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On *M*-Asymmetric Irresolute Multifunctions in Bitopological Spaces

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Abstract

In this paper, our focus is to introduce and investigate a class of mappings called *M*-asymmetric irresolute multifunctions defined between bitopological structural sets satisfying certain minimal properties. *M*-asymmetric irresolute multifunctions are point-to-set mappings defined using *M*-asymmetric semiopen and semiclosed sets. Some relations between *M*-asymmetric semicontinuous multifunctions and *M*-asymmetric irresolute multifunctions are established. This notion of *M*-asymmetric irresolute multifunctions is analog to that of irresolute multifunctions in the general topological space and, upper and lower *M*-asymmetric irresolute multifunctions in minimal bitopological spaces, but mathematically behaves differently.

Keywords

Asymmetric-Semiopen Sets, *m*-Space, *m*-Asymmetric Semiopen Sets, Irresolute Multifunctions, Upper (Lower) *M*-Asymmetric Irresolute Multifunctions, *M*-Asymmetric Irresolute Multifunctions

1. Introduction

The concept of a continuous function in topological spaces and multifunction continuity of a basic concept in the theory of classical point set topology have received considerable attention by several authors not only in the field of functional analysis but also in other branches of applied sciences such as mathematical economics, control theory and fuzzy topology. With regard to this, several scholars have generalized the notion of continuity in (bi-) topological spaces by use of more weaker forms of open and closed sets called semiopen and semiclosed sets: [1]-[9].

The fundamental notion of semiopen sets and continuity of functions on such sets was introduced by Levine [4] in the realm of topological spaces. These concepts have then been generalized to bitopological spaces by Maheshwari and Prasad [5], and as well by Bose [10]. Berge [11] on the other hand introduced the notion of upper and lower continuous multifunctions and lately, the concept got generalized to bitopological spaces by Popa [6], in which he investigated how multifunctions preserved the conserving properties of connectedness, compactness and paracompactness between bitopological spaces.

Noiri and Popa [12] in 2000, investigated the notions of upper (lower) continuous multifunction and M-continuous function deal to Berge [11] and, Popa and Noiri [8] respectively, and extended these notions to upper and lower M-continuous multifunctions. They observed that, upper (lower) continuous multifunctions have properties similar to those of upper (lower) continuous functions and multifunctions between topological spaces. Recently, Matindih and Moyo [9] have generalized the ideas of [12] and studied upper and lower M-asymmetric semicontinuous multifunctions from which they showed that, semicontinuous multifunctions have quasi-properties to those for upper and lower continuous functions and M-continuous multifunctions between topological spaces, with the only difference that, the semiopen sets in use are bitopologgically structured.

Irresolute mappings, quasi to continuity on the other hand, have received considerable attention by various scholars. This notion of irresolute functions and their fundamental properties were first introduced and investigated in 1972 by Crossley and Hildebrand [3]. They showed that, irresolute functions are not necessarily continuous and neither are continuous functions necessarily irresolute. As a generalization to this idea, Ewert and Lipski [13], then studied the concept of upper and lower irresolute multivalued mappings, followed by Popa [14] who looked at some characterizations of upper and lower irresolute multifunctions. Popa in [1] further extended his investigation to studying the structural properties and relationship of irresolute multifunctions to continuous functions and multifunctions in topological spaces. As a generalization to Popa's [14] idea, Matindih *et al.* [2] have recently studied upper and lower M-asymmetric irresolute multifunctions in bitopologgical spaces with sets satisfying certain minimal structures. They clearly showed that, upper and lower M-asymmetric irresolute multifunctions have properties similar to those of upper and lower irresolute multifunctions [1] defined between topological spaces. Further, with the aid of counter examples, it was shown that, upper and lower M-asymmetric irresolute multifunctions are respectively upper and lower M-asymmetric semicontinuous; however, the converse was not in general true.

In this present paper, we introduce and investigate the notion of M-asymmetric irresolute multifunctions as an extension from Matindih *et al.*, [2] and, a generalization of results by Popa [1].

Our paper is organized as follows: Section 2 presents necessary concepts concerning semiopen sets, m-asymmetric semiopen sets and upper and lower

M-asymmetric irresolute multifunctions [2]. In Section 3, we present and discuss characterizations of *M*-asymmetric irresolute multifunctions defined between bitopological spaces with sets satisfying certain minimal structure. Section 4 gives concluding remarks.

2. Preliminaries and Basic Properties

We present in this section some properties and notations to be used throughout this paper. For more details, we refer the reader to ([2] [5] [6] [7] [9] [10] [12] [14] [15]).

By a bitopological space $(X,\mathscr{T}_1,\mathscr{T}_2)$, we mean a nonempty set X on which are defined two topologies $\mathscr{T}_1$ and $\mathscr{T}_2$. The concept was first introduced by Kelly [15]. In the sequel, $(X,\mathscr{T}_1,\mathscr{T}_2)$ or in short X will denote a bitopological space unless clearly stated. For a bitopological space $(X,\mathscr{T}_i,\mathscr{T}_j)$, $i,j=1,2$; $i\neq j$, we shall denote the interior and closure of a subset A of X with respect to the topology $\mathscr{T}_i=\mathscr{T}_j$ by $Int_{\mathscr{T}_i}(A)$ and $Cl_{\mathscr{T}_i}(A)$ respectively.

Definition 2.1. [10] [15] Let $(X,\mathscr{T}_i,\mathscr{T}_j)$, $i,j=1,2$; $i\neq j$ be a bitopological space and A be any nonvoid subset of X.

1) A is said to be $\mathscr{T}_i\mathscr{T}_j$-open if $A\in\mathscr{T}_i\cup\mathscr{T}_j$; *i.e.*, $A=A_i\cup A_j$ where $A_i\in\mathscr{T}_i$ and $A_j\in\mathscr{T}_j$. The complement of an $\mathscr{T}_i\mathscr{T}_j$-open set is a $\mathscr{T}_i\mathscr{T}_j$-closed set.

2) The $\mathscr{T}_i\mathscr{T}_j$-interior of A denoted by $Int_{\mathscr{T}_i}(Int_{\mathscr{T}_j}(A))$ (or $\mathscr{T}_i\mathscr{T}_j$-$Int(A)$) is the union of all $\mathscr{T}_i\mathscr{T}_j$-open subsets of X contained in A. Evidently, if $A=Int_{\mathscr{T}_i}(Int_{\mathscr{T}_j}(A))$, then A is $\mathscr{T}_i\mathscr{T}_j$-open.

3) The $\mathscr{T}_i\mathscr{T}_j$-closure of A denoted by $Cl_{\mathscr{T}_i}(Cl_{\mathscr{T}_j}(A))$ is defined to be the intersection of all $\mathscr{T}_i\mathscr{T}_j$-closed subsets of X containing A. Observe that asymmetrically, $Cl_{\mathscr{T}_i}(Cl_{\mathscr{T}_j}(A))\subseteq Cl_{\mathscr{T}_i}(A)$ and $Cl_{\mathscr{T}_i}(Cl_{\mathscr{T}_j}(A))\subseteq Cl_{\mathscr{T}_j}(A)$.

