{\sum_{m=1}^{n} p_m^0 C_{mi}(p_0)} = b_i^1(p_0), \quad i = \overline{1,l}. \tag{137}$$

Taking into account the representation

$$\sum_{m=1}^{l} C_{im}(p) b_{mj}^1(p_0) = b_{ij}(p_0), \quad i = \overline{1,n}, \quad j = \overline{1,l}, \tag{138}$$

and summing up over the index *j*, we obtain

$$\sum_{m=1}^{l} C_{im}(p_0) b_m^1(p_0) = \sum_{j=1}^{l} b_{ij}(p_0). \tag{139}$$

From the last and (137) we have

$$\sum_{m=1}^{l} C_{im}(p_0) \frac{\langle b_m(p_0), p_0 \rangle}{\langle C_m(p_0), p_0 \rangle} = \sum_{j=1}^{l} b_{ij}(p_0), \quad i = \overline{1,n}. \tag{140}$$

Theorem 11 is proved.

Theorem 12. *Suppose that the demand vectors* $C_i(p) = \{C_{ki}(p)\}_{k=1}^{n}, i = \overline{1,l}$ *belong to the interior of the nonnegative cone created by the supply vectors* $b_i(p) = \{b_{ki}(p)\}_{k=1}^{n}, i = \overline{1,l}$, *and the inequalities* $\sum_{j=1}^{n} C_{jk}(p) > 0, k = \overline{1,l}$, *are true on the set*

$$P = \left\{ p = \{p_i\}_{i=1}^{n}, p_i \geq 0, i = \overline{1,n}, \sum_{i=1}^{n} p_i = 1 \right\}.$$

Then, for the matrix $C(p)$ *the representation* $C(p) = B(p)D(p)$ *is valid, where the square matrix* $D(p)$ *is nonnegative one. Let us assume that the matrix* $D(p)$ *has an inverse matrix* $D^{-1}(p) = B_1(p) = \left\| b_{ij}^1(p) \right\|_{ij=1}^{l}$ *such that* $\sum_{j=1}^{l} b_{ij}^1(p) = b_i^1(p) > 0, i = \overline{1,l}$, *and for the matrix* $D(p)$ *the representation* $D(p) = \tau(p)C(p)$ *is true. If the matrix* $\left\| \sum_{k=1}^{l} C_{jk}(p) \tau_{ki}(p) b_k^1(p) \right\|_{j,i=1}^{l}$ *is nonnegative and indecomposable, then the problem*

$$\sum_{m=1}^{l} C_{im}(p)\frac{\langle b_m(p), p\rangle}{\langle C_m(p), p\rangle} = \sum_{j=1}^{l} b_{ij}(p), \quad i=\overline{1,n}, \tag{141}$$

has a strictly positive solution p_0 *relative to the vector p, where*

$$\tau(p) = \left\|\tau_{ki}(p)\right\|_{k=1,i=1}^{l,n}, \quad C(p) = \left\|C_{im}(p)\right\|_{i=1,m=1}^{n,l}.$$

Proof. Let us consider the nonlinear map

$$H(p) = \left\{H_i(p)\right\}_{i=1}^{n}, \tag{142}$$

$$H_i(p) = \frac{p_i + \sum_{j=1}^{n} p_j\left(\sum_{k=1}^{l} C_{jk}(p)\tau_{ki}(p)b_k^1(p)\right)}{1 + \sum_{i=1}^{n}\sum_{j=1}^{n} p_j\left(\sum_{k=1}^{l} C_{jk}(p)\tau_{ki}(p)b_k^1(p)\right)}, \quad i=\overline{1,n}, \tag{143}$$

on the set $P = \left\{p = \{p_i\}_{i=1}^{n}, p_i \ge 0, i=\overline{1,n}, \sum_{i=1}^{n} p_i = 1\right\}$. It is continuous on the set P and maps it into itself. It means that the problem (143) has fixed point $p_0 \in P$ such that it solves the problem

