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The Computing Formula of Number of Primes No More than Any Given Positive Integer

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Abstract

In this paper, we give out the formula of number of primes no more than any given n ($n \in Z^+$, $n > 2$). At the same time, we also show the principle, derivation process of the formula and application examples, it is usually marked with $\pi(n)$, which is:

$$\pi(n)=\begin{cases} 2\left[\dfrac{n-1}{6}\right]+\left[\dfrac{r+1}{6}\right]+2 \quad (2<n<25); \\ 2\left[\dfrac{n-1}{6}\right]+\left[\dfrac{r+1}{6}\right]+2 \\ +\sum\limits_{i=1}^{[\log_5 n]-1}(-1)^i \cdot \sum\limits_{\sum s_x=i}\left(\left[\dfrac{\dfrac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]\right. \\ \left.+\left[\dfrac{\dfrac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\right) \quad (n\ge 25). \end{cases}$$

that is:

1) $\pi(n)=2\left[\dfrac{n-1}{6}\right]+\left[\dfrac{r+1}{6}\right]+2$ ($2<n<25$);

2) $$\pi(n)=2\left[\dfrac{n-1}{6}\right]+\left[\dfrac{r+1}{6}\right]+2 + \sum_{i=1}^{[\log_5 n]-1}(-1)^i \cdot \sum_{\sum s_x=i}\left(\left[\dfrac{\dfrac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]\right.$$

$$+\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\Bigg)\quad (n\geq 25).$$

where "[ ]" denotes taking integer. $r=1,2,3,4,5,6$; $s_x=s_1,s_2,\cdots,s_j,s_h$; $s_1,s_2,\cdots,s_j,s_h=0,1,2,3,\cdots$. As $i\geq 2$, $2\leq s_x\leq i-1$ ($x=1,2,\cdots,j,h$).

Keywords

Positive Integer Numbers Spectrum, Row, Column, Composition, Prime

1. Introduction

In the long history of mathematics, it has always been believed that the distribution of prime numbers is irregular, which is sometimes more or less in positive integers. So is there really no law in the distribution of prime numbers [1]?

For any given positive integer n, how many primes are there less than n? Until the 18th century, it was still not known [2]. This is one of the important and interesting essential problems, mathematicians have been puzzled since the Euclid era in 300 BC. It has been explored continuously, some mathematicians have only found some approximate formulas.

In the late eighteenth century, some mathematicians examined tables of prime numbers created by using hand calculations. With these values, they looked for functions that estimated $\pi(n)$. In 1798, French mathematician Adrien-Marie Legendre used tables of primes up to 400,031, computed by Jurij Vega, to note that $\pi(n)$ could be approximated by the function:

$$\pi(n)\approx\frac{n}{\ln n-1.08366}\quad [3].$$

The great German mathematician Karl Friedrich Gauss conjectured that $\pi(n)$ increases at the same rate as the functions:

$$\pi(n)=\frac{n}{\ln n}\quad\text{and}\quad \mathrm{Li}(n)\approx\int_2^n\frac{\mathrm{d}t}{\ln t}.$$

(where $\int_2^x\frac{\mathrm{d}t}{\ln t}$ represents the area under the curve $y=1/\ln n$ and above the t-axis from $t=2$ to $t=x$. Li is an abbreviation of logarithmic integral) [3].

Despite their efforts to explore, mathematicians finally failed to find an accurate formula for the calculation of prime numbers. However, Legendre, a famous mathematician at the time, wrongly asserted rational expression of $\pi(n)$ did not exist!

The discovery of the positive integer spectrum finally solves this problem. It is a powerful tool to solute prime problems, which reflects the inner law of the distribution of prime numbers, the $\pi(n)$ formula was derived from that.

For given 100, 000, 000, 000, 000, 000, 000, someone has calculated out the number of primes less than that is 2,220,819,602,560,918,840 by using the computer. If you program the same problem according to our given $\pi(n)$ formula, it would save much time.

It is a great discovery to use continuous quantity to express discrete quantity, this determines the extraordinary theoretical significance of the formula. By using the $\pi(n)$ formula, we can verify some proven theorems and conjectures, such as prime theorem, Bertrand conjecture. We might prove some unproved conjectures, such as Brocard conjecture, Crame conjecture, Jeboff conjecture (prime interval A conjecture) and Oberman conjecture (prime interval B conjecture), which can also clarify many other problems about prime distribution that was not understood before. For example, whether there is at least one prime number between the successive triangle numbers $\frac{n(n+1)}{2}$. It will become a basic tool for anyone to study and solve the problems of the prime number and prime distribution. It may also be an important tool to solve Goldbach conjecture and twin prime conjecture.