Definition 2.2. [5] [10] Let $(X,\mathscr{T}_i,\mathscr{T}_j)$, $i,j=1,2$; $i\neq j$ be a bitopological space and, let A and B be non-void subsets of X.

1) A is said to be $\mathscr{T}_i\mathscr{T}_j$-semiopen in X provided there is a $\mathscr{T}_i$-open subset O of X such that $O\subseteq A\subseteq Cl_{\mathscr{T}_j}(O)$, equivalently $A\subseteq Cl_{\mathscr{T}_j}(Int_{\mathscr{T}_i}(A))$. It's complement is said to be $\mathscr{T}_i\mathscr{T}_j$-semiclosed.

2) The $\mathscr{T}_i\mathscr{T}_j$-semiinterior of A denoted by $\mathscr{T}_i\mathscr{T}_j$-$sInt(A)$ is defined to be the union of $\mathscr{T}_i\mathscr{T}_j$-semiopen subsets of X contained in A. The $\mathscr{T}_i\mathscr{T}_j$-semiclosure of A denoted by $\mathscr{T}_i\mathscr{T}_j$-$sCl(A)$, is the intersection of all $\mathscr{T}_i\mathscr{T}_j$-semiclosed sets of X containing A.

3) B is said to be a $\mathscr{T}_i\mathscr{T}_j$-semi-neighbourhood of a point $x\in X$ provided there is a $\mathscr{T}_i\mathscr{T}_j$-semiopen subset O of X such that $x\in O\subseteq B$.

The family of all $\mathscr{T}_i\mathscr{T}_j$-semiopen and $\mathscr{T}_i\mathscr{T}_j$-semiclosed subsets of X will be denote by $\mathscr{T}_i\mathscr{T}_j sO(X)$ and $\mathscr{T}_i\mathscr{T}_j sC(X)$ respectively.

Definition 2.3. [7] [12] A subfamily m_X of a power set $\mathscr{P}(X)$ of a nonevoid set X is said to be a minimal structure (briefly m-structure) on X if both $\varnothing$ and X lies in m_X.

The pair (X,m_X) is called an m-space and the members of (X,m_X) are said to be m_X-open sets.

Definition 2.4. [9] Let $(X, \mathscr{T}_i, \mathscr{T}_j)$, $i, j = 1,2$; $i \neq j$ be a bitopological space and m_X a minimal structure on X generated with respect to m_i and m_j. An ordered pair $((X, \mathscr{T}_i, \mathscr{T}_j), m_X)$ is called a minimal bitopological space.

As our minimal structure m_X on X is determined by the two minimal structures m_i and m_j, $i, j = 1,2$; $i \neq j$ generated by the two topologies $\mathscr{T}_i$ and $\mathscr{T}_j$ respectively, we shall denote it in the sense of Matindih and Moyo [9] by $m_{ij}(X)$ (or simply m_{ij}) and call the pair $((X, \mathscr{T}_i, \mathscr{T}_j), m_{ij})$ (or (X, m_{ij})) a minimal bitopological space unless explicitly defined.

Definition 2.5. [9] A minimal structure m_X on a bitoplogical space $(X, \mathscr{T}_i, \mathscr{T}_j)$, $i, j = 1,2$; $i \neq j$; is said to have property ($\mathscr{B}$) in the sense of Maki [7] if the union of any collection of $m_{ij}(X)$-open subsets of X belongs to m_X.

Definition 2.6. [9] Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be a minimal bitopological space and A a subset of X. A is said to be:

1) m_{ij}-semiopen in X if there exists an m_i-open set O such that $O \subseteq A \subseteq Cl_{m_j}(O)$, that is, $A \subseteq Cl_{m_j}(Int_{m_i}(A))$.

2) m_{ij}-closed in X if there exists an m_i-open set O such that $Cl_{m_j}(A) \subseteq O$ whenever $A \subseteq O$, that is, $Int_{m_j}(Cl_{m_i}(A)) \subseteq A$ or equivalently, there exists an m_i-closed set K in X such that $Int_{m_j}(K) \subseteq A \subseteq K$.

We shall denote the collection of all m_{ij}-semiopen and m_{ij}-semiclosed sets in (X, m_{ij}) by $m_{ij}sO(X)$ and $m_{ij}sC(X)$ respectively.

Remark 2.7. [9] Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be a minimal bitopological space.

1) if $m_i = \mathscr{T}_i$ and $m_j = \mathscr{T}_j$, the any m_{ij}-semiopen set is $\mathscr{T}_i\mathscr{T}_j$-semiopen.

2) every m_{ij}-open (resp. m_{ij}-closed) set is m_{ij}-semiopen (resp. m_{ij}-semiclosed), but the converse is not generally true, see Examples 3.5 [9].

The m_{ij}-open sets and the m_{ij}-semiopen sets are generally not stable for the union condition. However, for certain m_{ij}-structures, the class of m_{ij}-semiopen sets are stable under union of sets as in the Lemma below:

Lemma 2.8. [9] *Let* (X, m_{ij}), $i, j = 1,2$; $i \neq j$ *be an* m_{ij}*-space and* $\{A_\gamma : \gamma \in \Gamma\}$ *be a family of subsets of X. Then, the properties below hold:*

1) $\bigcup_{\gamma \in \Gamma} A_\gamma \in m_{ij}sO(X)$ provided for all $\gamma \in \Gamma$, $A_\gamma \in m_{ij}sO(X)$.

2) $\bigcap_{\gamma \in \Gamma} A_\gamma \in m_{ij}sC(X)$ provided for all $\gamma \in \Gamma$, $A_\gamma \in m_{ij}sC(X)$.

Remark 2.9. It should be generally noted that, the intersection of any two m_{ij}-semiopen sets may not be m_{ij}-semiopen in a minimal bitopological space (X, m_{ij}), as Example 3.9 of [9] illustrates.

Definition 2.10. [9] Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be an m_{ij}-space and let O be a subset of X. Then,

1) O is an m_{ij}-semineighborhood of a point x of X if there exists an m_{ij}-semiopen subset U of X such that $x \in U \subseteq O$.

2) O is an m_{ij}-semineighborhood of a subset A of X if there exists an m_{ij}-semiopen subset U of X such that $A \subseteq U \subseteq O$.

3) O is an m_{ij}-semineighbourhood which intersects a subset A of X if there exists a semiopen subset U of X such that $U \subseteq O$ and $U \cap A \neq \varnothing$.

Definition 2.11. [9] Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be an m_{ij}-space and A a none-void subset of X. Then, we denote and defined the m_{ij}-semiinterior and m_{ij}-semiclosure of A respectively by:

1) $m_{ij}sInt(A) = \bigcup \{U : U \subseteq A \text{ and } U \in m_{ij}sO(X)\}$,

2) $m_{ij}sCl(A) = \bigcap \{F : A \subseteq F \text{ and } F \in m_{ij}sC(X)\}$,

Remark 2.12. [9] For any bitopological spaces $(X, \mathscr{T}_1, \mathscr{T}_2)$;

1) $\mathscr{T}_i\mathscr{T}_j sO(X)$ is a minimal structure on X.

