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A Method for the Squaring of a Circle

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Abstract

This paper presents a Method for the squaring of a circle (*i.e.*, constructing a square having an area equal to that of a given circle). The construction, when applied to a given circle having an area of 12.7 cm^2, it produced a square having an area of 12.7 cm^2, **using only an unmarked ruler and a compass.** This result was a clear demonstration that not only is the construction valid for the squaring of a circle but also for achieving absolute results (independent of the number pi (π) and in a finite number of steps) when carried out with precision.

Keywords

Famous Problems in Mathematics, Archimedes, College Mathematics, Cycloidal Construction, Mean Proportional Principle, Squaring the Circle, Quadrature, Geometer's Sketch Pad, College Geometry

1. Introduction

The **quadrature of a circle** (*i.e.*, the **squaring** of **a circle** or finding the square whose area is exactly equal to that of a given circle) is one of the three famous geometric problems that have intrigued mathematicians for centuries, dating back to the days of the ancient Greeks such as Hippocrates, Plato, and the creator of the number pi (π), Archimedes. The other two famous problems are **the trisection of an angle** and **the doubling of a cube** [1] [2] [3].

Despite tireless efforts on the part of these mathematicians, this problem not only has remained unsolved until now, but proofs have been offered notably by Ferdinand Von Lindermann (1882) and others, who applied algebraic methods to geometry to show that the problem cannot be solved **using a straightedge (or unmarked ruler) and a compass alone.** The basis for that conclusion is the fact that the number pi (π), a function of the area of a circle, is not an algebraic number, nor is it constructible by a straightedge and a compass.

Despite the formidability of the quadrature problem, the construction presented in this paper will demonstrate how a construction, **using only an un-**

marked straightedge and a compass, can be developed to produce, in a finite number of steps, a square whose area is **equivalent** to that of a given circle.

2. Example Problem

Given a circle having an area of 12.7 cm^2, construct a square whose area is equivalent to that of the circle**, using an unmarked straightedge and compass only.**

Procedure

Since the procedure being presented is based on the mechanics of the rolling wheel, it is, therefore, instructive to proceed with the following steps:

1) Develop a typical cycloid construction for a circle in Figure 1.

This construction illustrates the periodic locations of a point on the circumference of the given circle as it completes one revolution while rolling without slipping on a flat surface [4]. For example, if one were to imagine a bug that gets attached to the rim of a wheel at point A, which is coincident with station L0; then, as the wheel rolls on towards the right, the points 0, 1, 2, 3, 4, 5, and 6 represent the positions of the bug on the wheel at stations L0, L1, L2, L3, L4, L5, and L6 (or point B), where the bug is reunited with the surface. Therefore, the distance covered by the bug between contacts with the flat surface is equivalent to the circumference of the circle, which is denoted by the straight line AB.

It should be noted that while points 1, 2, 3, 4, 5, and 6 are necessary for the layout of a cycloidal path, in this construction, they are required here *only* for determining the distance covered, AB, between the points of contact after one revolution, Hence, there is no need for any additional tool besides the straightedge and compass for this construction.

For further details on cycloid construction, see reference [4].

2) Convert the circumference of the circle to a rectangle. See Figure 1.

Knowing that the circumference is $2\pi r$ and the area of a circle is πr^2, then we can think of this area as being represented by the rectangle ABCD where the side AB is known to be 2πr, and the other side BC is unknown but can be determined since this distance is related to the circumference of the circle.

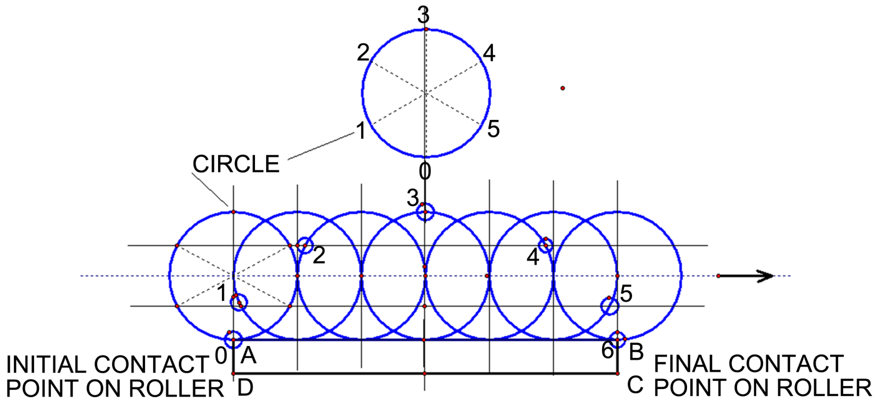

Figure 1. Typical cycloid construction (plotted points only) showing one revolution of a point on rolling circle.