At present, many scholars are studying the distribution law and number calculation of prime numbers [4]-[9]. Everyone uses different principles and tools, and the applicable situations of the formula may be different [10] [11]. In comparison, the $\pi(n)$ formula we given is the most convenient computing formula that only depends on n without any other conditions.

In this paper, we take the positive integer spectrum as the basic tool and deduce the formula based on the properties.

The positive integer number spectrum is the special arrangement with infinite rows and six columns by all positive integers put in order of natural numbers.

	The positive integer number spectrum					
	1st column	2nd column	3rd column	4th column	5th column	6th column
1st row	1	2	3	4	5	6
2nd row	7	8	9	10	11	12
3rd row	13	14	15	16	17	18
4th row	19	20	21	22	23	24
5th row	25	26	27	28	29	30
6th row	31	32	33	34	35	36
…	…	…	…	…	…	…
n-th row	$6(n-1)+1$	$6(n-1)+2$	$6(n-1)+3$	$6(n-1)+4$	$6(n-1)+5$	$6(n-1)+6$
…	…	…	…	…	…	…

The expression of the row, column and elements of the positive integer number spectrum.

The positive integer number expresses that any number X can be only expressed into the type of $6(n-1)+r$, ($n \in Z^+$, $n \neq 1$; $r = 1,2,3,4,5,6$), n is row ordinal number, r is a column ordinal number.

2. Main Conclusions

The whole process contains 3 parts. Part 1 is the positive integer number spectrum and its properties. Part 2 is the formula $\pi(n)$. Part 3 is the examples of the $\pi(n)$ formula application.

Part 1

In this part we introduce the positive integer number spectrum and its some properties firstly in the following.

2.1. The Positive Integer Number Spectrum and Its Some Properties

2.1.1. The Positive Integer Number Spectrum

1) Definition: The positive number spectrum is the special arrangement with infinite rows and six columns in all positive integers put in order of integer numbers.

	Integer number spectrum					
	1st column	2nd column	3rd column	4th column	5th column	6th column
1st row	1	2	3	4	5	6
2nd row	7	8	9	10	11	12
3rd row	13	14	15	16	17	18
4th row	19	20	21	22	23	24
5th row	25	26	27	28	29	30
6th row	31	32	33	34	35	36
…	…	…	…	…	…	…
n-th row	$6(n-1)+1$	$6(n-1)+2$	$6(n-1)+3$	$6(n-1)+4$	$6(n-1)+5$	$6(n-1)+6$
…	…	…	…	…	…	…

2) The expression of columns, line and elements of the positive integer number spectrum

The positive integer number spectrum indicates that any number X can be only expressed into the type of $6(n-1)+r$, ($n \in Z^+$, $n \neq 1$; $r = 1,2,3,4,5,6$), n is row ordinal number, r is column ordinal number.

A number of the r-th ($r = 1,2,3,4,5,6$) is denoted with q_r, the set of all numbers of the r-th column is denoted with Q_r.

2.1.2. Multiplication Law and Some Properties of the Positive Integer Number Spectrum

1) Multiplication law of the positive integer number spectrum

We can easily prove the positive integer number spectrum satisfying the following laws according to it's formula of general term $6(n-1)+r$.

Multiplication law:

$q_1 \cdot q_1 \in Q_1$, $q_1 \cdot q_2 \in Q_2$, $q_1 \cdot q_3 \in Q_3$, $q_1 \cdot q_4 \in Q_4$, $q_1 \cdot q_5 \in Q_5$, $q_1 \cdot q_6 \in Q_6$

$q_2 \cdot q_2 \in Q_4$, $q_2 \cdot q_3 \in Q_6$, $q_2 \cdot q_4 \in Q_2$, $q_2 \cdot q_5 \in Q_4$, $q_2 \cdot q_6 \in Q_6$

$q_3 \cdot q_3 \in Q_3$, $q_3 \cdot q_4 \in Q_6$, $q_3 \cdot q_5 \in Q_3$, $q_3 \cdot q_6 \in Q_6$

$q_4 \cdot q_4 \in Q_4$, $q_4 \cdot q_5 \in Q_2$, $q_4 \cdot q_6 \in Q_6$

$q_5 \cdot q_5 \in Q_1$, $q_5 \cdot q_6 \in Q_6$

$q_6 \cdot q_6 \in Q_6$

2) Some properties of the positive integer number spectrum

Property 1: Any composition of the first column can be written into $q_1 \cdot q_1$ or $q_5 \cdot q_5$.