2) In the following, we denote by $m_{ij}(X)$ a minimal structure on X as a generalization of $\mathscr{T}_i$ and $\mathscr{T}_j$. For a none-void subset A of X, if $m_{ij}(X) = \mathscr{T}_i\mathscr{T}_j sO(X)$, then by Definition 2.11;

a) $m_{ij}Int(A) = \mathscr{T}_i\mathscr{T}_j sInt(A)$,

b) $m_{ij}Cl(A) = \mathscr{T}_i\mathscr{T}_j sCl(A)$.

Lemma 2.13. [9] *For any m_{ij}-space (X, m_{ij}), $i, j = 1,2$; $i \neq j$ and nonevoid subsets A and B of X, the following properties of m_{ij}-semiclosure and m_{ij}-semiinterior holds:*

1) $m_{ij}sInt(A) \subseteq A$ and $m_{ij}sCl(A) \supseteq A$.

2) $m_{ij}sInt(A) \subseteq m_{ij}sInt(B)$ and $m_{ij}sCl(A) \subseteq m_{ij}sC(B)$ provided $A \subseteq B$.

3) $m_{ij}sInt(\varnothing) = \varnothing$, $m_{ij}sInt(X) = X$, $m_{ij}sCl(\varnothing) = \varnothing$ and $m_{ij}sCl(X) = X$.

4) $A = m_{ij}Int(A)$ provided $A \in m_{ij}sO(X)$.

5) $A = m_{ij}sCl(A)$ provided $X \setminus A \in m_{ij}sO(X)$.

6) $m_{ij}sInt(m_{ij}sInt(A)) = m_{ij}sInt(A)$ and $m_{ij}sCl(m_{ij}sCl(A)) = m_{ij}sCl(A)$

Lemma 2.14. [9] *Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be an m_{ij}-space and A be a nonevoid subset of X. For each $U \in m_{ij}sO(X)$ containing x_o, $U \cap A \neq \varnothing$ if and only if $x_o \in m_{ij}sCl(A)$.*

Lemma 2.15. [9] *For an m_{ij}-space (X, m_{ij}), $i, j = 1,2$; $i \neq j$ and any none-void subset A of X, the properties below holds:*

1) $m_{ij}sCl(X \setminus A) = X \setminus (m_{ij}sInt(A))$,

2) $m_{ij}sInt(X \setminus A) = X \setminus (m_{ij}sCl(A))$.

Lemma 2.16. [9] *For an m_{ij}-space (X, m_{ij}), $i, j = 1,2$; $i \neq j$ and any none-void subset A of X, the properties below are true:*

1) $m_{ij}sCl(A) = Int_{m_j}(Cl_{m_i}(A)) \cup A$.

2) $m_{ij}sCl(A) = Int_{m_j}(Cl_{m_i}(A))$ provided $A \in m_{ij}O(X)$. The converse to this assertion is not necessarily true as was shown in Example 3.17 of [9].

Remark 2.17. [9] For a bitopological space $(X, \mathscr{T}_i, \mathscr{T}_j)$, $i, j = 1,2$; $i \neq j$ the families $\mathscr{T}_i\mathscr{T}_j O(X)$ and $m_{ij}sO(X)$ are all m_{ij}-structures of X satisfying property $\mathscr{B}$.

Lemma 2.18. [9] *For an m_{ij}-space (X, m_{ij}), $i, j = 1,2$; $i \neq j$ with m_{ij} satisfying property ($\mathscr{B}$) and subsets A and F of X, the properties below holds:*

1) $m_{ij}sInt(A) = A$ provided $A \in m_{ij}sO(X)$.

2) If $X \setminus F \in m_{ij}sO(X)$, then $m_{ij}sCl(F) = F$.

Lemma 2.19. [9] *Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be an m_{ij}-space with m_{ij} sa-*

tisfying property $\mathscr{B}$ and A be a nonevoid subset of X. Then the properties given below holds:

1) $A = m_{ij}sInt(A)$ if and only if A is an $m_{ij}(X)$-semiopen set.
2) $A = m_{ij}sCl(A)$ if and only if $X\backslash A$ is an m_{ij}-semiopen set.
3) $m_{ij}sInt(A)$ is m_{ij}-semiopen.
4) $m_{ij}sCl(A)$ is m_{ij}-semiclosed.

Lemma 2.20. [9] *Let (X, m_{ij}), $i, j = 1,2$; $i \neq j$ be an m_{ij}-space with m_{ij} satisfying property $\mathscr{B}$ and let $\{A_\gamma : \gamma \in \Gamma\}$ be an arbitrary collection of subsets of X. If $A_\gamma \in m_{ij}sO(X)$ for every $\gamma \in \Gamma$, then $\bigcup_{\gamma\in\Gamma} A_\gamma \in m_{ij}sO(X)$.*

Lemma 2.21. [9] *Let $((X, \mathscr{T}_i, \mathscr{T}_j), m_{ij})$, $i, j = 1,2$; $i \neq j$ be an m_{ij}-space and A a nonvoid subset of X. If m_{ij}-satisfy property $\mathscr{B}$, there holds;*

1) $m_{ij}sCl(A) = A \cup Int_{m_i}(Cl_{m_i}(A))$, and
2) $m_{ij}sInt(A) = A \cap Cl_{m_i}(Int_{m_i}(A))$.

If the property $\mathscr{B}$ of Make is removed in the previous Lemma, the equality does not necessarily hold, refer to Example 3.23 [9]

***Lemma* 2.22.** [9] *For a minimal bitopological space $((X, \mathscr{T}_i, \mathscr{T}_j), m_{ij}(X))$, $i, j = 1,2$; $i \neq j$, and any subset U of X, the properties below holds:*

1) $m_{ij}sInt(U) \subseteq Cl_{m_j}(Int_{m_i}(m_{ij}sInt(U))) \subseteq Cl_{m_j}(Int_{m_i}(U))$.
2) $Int_{m_j}(Cl_{m_i}(U)) \subseteq Int_{m_j}(Cl_{m_i}(m_{ij}sCl(U))) \subseteq m_{ij}sCl(U)$.

Definition 2.23. [12] A point-to-set correspondence $F: X \to Y$ from a topological space X to a topological space Y such that for every point x of X, $F(x)$ is a nonevoid subset of Y is called a multifunction.