3) Determine BC

BC is found by equating the area of the rectangle to the area of the circle.

$$\mathrm{AB} \times \mathrm{BC} = \pi r^2 \qquad \text{Equation} \qquad (1)$$

which yields

$$\mathrm{BC} = \frac{\pi r^2}{2\pi r}$$

BC = $r/2$ … half radius.

Therefore, from Equation (1), the rectangle ABCD can be written as

$$\mathrm{AB} \times r/2 = \text{Area of given circle} \qquad \text{Equation} \qquad (2)$$

$$(2\pi r) \times r/2 = \text{Area of given circle} \qquad \text{Equation} \qquad (3)$$

4) Determine the equivalent rectangular area to that of the circle

Applying Equation (3) to the problem at hand, given that the area of the circle is equal to 12.7 cm^2, this area can be represented as a rectangle ABCD, **Figure 2**, where side AB = 12.7 cm (See **Figure A1** for construction), and the other side BC = 1 cm.

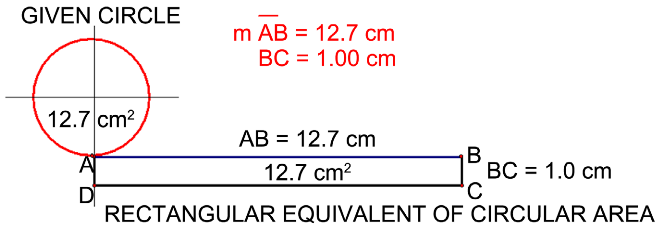

Figure 2. Rectangular equivalent of circular area.

5) Convert the rectangle ABCD to a square, using the Mean Proportional Principle [5] [6]. See Figure 3.

a) Referring to rectangle ABCD, with center at C and radius CB, describe an arc cutting DC (ext'd) at K.

b) With KD as the base, construct a semicircle cutting BC (extended) at point F.

This point will define on line BC (extended) at F, there defining the segment CF as one side of the required square.

c) Complete the required square C I J F.

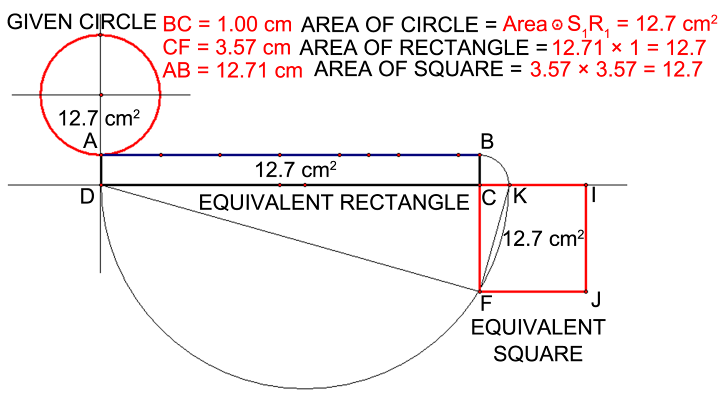

Figure 3. The Quadrature.

3. Proof

The proof of this construction is in the final measurements of the rectangle ABCD and square C I J F.

Final Construction Results

Area of Circle = 12.7 cm^2 (Given);

Area of Rectangle = 12.7 cm^2;

Area of square = 12.7 cm^2.

4. To Summarize

Given the area of a circle, the quickest way to convert this area to that of a square is to do the following:

1) Form a rectangle ABCD, as in the present example, with one side AB = 12.7 cm and the other side BC = 1 cm, so that the area of the rectangle formed agrees with the given area, which is 12.7 cm^2.