Proof: Denoting the composition of the first column with H_1, then: $H_1 = q_1 \cdot q_1$ or $H_1 = q_5 \cdot q_5$.

Because the composition $6(m-1)+1$ can and only can be discomposed into $[6(n-1)+1][6(s-1)+1]$ ($n \le s < m$) or $[6(n-1)+5][6(s-1)+5]$ ($n \le s < m$); thereby it can not be discomposed into $[6(n-1)+a][6(s-1)+b]$ ($a,b = 2,3,4,6$).

Therefore, $H_1 = q_1 \cdot q_1$ or $H_1 = q_5 \cdot q_5$.

Property 2: Any composition of the fifth column can be written into p_1p_5.

Proof: Denoting the composition of the first column is H_5, then $H_5 = q_1 \cdot q_5$.

Because the composition $6(m-1)+5$ can and only can be discomposed into $[6(n-1)+1][6(s-1)+5]$ ($n \le s < m$); thereby it can not be discomposed into $[6(n-1)+a][6(s-1)+b]$ ($a,b = 2,3,4,6$).

Therefore, $H_5 = q_1 \cdot q_5$.

Part 2

In this part we introduce the $\pi(n)$ formula.

2.2. The $\pi(n)$ Formula and Its Computing Theorem and Proof

2.2.1. The Principle of Deducing $\pi(n)$ and the Computing Theorem of the $\pi(n)$ Formula

1) The principle of deducing $\pi(n)$

$\pi(n)$ is equal to the sum of the number of primes of the first column and the number of primes of the fifth column and 2.

The number of primes of the first column is denoted with m_1, the number of primes of the fifth column is denoted with m_5. m_1 is equal to the number of the first column which is less than n abstracting the number of composite number and abstracting 1. m_5 is equal to the number of the fifth column which is less than n abstract the number of composite number.

2) The computing theorem of the $\pi(n)$ formula

Number of primes no more than any given positive integer n is the sum of three parts, which are prime numbers of first column and fifth column, and the number 2 of number prime 2 and 3.

2.2.2. The $\pi(n)$ Formula and Its Conditions

1) $\pi(n) = 2\left[\frac{n-1}{6}\right] + \left[\frac{r+1}{6}\right] + 2$ ($2 < n < 25$);

2) $\pi(n)=2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2$

$$+\sum_{i=1}^{[\log_5 n]-1}(-1)^i\cdot\sum_{\sum s_x=i}\left(\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]\right.$$

$$\left.+\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\right)\ (n\geq 25).$$

where "[ ]" denotes taking integer.

$$r=1,2,3,4,5,6;\ s_x=s_1,s_2,\cdots,s_j,s_h;\ s_1,s_2,\cdots,s_j,s_h=0,1,2,3,\cdots$$

As $i\geq 2$, $2\leq s_x\leq i-1$ ($x=1,2,\cdots,j,h$).

2) The conditions of the $\pi(n)$ formula

As i is determined, k takes n and $\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R\geq 6$.

The regulations of R,R' in the following:

The base number of the power of the denominator of the fraction $\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}$ are possible to take a part or all of the following numbers $5,7,\cdots,(6k-1),(6k+1)$, setting the minimum of the part or the whole obtained numbers is p.

As p is the number of the fifth column, $R=P-6$, $R'=P-4$;

As p is the number of the first column, $R=P-6$, $R'=P-2$.

2.2.3. The Reasoning Process of the $\pi(n)$ Formula

1) As $2<n<25$, the expression of $\pi(n)$ formula is:

$$\pi(n)=2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2.$$

2) As $n\geq 25$, the expression of $\pi(n)$ formula makes up of three parts:

$$\pi(n)=2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2$$

$$+\sum_{i=1}^{[\log_5 n]-1}(-1)^i\cdot\sum_{\sum s_x=i}\left(\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]\right.$$

$$\left.+\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\right)$$

The first one is $2\left[(n-1)/6\right]+\left[(r+1)/6\right]$, we set its value to Z, the number of first column is n_1, the number of fifth column is n_5, then $Z = n_1 + n_5 - 1$.