In the sense of Berge [11], we shall denote and define the upper inverse and lower inverse of a non-void subset H of Y with respect to a multifunction F respectively by:

$$F^+(H) = \{x \in X : F(x) \subseteq H\} \text{ and } F^-(H) = \{x \in X : F(x) \cap H \neq \varnothing\}.$$

Generally for F^- and F^+ between Y and the power set 2^X,
$F^-(y) = \{x \in X : y \in F(x)\}$ provided $y \in Y$. Clearly for a nonvoid subset H of Y, $F^-(H) = \bigcup\{F^-(y) : y \in H\}$ and also,

$$F^+(H) = X \setminus F^-(Y \setminus H) \text{ and } F^-(H) = X \setminus F^+(Y \setminus H)$$

For any nonvoid subsets A and H of X and Y respectively, $F(A) = \bigcup_{x\in A} F(x)$ and $A \subseteq F^+(F(A))$ and also, $F(F^+(H)) \subseteq H$.

Definition 2.24. [13] [14] A multifunction $F:(X, \mathscr{I}) \to (Y, \mathscr{O})$, between topological spaces X and Y is said to be:

1) upper irresolute at a point x_o of X provided for any semiopen subset H of Y such that $H \supseteq F(x_o)$, there exists a semiopen subset O of X with $x_o \in O$ such that $H \supseteq F(O)$, whence, $F^+(H) \supseteq O$.

2) lower irresolute at a point x_o of X provided for any semiopen subset G of Y such that $H \cap F(x_o) \neq \varnothing$, there exists a semiopen subset O of X with $x_o \in O$ such that $H \cap F(x) \neq \varnothing$ for all $x \in O$, whence, $F^-(G) \supseteq O$.

3) upper (resp lower) irresolute provided it is upper (resp lower) irresolute at all points x_o of X.

Definition 2.25. [1] A multifunction $F:(X,\mathscr{I})\to(Y,\mathscr{O})$, between topological spaces X and Y is said to be irresolute at a point $x_o\in X$ if for any semiopen sets $H_1,H_2\subseteq Y$ such that $H_1\supseteq F(x_o)$ and $H_2\cap F(x_o)\neq\varnothing$ there exists a semiopen set $U\subseteq X$ containing x_o such that $G_1\supseteq F(U)$ and $H_2\cap F(x)\neq\varnothing$ for every $x\in U$.

A multifunction $F:X\to Y$ is irresolute if it is irresolute at every point $x_o\in X$.

Definition 2.26. [9] Let $((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X))$ and $((Y,\mathscr{T}_1,\mathscr{T}_2),m_{ij}(Y))$, $i,j=1,2$; $i\neq j$ be minimal bitopological spaces. A multifunction $F:((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X))\to((Y,\mathscr{C}_j,\mathscr{C}_i),m_{ij}(Y))$ is said to be:

1) Upper m_{ij}-semi-continuous at some point $x_o\in X$ provided for any $m_{ij}(Y)$-open set V satisfying $V\supseteq F(x_o)$, there is an $m_{ij}(X)$-semiopen set O with $x_o\in U$ for which $V\supseteq F(O)$, whence, $F^{+}(V)\supseteq O$.

2) Lower m_{ij}-semi-continuous at some point $x_o\in X$ provided for each $m_{ij}(Y)$-open set V satisfying $V\cap F(x_o)\neq\varnothing$, we can find an $m_{ij}(X)$-semiopen set O with $x_o\in O$ such that for all $x\in O$, $V\cap F(x)\neq\varnothing$.

3) Upper (resp Lower) m_{ij}-semi continuous if it is Upper (resp Lower) m_{ij}-semi continuous at each and every point of X.

Definition 2.27. [2] A multifunction $F:((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X))\to((Y,\mathscr{C}_i,\mathscr{C}_j),m_{ij}(Y))$ between minimal bitopological spaces $((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X))$ and $((Y,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(Y))$, $i,j=1,2$; $i\neq j$ said to be:

1) upper M-asymmetric irresolute at a point $x_o\in X$ provided for any $m_{ij}(Y)$-semiopen set H such that $H\supseteq F(x_o)$, there exists an $m_{ij}(X)$-semiopen set O with $x_o\in O$ such that $H\supseteq F(O)$ whence, $F^{+}(H)\supseteq O$.

2) lower M-asymmetric irresolute at a point $x_o\in X$ provided for any $m_{ij}(Y)$-semiopen set H that intersects $F(x_o)$, there exists a $m_{ij}(X)$-semiopen set O with $x_o\in O$ such that $H\cap F(x)\neq\varnothing$ for all $x\in O$ whence, $F^{-}(H)\supseteq O$.

3) upper (resp lower) M-asymmetric irresolute provided it is upper (resp lower) M-Asymmetric irresolute at each and every point x_o of X.

3. Some Characterization on *M*-Asymmetric Irresolute Multifunctions

We now study a special kind of Asymmetric-multifunction F for which the inverse image of any m_{ij}-asymmetric semiopen set under F is as well an m_{ii}-asymmetric semiopen set.

Definition 3.1. A multifunction $F:((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X))\to((Y,\mathscr{C}_i,\mathscr{C}_j),m_{ij}(Y))$ $i,j=1,2$; $i\neq j$, between bitopological spaces X and Y having certain minimal conditions is said to be M-Asymmetric irresolute at a point $x_o\in X$ if for any $m_{ij}(Y)$-semiopen sets H_1 and H_2 such that $H_1\supseteq F(x_o)$ and $H_2\cap F(x_o)\neq\varnothing$, there exists an

$m_{ij}(X)$-semiopen set O containing x_o such that $H_1 \supseteq F(O)$ and $H_2 \cap F(x) \neq \varnothing$ for every $x \in O$.

The multifunction F is M-Asymmetric irresolute if it is M-Asymmetric irresolute at every point $x_o \in X$.

Remark 3.2. Clearly, we can noted that, provided a multifunction is both upper and lower M-Asymmetric irresolute, then it is M-asymmetric irresolute and vice-versa, as we illustrate in Example 3.3 below.

Example 3.3. *Define a multifunction* $F:((X,\mathscr{T}_1,\mathscr{T}_2),m_{ij}) \to ((Y,\mathscr{Q}_1,\mathscr{Q}_2),m_{ij})$ *by:*

$$F(x)=\begin{cases}\{3\}, & \text{when } x=a\\ \{-3,-2\}, & \text{when } x=c\\ \{1,3\}, & \text{when } x=f\end{cases}$$

where $X=\{a,b,c,d,e,f\}$ with minimal structures defined by
$m_1(X)=\{\varnothing,\{a\},\{c\},\{f\},\{c,d,f\},X\}$ and
$m_2(X)=\{\varnothing,\{b\},\{d\},\{e\},\{c,e,f\},X\}$ and, $Y=\{-3,-2,-1,0,1,2,3\}$ with minimal structures given by
$m_1(Y)=\{\varnothing,\{-2\},\{2\},\{3\},\{-3,-1\},\{1,3\},\{-2,0,2,3\},Y\}$ and
$m_2(Y)=\{\varnothing,\{1\},\{-2,0\},\{1,3\},\{-2,0,2,3\},Y\}$. Clearly, F is M-asymmetric irresolute at $a \in X$ and hence, is both upper and lower M-asymmetric irresolute.