$$\sum_{j=1}^{n} p_j^0\left(\sum_{k=1}^{l} C_{jk}(p_0)\tau_{ki}(p_0)b_k^1(p_0)\right) = \lambda p_i^0, \quad i=\overline{1,n}, \tag{144}$$

where $\lambda = \sum_{i=1}^{n}\sum_{j=1}^{n} p_j^0\left(\sum_{k=1}^{l} C_{jk}(p_0)\tau_{ki}(p_0)b_k^1(p_0)\right)$. As in the Theorem 8, it is proved that $\lambda > 0$ and the vector p_0 is a strictly positive one due to the Theorem 10 conditions. Let us introduce the vector $d = \{d_k\}_{k=1}^{l}$, where $d_k = \sum_{j=1}^{n} C_{jk}(p_0)p_j^0, k=\overline{1,l}$. Then, from the Equalities (144) we obtain

$$p_i^0 = \frac{1}{\lambda}\sum_{k=1}^{l} d_k\tau_{ki}(p_0)b_k^1(p_0). \tag{145}$$

Or, multiplying on $C_{im}(p_0)$ the left and right hand sides of the equalities (145) and summing up relative to the index *i* from 1 to *n*, we obtain

$$\begin{aligned} d_m &= \sum_{i=1}^{n} C_{im}(p_0)p_i^0 = \frac{1}{\lambda}\sum_{i=1}^{n} C_{im}(p_0)\sum_{k=1}^{l} d_k\tau_{ki}(p_0)b_k^1(p_0) \\ &= \frac{1}{\lambda}\sum_{k=1}^{l} d_k b_k^1(p_0)\sum_{i=1}^{n}\tau_{ki}(p_0)C_{im}(p_0) \\ &= \frac{1}{\lambda}\sum_{k=1}^{l} d_k b_k^1(p_0)D_{km}(p_0), \quad m=\overline{1,l}, \end{aligned} \tag{146}$$

$$\sum_{m=1}^{l} d_m D_{mk}^{-1}(p_0) = \sum_{m=1}^{l} d_m B_{mk}^1(p_0) = \frac{1}{\lambda}b_k^1(p_0)d_k, \quad k=\overline{1,l}. \tag{147}$$

So, we obtain that there exists a strictly positive solution to the problem

$$\sum_{m=1}^{l} d_m B_{mk}^1(p_0) = \frac{1}{\lambda}d_k b_k^1(p_0), \quad k=\overline{1,l}. \tag{148}$$

Summing up the left and right hand sides of Equalities (148) over the index *k* we have

$$\sum_{m=1}^{l} d_m b_m^1(p_0) = \frac{1}{\lambda}\sum_{k=1}^{l} d_k b_k^1(p_0). \tag{149}$$

So, $\lambda = 1$. From here we obtain that the problem

$$\sum_{m=1}^{l} d_m B_{mk}^1(p_0) = d_k b_k^1(p_0), \quad k = \overline{1,l}, \tag{150}$$

has strictly positive solution belonging to the cone created by the columns of the matrix $C^{\mathrm{T}}(p_0)$, due to the fact that the representations for $d_m = \sum_{i=1}^{n} C_{im}(p_0) p_i^0, m = \overline{1,l}$, is true. This proves Theorem 12, since the rest statement is proved as above in previous Theorems. □

Theorem 13. *In the economy system, let m firms function, technologies of which is described by technological mappings* $F_i(x_i), x_i \in X_i$ *from CTM class. Suppose that the firms realized the productive processes* $(x_i, y_i), x_i \in X_i, y_i \in F_i(x_i)$ *and* $\pi = \{\pi_i\}_{i=1}^{m}, 0 < \pi_i < 1, i = \overline{1,m}$, *is a taxation vector. Suppose that the matrices* $C = \|C_{ki}\|_{k,i=1}^{n,m}$, $B = \|b_{ki}\|_{k,i=1}^{n,m}$ *satisfy either the conditions of Theorem* 8 *or the conditions of the Theorem* 10, *that is, for the matrix B the representation* $B = CB_1$ *is true and the matrix* $B_1 = \|b_{ki}\|_{k=1,i=1}^{n,m}$ *satisfies either the conditions of Theorem* 8 *or the conditions of Theorem* 10, *where* $C_{ki} = x_{ki}$, $x_i = \{x_{ki}\}_{k=1}^{n}$, $b_{ki} = (1-\pi_k) y_{ki}$, $y_i = \{y_{ki}\}_{k=1}^{n}$. *Then there exists a strictly positive price vector* p_0 solving the set of equations (58). *If* $b_i^1 > (1-\pi_i)$, $b_i^1 = \sum_{k=1}^{m} b_{ik}^1, i = \overline{1,m}$, *then every firm is profitable, that is,* $\langle y_i - x_i, p_0 \rangle > 0, i = \overline{1,m}$.