The second one is the later expression of sum, setting the value of that is H, H is equal to the number of the compositions of first column and the fifth column.

Set $n = 6(k-1) + r$, where k is the row, r is column.

$$\begin{aligned} Z &= 2\left[(n-1)/6\right] + \left[(r+1)/6\right] \\ &= 2\left[\left(6(k-1)+r-1\right)/6\right] + \left[(r+1)/6\right] \\ &= 2\left[(k-1)+(r-1)/6\right] + \left[(r+1)/6\right] \\ &= 2(k-1) + 2\left[(r-1)/6\right] + \left[(r+1)/6\right] \end{aligned}$$

1) As $r = 1,2,3,4$

$$\left[\frac{r-1}{6}\right] = 0,\ \left[\frac{r+1}{6}\right] = 0, \text{ then } Z = 2(k-1).$$

Because the row is k, the numbers of the first column number is k in all, except1, the plus is $k - 1$, therefore the numbers of the first column number and the fifth column number which is less or equal to n except 1 is equal to $(k-1)+(k-1) = 2(k-1)$, so the value of Z is right.

2) As $r = 5,6$

$$\left[(r-1)/6\right] = 0,\ \left[(r+1)/6\right] = 1, \text{ then } Z = 2(k-1)+1 = 2k-1.$$

Because the row is k, the numbers of the first column number is k in all, except 1, the plus is $k - 1$, the numbers of the fifth column number is $k - 1$ in all, therefore the numbers of the first column number and the fifth column number which is less or equal to n except 1 is equal to $(k-1)+k = 2k-1$, so the value of Z is right.

Let us deduce the value of H in the following.

The expansion of H is:

$$H = -\left[\frac{\frac{n}{5}+1}{6}\right] - \left[\frac{\frac{n}{5}-1}{6}\right] - \left[\frac{\frac{n}{7}-1}{6}\right] - \left[\frac{\frac{n}{7}-5}{6}\right] - \cdots$$

$$+\left[\frac{\frac{n}{5\times7}+1}{6}\right] + \left[\frac{\frac{n}{5\times7}-1}{6}\right] + \left[\frac{\frac{n}{5\times11}-1}{6}\right] + \left[\frac{\frac{n}{5\times11}-1}{6}\right] + \cdots$$

$$+\left[\frac{\frac{n}{7\times11}-1}{6}\right] + \left[\frac{\frac{n}{7\times11}-5}{6}\right] + \left[\frac{\frac{n}{7\times13}-1}{6}\right] + \left[\frac{\frac{n}{7\times11}-5}{6}\right] + \cdots$$

$$+\cdots$$

$$-\left[\frac{\frac{n}{5\times5\times7}+1}{6}\right] - \left[\frac{\frac{n}{5\times5\times7}-1}{6}\right] - \left[\frac{\frac{n}{5\times5\times11}+1}{6}\right] - \left[\frac{\frac{n}{5\times5\times11}-1}{6}\right] - \cdots$$

$$-\cdots$$

$$+\left[\frac{\frac{n}{5\times5\times5\times7}+1}{6}\right]+\left[\frac{\frac{n}{5\times5\times5\times7}-1}{6}\right]+\left[\frac{\frac{n}{5\times5\times5\times11}+1}{6}\right]+\left[\frac{\frac{n}{5\times5\times5\times11}-1}{6}\right]+\cdots$$

$+\cdots$

$-\cdots$

$\cdots$

$$+(-1)^{\left(\left[\log_5 n\right]-1\right)}\left[\frac{\frac{n}{5^{s_1}\cdot 7}+1}{6}\right]+(-1)^{\left(\left[\log_5^n\right]-1\right)}\left[\frac{\frac{n}{5^{s_1}\cdot 7}-1}{6}\right]+\cdots$$

$-\cdots$

$$+(-1)^{\left(\left[\log_5 n\right]-1\right)}\left[\frac{\frac{n}{\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}+R}{6}\right]$$

$$+(-1)^{\left(\left[\log_5 n\right]-1\right)}\left[\frac{\frac{n}{\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}+R'}{6}\right]+\cdots$$

1) As $i = 1$, (set $p \in P_5$),

$\left[\frac{\frac{n}{p}-(p-6)}{6}\right]+\left[\frac{\frac{n}{p}-(p-4)}{6}\right]$ expresses the number of compositions of the first column and fifth column that is less than or equal to n and contain p and more than or equal to p^2.