Indeed, $F(a) \subseteq \{-1,2,3\} \in m_{ij}sO(Y)$ and $F(a) \cap \{-3,1,3\} \neq \varnothing$ for the $m_{ij}(Y)$-semiopen sets $\{-1,2,3\}$ and $\{-3,1,3\}$ whence, $F(\{a\}) \subset \{-1,2,3\}$ and $F(\{a\}) \cap \{-3,1,3\} \neq \varnothing$. This also holds for some $m_{ij}(X)$-semiopen sets containing c and f respectively.

We now discuss some characterizations of M-asymmetric irresolute multifunctions and look at some of the relationship to.

Theorem 3.4. A multifunction
$F:((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X)) \to ((Y,\mathscr{Q}_i,\mathscr{Q}_j),m_{ij}(Y))$, $i,j=1,2$; $i \neq j$ for which $((Y,\mathscr{Q}_i,\mathscr{Q}_j),m_{ij}(Y))$ satisfies property $\mathscr{B}$, is said to be M-asymmetric irresolute at some point x_o of X if and only if for any $m_{ij}(Y)$-semiopen sets H_1 and H_2 such that $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$, there holds the relation:

$$x_o \in Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1) \cap F^-(H_2)\right)\right).$$

Proof. For necessity, suppose $x_o \in Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1) \cap F^-(H_2)\right)\right)$ for any $H_1,H_2 \in m_{ij}so(Y)$ with $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$. By Definition 2.10 (iii) and Lemma 2.14, there exists an $m_{ij}(X)$-semiopen neighborhood O of x_o such that $x_o \in O \subseteq F^+(H_1) \cap F^-(H_2)$. Thus, $x_o \in O \subset F^+(H_1)$ and $x_o \in O \subseteq F^-(H_2)$. Since the sets H_1 and H_2 are $m_{ij}(Y)$-semiopen, we have $F(O) \subseteq H_1 \subseteq Cl_{m_j}\left(Int_{m_i}(H_1)\right)$ and $H_2 \cap F(x) \neq \varnothing$ for all $x \in O$. Because O is $m_{ij}(X)$-semiopen, we infer F to be an M-asymmetric irresolute multifunction at a point x_o of X.

For sufficiency, suppose F is an M-asymmetric irresolute multifunction at a point x_o of X, let H_1 and H_2 be any $m_{ij}(Y)$-semiopen sets such that $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$. By Definition 3.1, there exists an $m_{ij}(X)$-semiopen set O

with $x_2 \in O$ for which $H_1 \supseteq F(O)$ and for all $x \in O$, $H_2 \cap F(x) \neq \varnothing$. And so, $Cl_{m_j}\left(Int_{m_i}(H_1)\right) \supseteq H_1 \supseteq F(O)$ and $Cl_{m_j}\left(Int_{m_i}(H_2 \cap F(x))\right) \supseteq H_2 \cap F(x) \supseteq f(x_o)$. Because F is a multifunction, $x_o \in O \subseteq F^+(H_1)$ and $x_o \in O \subseteq F^-(H_2)$ so that $x_o \in O \subseteq F^+(H_1) \cap F^-(H_1)$. Since, O is an $m_{ij}(X)$-semiopen set, $O = m_{ij}sInt(O) \subset Cl_{m_j}\left(Int_{m_i}(O)\right)$. Thus, since Y satisfies property $\mathscr{B}$, we have by applying Lemmas 2.18, 2.19 and 2.20 that,

$$x_o \in O = m_{ij}sInt(O) \subseteq Cl_{m_j}\left(Int_{m_i}(O)\right) \subseteq Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1) \cap F^-(H_1)\right)\right).$$

Theorem 3.5. Let $\left((Y, \mathscr{C}_j, \mathscr{C}_i), m_{ij}(Y)\right)$, $i, j = 1, 2$; $i \neq j$ satisfy property $\mathscr{B}$. A multifunction $F: \left((X, \mathscr{T}_i, \mathscr{T}_j), m_{ij}(X)\right) \to \left((Y, \mathscr{C}_i, \mathscr{C}_j), m_{ij}(Y)\right)$ is M-asymmetric irresolute at a point x_o of X if and only if for any $m_{ij}(X)$-semiopen set O containing x_o and any $m_{ij}(Y)$-semiopen sets H_1 and H_2 with $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$, there exists a nonempty $m_{ij}(X)$-open set $U_O \subseteq O$ such that $H_1 \supseteq F(U_O)$ and $H_2 \cap F(x) \neq \varnothing$ for all $x \in U_O$.

Proof. For necessity, let, $\{O_{x_o}\}$ be a family of all $m_{ij}(X)$-open neighbourhoods of a point x_o. Then, for any $m_{ij}(X)$-open set $O \in \{O_{x_o}\}$ and any $m_{ij}(Y)$-semiopen sets H_1 and H_2 with $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$, we can find some non-empty $m_{ij}(X)$-open set U_O contained in O for which $H_1 \supseteq F(U_O)$ and $H_2 \supseteq F(x)$ for every $x \in U_O$. Put $Q = \bigcup_{O \in O_{x_o}} U_O$, then Q is an $m_{ij}(x)$-open set, $x_o \in Cl_{m_i}\left(Cl_{m_j}(Q)\right)$ by Theorem 3.4 and $H_1 \supseteq F(Q)$ and $H_1 \cap F(q) \neq \varnothing$ for all $q \in Q$. Set $R = \{x_o\} \cup Q$, then

$$Q \subseteq R \subseteq Cl_{m_i}\left(Cl_{m_j}(Q)\right).$$

Hence, Q is an $m_{ij}(X)$-semiopen set, $x_o \in R$ and $H_1 \supseteq F(R)$ and $H_2 \cap F(r) \neq \varnothing$ for all $r \in R$, whence, $x_o \in R \subseteq F^+(H_1)$ and $x_o \in R \subseteq F^-(H_2)$ as R is an $m_{ij}(X)$-semiopen set by Definition 2.6. Therefore, F is an M-asymmetric irresolute multifunction at a point x_o of X.