Theorem 14. *In the economy system, let m firms function, technologies of which are described by technological mappings* $F_i(x_i), x_i \in X_i$ *from CTM class* (see [11]). *Suppose that the firms realized the continuous strategies of firms behaviour* $(x_i(p), y_i(p)), x_i(p) \in X_i, y_i(p) \in F_i(x_i(p))$ *and* $\pi = \{\pi_i\}_{i=1}^{m}, 0 < \pi_i < 1, i = \overline{1,m}$, *is a taxation vector. Suppose that the matrices* $C(p) = \|C_{ki}(p)\|_{k,i=1}^{n,m}$, $B(p) = \|b_{ki}(p)\|_{k,i=1}^{n,m}$ *satisfy either the conditions of Theorems* 11, *or the conditions of Theorem* 12, *that is, for the matrix* $B(p)$ *the representation* $B(p) = C(p)B_1(p)$ *is true and either the matrix* $B_1(p) = \|b_{ki}(p)\|_{k=1,i=1}^{n,m}$ *satisfies the conditions of Theorem* 11, *or the conditions of Theorem* 12, *where* $C_{ki}(p) = x_{ki}(p)$, $x_i(p) = \{x_{ki}(p)\}_{k=1}^{n}$, $b_{ki}(p) = (1-\pi_i) y_{ki}(p)$, $y_i(p) = \{y_{ki}(p)\}_{k=1}^{n}$. *Then, there exists a strictly positive price vector* p_0 solving the set of equations (61). *If* $b_i^1(p) > (1-\pi_i)$, $b_i^1(p) = \sum_{k=1}^{m} b_{ik}^1(p), i = \overline{1,m}, p \in P$, *then every firm is profitable, that is,* $\langle y_i(p_0) - x_i(p_0), p_0 \rangle > 0, i = \overline{1,m}$, where the vector $(1-\pi) = \{1-\pi_i\}_{i=1}^{m}$.

5. Conclusion

Section 1 lists the main results. Section 2 contains results that make it possible to build equilibrium states of the economic system, which can be useful in the process of planning the output of firms based on information about the structure of demand. Section 3 is devoted to the study of the phenomenon of recession. Proved Theorems in which it is established that the phenomenon of recession occurs when firms produce goods that do not find a buyer. The concepts of the vector of real consumption and the generalized equilibrium vector of prices are introduced. A corresponding parameter was introduced to describe the depth of the recession. It varies from zero to unity. The greater it is, the greater the depth of the recession. Section 4 is devoted to the conditions under which sustainable development of the economy is possible. For the input-output production model, the necessary and sufficient conditions under which each industry is profitable have been established. The latter occurs when some vector built on the basis of the vector of gross output and the vector of taxation falls into the cone formed by the columns of the matrix, which is the product of the matrix of direct and total costs. The rest of the results of Section 4 are devoted to finding out the conditions for the production processes of firms, which are described by technological mappings from the CTM class, there is an equilibrium price vector that ensures complete clearing of markets and each firm is profitable.

Funding

This work was partially supported by the Program of Fundamental Research of the Department of Physics and Astronomy of the National Academy of Sciences of Ukraine Construction and research of financial market models using the methods of non-equilibrium statistical physics and the physics of nonlinear phenomena N0123U100362.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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