Let $\left[\frac{n}{p}\right]=t$, the t numbers that are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, …, include these numbers 5, 11, 17, 23, …, the method of calculating the numbers is, every six numbers is cut into a segment from former to later with starting q, the last number or numbers from 1 to q as a last segment, then there are $\left[\frac{\left[\frac{n}{p}\right]-(p-1)+5}{6}\right]=\left[\frac{\frac{n}{p}-(p-6)}{6}\right]$ segment in all.

Therefore, the first number of every segment is the number of the fifth column, so the numbers of the composition of the fifth column which contain $q_1, q_2, q_3, \cdots, q_n$ and is less than or equal to n and is more than or equal to p^2 is:

$$\left[\frac{\frac{n}{p}-(p-6)}{6}\right].$$

We can calculate the number of compositions of the first column and the fifth column, which is less or equal to n and more than or equal to p^2 and contain the factor p is:

$$\left[\frac{\left[\frac{n}{p}\right]-(p+1)+5}{6}\right]=\left[\frac{\frac{n}{p}-(p-4)}{6}\right] \text{ in all.}$$

Because of composition:

$p\cdot t'\left(t'\in\{1,2,\cdots,t\}\right)$ factor $t'\geq p$, so the composition $p\cdot t'\geq p^2$.

Because for the factor t, We can prove the following conclusion with same method that $\left[\frac{\frac{n}{p}-(p-6)}{6}\right]+\left[\frac{\frac{n}{p}-(p-2)}{6}\right]$ expresses the number of compositions of the first column and fifth column that is less than or equal to n and contain p and more than or equal to p^2, where p is the number of the first column, and $p\neq 1$.

2) As $i\geq 2$,

$$\{p_1,p_2,\cdots,p_j,p_h\}\subseteq\{5,7,11,13,\cdots,(6k-1),(6k+1)\}$$

and:

$$p_1<p_2<\cdots<p_j<p_h.$$

Set p_1 is the number of the fifth column number, then:

$$\left[\frac{\frac{n}{p_1^{s_1}\cdot p_2^{s_2}\cdots p_j^{s_j}\cdot p_h^{s_h}}-(p_1-6)}{6}\right]+\left[\frac{\frac{n}{p_1^{s_1}\cdot p_2^{s_2}\cdots p_j^{s_j}\cdot p_h^{s_h}}-(p_1-4)}{6}\right]$$

expresses the number of compositions that is less than or equal to n and contain $p_1,p_2,\cdots,p_j,p_h$ and more than or equal to $p_1^{s_1+1}\cdot p_2^{s_2}\cdots p_j^{s_j}\cdot p_h^{s_h}$, where p_1 is the number of the first column.

Set $\left[\frac{n}{p_1^{s_1}\cdot p_2^{s_2}\cdots p_j^{s_j}\cdot p_h^{s_h}}\right]=t$, the t numbers that are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, …, include these numbers 5, 11, 17, 23, …, the method of calculating the numbers is: every six numbers is cut into a segment from former to later with starting q, the last number or numbers from 1 to q as a last segment, then there are:

$$\left[\frac{t-(p-1)+5}{6}\right]=\left[\frac{\frac{n}{p_1^{s_1}\cdot p_2^{s_2}\cdots p_j^{s_j}\cdot p_h^{s_h}}-(p_1-6)}{6}\right] \text{ segments in all.}$$

Therefore, the first number of every segment is the number of the fifth column, so the numbers of the composition of the fifth column which contain $q_1, q_2, q_3, \cdots, q_n$, and is less than or equal to n and is more than or equal to $p_1^{s_1+1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}$ is:

$$\left[\frac{\frac{n}{p_1^{s_1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}} - (p_1 - 6)}{6}\right] \text{ in all.}$$

We can prove the following conclusion with the same method as that:

$$\left[\frac{\frac{n}{p_1^{s_1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}} - (p_1 - 6)}{6}\right] + \left[\frac{\frac{n}{p_1^{s_1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}} - (p_1 - 2)}{6}\right]$$

Express the number of compositions that is less than or equal to n and contain $p_1, p_2, \cdots, p_j, p_h$ and more than or equal to $p_1^{s_1+1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}$,where p_1 is the number of the first column.