For sufficiency, suppose F is M-asymmetric irresolute at a point x_o of X, let H_1 and H_2 be $m_{ij}(Y)$-semiopen sets with $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$. Then, $x_o \in F^+(G) \subseteq Cl_{m_j}\left(Int_{m_i}\left(F^+(G)\right)\right)$ by Theorem 3.4. Let O be an $m_{ij}(X)$-open neighbourhood of x_o. From Remark 2.7 (2), we infer $F(O) \subseteq H_1$ and $F(O) \subseteq H_2$, whence respectively $O \subseteq F^+(G)$ and $O \subseteq F^-(H_2)$. Hence, $O \cap Int_{m_i}\left(Int_{m_j}\left(F^+(H_1) \cap F^+(H_2)\right)\right) \neq \varnothing$. But then,

$$\begin{aligned} &Int_{m_j}\left(Int_{m_i}\left(F^+(H_1) \cap F^+(H_2)\right)\right) \\ &\subseteq Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\right)\right) \cap Int_{m_j}\left(Int_{m_i}\left(F^+(H_2)\right)\right), \end{aligned}$$

and we then have, $O \cap \left[Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\right)\right) \cap Int_{m_j}\left(Int_{m_i}\left(F^+(H_2)\right)\right)\right] \neq \varnothing$. Since

$$Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1)\cap F^-(H_2)\right)\right)\supseteq F^+(H_1)\cap F^-(H_2)$$
$$\supseteq Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\right)\right)\cap Int_{m_j}\left(Int_{m_i}\left(F^+(H_2)\right)\right)$$
$$\supseteq Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\cap F^-(H_2)\right)\right),$$

we obtain from Lemma 2.14 that $O\cap Cl_{m_j}\left(Int_{m_i}\left(F^+(H_1)\cap F^-(H_2)\right)\right)\neq\varnothing$. Set $U_O=O\cap\left[Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\right)\right)\cap Int_{m_j}\left(Int_{m_i}\left(F^+(H_2)\right)\right)\right]$. Then, $U_O\neq\varnothing$, $U_O\subseteq O$, $U_O\subseteq Int_{m_j}\left(Int_{m_i}\left(F^+(H_1)\right)\right)\subseteq F^+(H_1)$, $U_O\subseteq Int_{m_j}\left(Int_{m_i}\left(F^-(H_2)\right)\right)\subseteq F^-(H_2)$ and so, U_O is an $m_{ij}(X)$-open set. Consequently, $F(U_O)\subseteq H_1$ and $H_2\cap F(x)\neq\varnothing$ for all $x\in U_O$.

Remark 3.6. Theorem 3.5 clearly, indicates that, every M-asymmetric irresolute multifunction is generally M-asymmetric semi-continuous however, the converse is nor generally true, as we shall see in Example 3.7 and Example 3.8 respectively.

Example 3.7. Recall in Example 3.3 that, the mapping $F:(X,m_{ij})\to(Y,m_{ij})$ defined by:

$$F(x)=\begin{cases}\{3\}, & \text{when } x=a\\ \{-3,-2\}, & \text{when } x=c\\ \{1,3\}, & \text{when } x=f\end{cases}$$

is M-asymmetric irresolute. However, F is not M-asymmetric semicontinuous henceforth not M-asymmetric continuous. Indeed, $F(a)\subseteq\{-2,0,3\}\in m_{ij}sO(Y)$ and $F(a)\cap\{-3,-1,1,3\}\neq\varnothing$ for the $m_{ij}(Y)$-semiopen sets $\{-2,0,3\}$ and $\{-3,-1,1,3\}$ whence, $F(\{a\})\subset\{-2,0,3\}$ and $F(\{a\})\cap\{-3,-1,1,3\}\neq\varnothing$. However, $\{-2,0,3\}$ and $\{-3,1,3\}$ are $m_{ij}(Y)$-open sets whose inverse images are $m_{ij}(X)$-semiopen, implying that F is M-asymmetric semicontinuous.

Example 3.8. Define a multifunction $F:\left((X,\mathscr{T}_1,\mathscr{T}_2),m_{ij}\right)\to\left((Y,\mathscr{Q}_1,\mathscr{Q}_2),m_{ij}\right)$ by:

$$F(x)=\begin{cases}\{2\}, & x=a\\ \{2,3\}, & x=b\\ \{1,4,5\}, & x=c\end{cases}$$

where $X=\{a,b,c\}$ on which are defined the minimal structures $m_1(X)=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and $m_2(X)=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and also $Y=\{1,2,3,4,5\}$ on which we have $m_1(Y)=\{\varnothing,\{1\},\{2\},\{2,3,4,5\},Y\}$ and $m_2(Y)=\{\varnothing,\{2\},\{2,3\},\{2,3,4,5\},Y\}$. Then, F is M-asymmetric semicontinuous but not M-asymmetric irresolute respectively since, $\{2\}\in m_{ij}O(Y)$ but $F^+(\{2\})=\{a\}\notin m_{ij}sO(X)$ and also $\{2,3\}\in m_{ij}sO(Y)$ but, $F^-(\{2,3\})=\{a,b\}\notin m_{ij}sO(X)$.

Theorem 3.9. Let $\left((Y,\mathscr{Q}_i,\mathscr{Q}_j),m_{ij}(Y)\right)$, $i,j=1,2$; $i\neq j$ satisfy property $\mathscr{B}$ and let $F:\left((X,\mathscr{T}_i,\mathscr{T}_j),m_{ij}(X)\right)\to\left((Y,\mathscr{Q}_i,\mathscr{Q}_j),m_{ij}(Y)\right)$ be a multifunction. Then, the properties below are equivalent:

1) F is M-asymmetric irresolute,

2) The set $F^{+}(H_1)\cap F^{-}(H_2)$ is $m_{ij}(X)$-semiopen for every $m_{ij}(Y)$-semiopen sets H_1 and H_2;

3) The set $F^{-}(K_1)\cup F^{+}(K_2)$ is $m_{ij}(X)$-semiclosed for any $m_{ij}(Y)$-semiclosed sets K_1 and K_2;

4) There holds the set inclusion

$$\begin{aligned}&Int_{m_j(X)}\left(Cl_{m_i(X)}\left(F^{-}(E_1)\cup F^{+}(E_2)\right)\right)\\&\subseteq F^{-}\left(m_{ij}(Y)sCl(E_1)\right)\cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right),\end{aligned}$$

for any subsets E_1 and E_2 of Y.

5) For any given subsets V_1 and V_2 of Y, there holds the set inclusion

$$\begin{aligned}&F^{-}\left(m_{ij}(Y)sCl(V_1)\right)\cup F^{+}\left(m_{ij}(Y)sCl(V_2)\right)\\&\supseteq m_{ij}(X)sCl\left(F^{-}(V_1)\cup F^{+}(V_2)\right);\end{aligned}$$

6) The relation

$$\begin{aligned}&F^{-}\left(m_{ij}(Y)sInt(Q_1)\right)\cap F^{+}\left(m_{ij}(Y)sInt(Q_2)\right)\\&\subseteq m_{ij}(X)sInt\left(F^{-}(Q_1)\cap F^{+}(Q_2)\right).\end{aligned}$$

holds true for any given subsets Q_1 and Q_2 of Y.

Proof. 1. (1) $\Rightarrow$ (2): Suppose (1) holds, let x_o be any point of X and, let H_1 and H_2 be any $m_{ij}(Y)$-semiopen set satisfying $F(x_o)\subseteq H_1$ and $H_1\cap F(x_o)\neq\varnothing$.