2) Then, using the Mean Proportional Principle [6] [7], convert the rectangle to a square.

In the example problem presented here where the segment AB = 12.7 cm is required, this construction is done graphically as shown in **Figure A1**. As follows:

1) Construct two segments—one AM that is 12 cm long and the other MO that is 1 cm long

2) Divide segment MO into 10 equal parts (See reference [5] on "Dividing A Line Segment Into Equal Parts,")

3) Select segment MN that is 7/10 th of MO, and add it to the 12 cm segment AM.

5. Practical Benefits

Apart from the mere satisfaction of an academic interest, which the quadrature of a circle problem has generated for centuries, there are several practical benefits which the construction presented here offers.

As with all other graphical solutions, it offers an alternative to the analytical approach normally used to solve the problem and, in so doing, it provides additional insight into a problem solution that otherwise may be too theoretical or abstract.

In the field of engineering mechanics, particularly that of kinematic analysis and synthesis, many graphical solutions rely on geometric constructions. In this context, the quadrature construction could be a part of a more complex graphical procedure, without which such a procedure cannot be considered totally graphical. Also, in the field of engineering mechanics, the construction can provide a means to make useful comparisons between area-dependent relationships such as those involving stress, flow and moment of inertia.

In addition, the construction provides a means whereby the number pi (π) can be generated as a precision scale for making or checking measurements that contain pi (π) as a factor.

Finally, what is significant to note is that it is difficult to foresee at this time the various ramifications that the solution of this problem could bring. But already, one could see that the area of a circle does not need to be expressed solely as a function of pi (π). That is, once the equivalent square for the circle is found, one can quickly determine the other side of an area equivalent rectangle once one side is specified.

6. Summary

A graphical method for constructing a square that is equivalent in area to that of a given circle, **using only an unmarked straightedge and a compass**, has been presented. The construction, when applied to a circle with a given area of 12.7 **cm**^{2}, it produced a square with an area of 12.7 **cm**^{2}, which is equivalent to that of the given circle. This equivalency clearly validates the logic of the method, which likes any mathematical formula; it guarantees that the required quadrature is achievable provided the construction is carried out with precision.

To be specific, despite the formidability of this age-old challenge, the construction presented has produced a square that is equivalent in area to that of a given circle, **using an unmarked straightedge and compass alone**. Also, by circumventing the use of the number pi (π), the construction has made it possible to achieve a complete solution in a finite number of steps (actually 5 steps), unlike other attempted solutions that this author has encountered in the literature. Therefore, for these reasons, one can only conclude that, finally, the age-old challenge of squaring the circle has been met. [1].

NOTE:

The use of the Geometer's Sketch Pad [7] *was solely for the layout of arcs and lines involved in building a presentable construction and not for measurements except for determining final results. Hence, its use is not a violation of the unmarked straightedge and compass rule.*

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

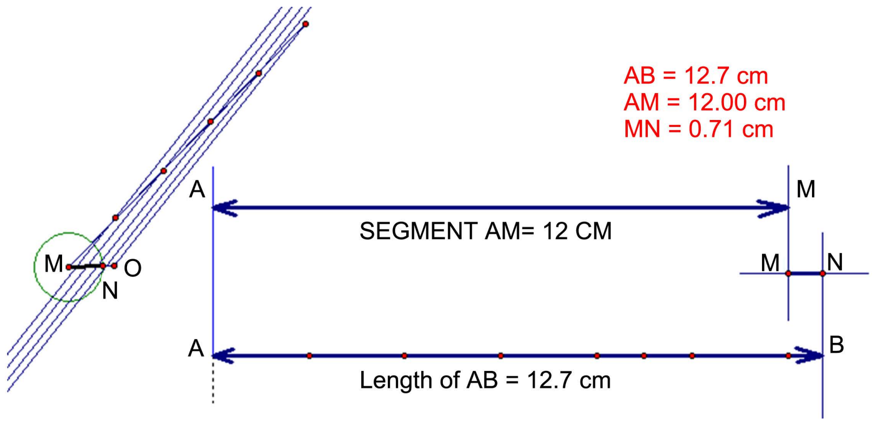

Figure A1. Construction of 12.7 cm segment.