By definition, $H_1\supseteq F(O)$ and $H_2\cap f(x_o)\neq\varnothing$ for some $m_{ij}(X)$-semiopen neighborhood O of x_o, containing all x. Thus, $F(x_o)\subset F(O)\subseteq H_1$ and $F(x_o)\subset F(O)\subseteq H_2$ and by hypothesis, $x_o\in O\subset F^{+}(H_1)\cap F^{-}(H_2)$. By Theorem 3.5 and 3.9 of [2], we have, $x_o\in F^{+}(H_1)\subseteq Cl_{m_i}\left(Int_{m_j}\left(F^{+}(H_1)\right)\right)$ and $x_o\in F^{-}(H_2)\subseteq Cl_{m_i}\left(Int_{m_j}\left(F^{-}(H_2)\right)\right)$ respectively. Thus, it follows from Theorem 3.4 that,

$$x_o\in Cl_{m_j}\left(Int_{m_i}\left(F^{+}(H_1)\cap F^{+}(H_1)\right)\right)$$

Since x_o is arbitrarily chosen in $F^{+}(H_1)\cap F^{-}(H_2)$, Definition 2.6 consequently implies $F^{+}(H_1)\cap F^{-}(H_2)$ is an $m_{ij}(X)$-semiopen set.

(2) $\Rightarrow$ (3): Suppose (2) holds. Let K_1 and K_2 be $m_{ij}(Y)$-semiclosed sets. Then, $Y\setminus K_1$ and $Y\setminus K_2$ are $m_{ij}(Y)$-semiopen by Lemma 2.13, and so, $F^{+}(Y\setminus K_1)=X\setminus F^{-}(K_1)$ and $F^{-}(Y\setminus K_2)=X\setminus F^{+}(K_2)$. Thus, by Lemma 2.15, we have

$$\begin{aligned}X\setminus m_{ij}(X)sCl\left(F^{-}(K_1)\right)&=m_{ij}(X)sInt\left(X\setminus F^{-}(K_1)\right)\\&=m_{ij}(X)sInt\left(F^{+}(Y\setminus K_1)\right)\\&=F^{+}(Y\setminus K_1)\\&=X\setminus F^{-}(K_1).\end{aligned}$$

and so, $m_{ij}(X)sCl\left(F^{-}(K_1)\right)=F^{-}(K_1)$. Similarly,

$$\begin{aligned} X \setminus m_{ij}(X)sCl\left(F^{+}(K_2)\right) &= m_{ij}(X)sInt\left(X \setminus F^{+}(K_2)\right) \\ &= m_{ij}(X)sInt\left(F^{-}(Y \setminus K_2)\right) \\ &= F^{-}(Y \setminus K_2) \\ &= X \setminus F^{+}(K_2), \end{aligned}$$

so that $m_{ij}(X)sCl\left(F^{+}(K_2)\right) = F^{+}(K_2)$. Thus,

$$\begin{aligned} & m_{ij}(X)sCl\left(F^{-}(K_1) \cup F^{+}(K_2)\right) \\ & \subseteq m_{ij}(X)sCl\left(F^{-}(K_1)\right) \cup m_{ij}(X)sCl\left(F^{+}(K_2)\right) \\ & = F^{-}(K_1) \cup F^{+}(K_2). \end{aligned}$$

But then, $m_{ij}(X)sCl\left(F^{-}(K_1) \cup F^{+}(K_2)\right) \supseteq F^{-}(K_1) \cup F^{+}(K_2)$. Consequently, $F^{-}(K_1) \cup F^{+}(K_2)$ is an $m_{ij}(X)$-semiclosed set.

(3) $\Rightarrow$ (4): Suppose (3) holds, let E_1 and E_2 be any arbitrary subsets of Y. Then $m_{ij}(Y)sCl(E_1)$ and $m_{ij}(Y)sCl(E_2)$ are $m_{ij}(Y)$-semiclosed sets and so, $F^{-}\left(m_{ij}(Y)sCl(E_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right)$ is an $m_{ij}(X)$-semiclosed set. Thus,

$$\begin{aligned} & Int_{m_j(X)}\left(Cl_{m_i(X)}\left(F^{-}\left(m_{ij}(Y)sCl(E_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right)\right)\right) \\ & \subseteq F^{-}\left(m_{ij}(Y)sCl(E_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right) \end{aligned}$$

But $E_1 \subseteq m_{ij}(Y)sCl(E_1)$ and $E_2 \subseteq m_{ij}(Y)sCl(E_2)$, thus,
$F^{-}(E_1) \subseteq F^{-1}\left(m_{ij}(Y)sCl(E_1)\right)$ and $F^{+}(E_2) \subseteq F^{+}\left(m_{ij}(Y)sCl(E_2)\right)$. As a result, Lemma 2.13 implies

$$\begin{aligned} & Int_{m_j(X)}\left(Cl_{m_i(X)}\left(F^{-}(E_1) \cup F^{+}(E_2)\right)\right) \\ & \subseteq Int_{m_j(X)}\left(Cl_{m_i(X)}\left(F^{-}\left(m_{ij}(Y)sCl(E_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right)\right)\right) \\ & \subseteq F^{-}\left(m_{ij}(Y)sCl(E_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(E_2)\right). \end{aligned}$$

(4) $\Rightarrow$ (5): Suppose (4) holds. Since $m_{ij}(Y)sCl(V_1)$ and $m_{ij}(Y)sCl(V_1)$ are all $m_{ij}(Y)$-semiclosed sets for any subsets V_1 and V_2 of Y and by Lemma 2.21, $m_{ij}(X)sCl(G) = G \cup Int_{m_j(X)}\left(Cl_{m_i(X)}(G)\right)$ for any $G \subseteq X$, it follows that,

$$\begin{aligned} & m_{ij}(X)sCl\left(F^{-}(V_1) \cup F^{+}(V_2)\right) \\ & = \left[F^{-}(V_1) \cup F^{+}(V_2)\right] \cup Int_{m_j(X)}\left(Cl_{m_i(X)}\left(F^{-}(V_1) \cup F^{+}(V_2)\right)\right) \\ & \subseteq \left[F^{-}(V_1) \cup F^{+}(V_2)\right] \cup F^{-}\left(m_{ij}(Y)sCl(V_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(V_2)\right) \\ & \subseteq F^{-}\left(m_{ij}(Y)sCl(V_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(V_2)\right) \end{aligned}$$

Hence,
$m_{ij}(X)sCl\left(F^{-}(V_1) \cup F^{+}(V_2)\right) \subseteq F^{-}\left(m_{ij}(Y)sCl(V_1)\right) \cup F^{+}\left(m_{ij}(Y)sCl(V_2)\right)$.

(5) $\Rightarrow$ (6): Suppose (5) is true. Since for each subsets Q_1 and Q_2 of Y, $m_{ij}(Y)sInt(Q_1)$ is $m_{ij}(Y)$-semiopen and $m_{ij}(Y)sInt(Q_1) = Y \setminus m_{ij}(Y)sCl(Y \setminus Q_1)$, we have from Lemma 2.15 that,

$$
\begin{aligned}
& X \setminus m_{ij}(X) sInt\left(F^{-}(Q_1) \cap F^{+}(Q_2)\right) \\
& = m_{ij}(X) sCl\left(X \setminus \left[F^{-}(Q_1) \cap F^{+}(Q_2)\right]\right) \\
& = m_{ij}(X) sCl\left(\left(X \setminus F^{-}(Q_1)\right) \cup \left(X \setminus F^{+}(Q_2)\right)\right) \\
& = m_{ij}(X) sCl\left(F^{+}(Y \setminus Q_1) \cup F^{-}(Y \setminus Q_2)\right) \\
& \subseteq F^{+}\left(m_{ij}(Y) sCl(Y \setminus Q_1)\right) \cup F^{-}\left(m_{ij}(Y) sCl(Y \setminus Q_2)\right) \\
& = F^{+}\left(Y \setminus m_{ij}(Y) sInt(Q_1)\right) \cup F^{-}\left(Y \setminus m_{ij}(Y) sInt(Q_2)\right) \\
& = \left[X \setminus F^{-}\left(m_{ij}(Y) sInt(Q_1)\right)\right] \cup \left[X \setminus F^{+}\left(m_{ij}(Y) sInt(Q_2)\right)\right] \\
& = X \setminus \left[F^{-}\left(m_{ij}(Y) sInt(Q_1)\right) \cap F^{+}\left(m_{ij}(Y) sInt(Q_2)\right)\right]
\end{aligned}
$$

Consequently,

$F^{-}\left(m_{ij}(Y) sInt(Q_1)\right) \cap F^{+}\left(m_{ij}(Y) sInt(Q_2)\right) \subseteq m_{ij}(X) sInt\left(F^{-}(Q_1) \cap F^{+}(Q_2)\right)$.

(6) $\Rightarrow$ (1): Suppose (6) holds, let x_o be any point of X and H_1 and H_2 be any $m_{ij}(Y)$-semopen sets such that $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$. Since Y satisfies property $\mathscr{B}$, we have from part (2) that,

$$
\begin{aligned}
F^{+}(H_1) \cap F^{-}(H_2) & \subseteq F^{+}\left(m_{ij}(Y) sInt(H_1)\right) \cap F^{-}\left(m_{ij}(Y) sInt(H_2)\right) \\
& \subseteq m_{ij}(X) sInt\left(F^{+}(H_1) \cap F^{-}(H_2)\right) \\
& \subseteq Cl_{m_j}\left(Int_{m_i}\left(F^{+}(G) \cap F^{-}(H_2)\right)\right)
\end{aligned}
$$

Thus, $F^{+}(H_1) \cap F^{-}(H_2)$ is an $m_{ij}(X)$-semiopen set containing x_o. Put $O = F^{+}(H_1) \cap F^{-}(H_2)$, then $H_1 \supseteq F(O)$ and $H_2 \cap F(x) \neq \varnothing$ for all $x \in O$. Since x_o is arbitrarily chosen, F is m_{ij}-asymmetric irresolute at x_o in X.

Theorem 3.10. Let $\left(\left(Y, \mathscr{Q}_i, \mathscr{Q}_j\right), m_{ij}(Y)\right)$, $i, j = 1, 2$; $i \neq j$ satisfy property $\mathscr{B}$ and let $F : \left(\left(X, \mathscr{T}_i, \mathscr{T}_j\right), m_{ij}(X)\right) \to \left(\left(Y, \mathscr{Q}_i, \mathscr{Q}_j\right), m_{ij}(Y)\right)$, be an M-asymmetric irresolute multifunction at an arbitrary point $x_o \in X$. Then, following properties holds:

1) For any arbitrary $m_{ij}(Y)$-semi neighbourhoods H_1 and H_2 with $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$, the set $F^{+}(H_1) \cap F^{-}(H_2)$ is an $m_{ij}(X)$-semi neighbourhood of x_o.

2) There exists an $m_{ij}(X)$-semi neighborhood O of x_o such that for each $m_{ij}(Y)$-semi neighbourhoods H_1 and H_2 with $H_1 \supseteq F(x_o)$ and $H_1 \cap F(x_o) \neq \varnothing$, $H_1 \supseteq F(O)$ and $H_2 \cap F(x_o) \neq \varnothing$ for all $x \in O$.

Proof. 1) Let x_o be any point in X and let H_1 and H_2 be an $m_{ij}(Y)$-semi neighbourhood of $F(x_o)$ with $H_1 \supseteq F(x_o)$ and $H_1 \cap F(x) \neq \varnothing$. By Definition 2.10, there exists two $m_{ij}(Y)$-semiopen sets V_1 and V_2 such that $H_1 \supseteq V_1 \supseteq F(x_o)$ and $\varnothing \neq H_2 \cap F(x_o) \supseteq V_2 \cap F(x_o)$ whence, $x_o \in F^{+}(V_1)$ and $x_o \in F^{-}(V_2)$. Thus, $x_o \in F^{+}(V_1) \cap F^{+}(V_2)$ and, since F is M-asymmetric irresolute, $F^{+}(V_1) \cap F^{+}(V_2)$ is an $m_{ij}(Y)$-semiopen set by Theorem 3.9 (2). Part (6) of

Theorem 3.9, then implies

$$\begin{aligned} x_o &\in F^+(V_1)\cap F^-(V_2) \\ &= F^+\left(m_{ij}(Y)sInt(V_1)\right)\cap F^-\left(m_{ij}(Y)sInt(V_2)\right) \\ &\subseteq m_{ij}(X)sInt\left(F^+(V_1)\cap F^-(V_2)\right) \\ &\subseteq m_{ij}(X)sInt\left(F^+(H_1)\cap F^-(H_2)\right) \\ &\subseteq F^+(H_1)\cap F^-(H_2) \end{aligned}$$

Because, $x_o \in m_{ij}(X)sInt\left(F^+(V_1)\cap F^-(V_2)\right)\subseteq F^+(H_1)\cap F^-(H_2)$, we consequently infer that, $F^+(H_1)\cap F^-(H_2)$ is a $m_{ij}(X)$-semi neighbourhood of x_o.

2) From (1), clearly holds (2). Indeed, for any point $x_o \in X$, let H_1 and H_2 be $m_{ij}(Y)$-semi neighbourhoods of $F(x_o)$ such that, $H_1 \supseteq F(x_o)$ and $H_2 \cap F(x_o) \neq \varnothing$. Set $O = F^+(H_1)\cap F^-(H_2)$. From (1), O is an $m_{ij}(X)$-semi neighbourhood of x_o and by the hypothesis, $H_1 \subseteq F(O)$ and $H_2 \cap F(x) \neq \varnothing$ for all x contained in O.

4. Conclusion

In this paper, a class of mappings called *M*-asymmetric irresolute multifunctions defined between bitopological structural sets satisfying certain minimal properties were introduced and investigated. *M*-asymmetric irresolute multifunctions were point-to-set mappings defined using M-asymmetric semiopen and semiclosed sets. Some relations between M-asymmetric semicontinuous multifunctions and *M*-asymmetric irresolute multifunctions were established.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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