In the following we prove: the composition of the first column and the fifth column $p_1^{s_1+1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}$ which is less or equal to n is unique, that is to calculate one time, where

$$\{p_1, p_2, \cdots, p_j, p_h\} \subseteq \{5, 7, 11, 13, \cdots, (6k-1), (6k+1)\}$$

and

$$p_1 < p_2 < \cdots < p_j < p_h, \quad s_1, s_2, \cdots, s_j, s_h \in N .$$

Because of $1 \le i \le [\log_5 n] - 1$, and as $\sum s_x = i \quad (x = 1, 2, \cdots, j, h)$, any composition of the first column and fifth column $p_1^{s_1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}$ which satisfy the condition $\sum s_x = i$ and which is less than or equal to n calculated, $s_1, s_2, \cdots, s_j, s_h \in N$.

And because $p_1 < p_2 < \cdots < p_j < p_h$. as it satisfy $\sum s_x = i$ and the values of $s_1, s_2, \cdots, s_j, s_h$ are determined, the composition $p_1^{s_1} \cdot p_2^{s_2} \cdots p_j^{s_j} \cdot p_h^{s_h}$ is unique.

We prove the maximum of i is $[\log_5 n] - 1$.

Because $n \ge 5^{s_1} \cdot 7^{s_2} \cdots (6k-1)^{s_j} \cdot (6k+1)^{s_h} > 5^{s_1} \cdot 5^{s_2} \cdots 5^{s_j} \cdot 5^{s_h} = 5^{\sum s_x}$,

Therefore the maximum of i is $[\log_5 n] - 1$.

We prove the code of every term is $(-1)^i$ in the following:

Because the composition with three factors can be regarded as the product of two factors, at the same cause, the composition with i factors can be regarded as the product of $i - 1$ factors. Therefore, we should take the composite of three factors into account when calculating a composite number containing two factors, and we should plus the number of composition with three factors. At the same cause, we should take the number of compositions with i factors into account when calculating the number of compositions with $i - 1$ factors, and we should plus the number of composition with i factors.

Therefore, as i is an odd, the code is negative, as i is an even, the code is positive, we can write them into the union form $(-1)^i$, so the code of every terms is $(-1)^i$.

2.2.4. The Last Conclusion

$$\pi(n)=\begin{cases} 2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2 \quad (2\le n<25); \\ 2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2 \\ +\sum_{i=1}^{[\log_5 n]-1}(-1)^i\cdot\sum_{\sum s_x=i}\left(\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]\right. \\ \left.+\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\right) \quad (n\ge 25). \end{cases}$$

that is:

1) $\pi(n)=2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2 \quad (2\le n<25)$;

2) $$\pi(n)=2\left[\frac{n-1}{6}\right]+\left[\frac{r+1}{6}\right]+2 +\sum_{i=1}^{[\log_5 n]-1}(-1)^i\cdot\sum_{\sum s_x=i}\left(\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R}{6}\right]+\left[\frac{\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R'}{6}\right]\right) \quad (n\ge 25).$$

where "[ ]" denotes taking integer.

$$r=1,2,3,4,5,6;$$

$$s_x=s_1,s_2,\cdots,s_j,s_h;$$

$$s_1,s_2,\cdots,s_j,s_h=0,1,2,3,\cdots$$

As $i\ge 2$, $2\le s_x\le i-1$ $(x=1,2,\cdots,j,h)$.

As i is determined, k take n and:

$$\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}-R\ge 6.$$

The regulations of R, R' in the following:

The base number of the power of the denominator of the fraction $\frac{n}{5^{s_1}\cdot 7^{s_2}\cdots(6k-1)^{s_j}\cdot(6k+1)^{s_h}}$ are possible take a part or all of the following numbers $5,7,\cdots,(6k-1),(6k+1)$, setting the minimum of the part or whole obtained numbers is p.

As p is the number of the fifth column, $R=P-6$, $R'=P-4$;

As p is the number of the first column, $R=P-6$, $R'=P-2$.

Part 3

In this part we introduce some examples of the $\pi(n)$ formula application.

2.3. Examples of Applying the π(n) Formula Calculation

2.3.1. Example No. 1

$$\begin{aligned}\pi(100)&=2\left[(100-1)/6\right]+2+\left[(4+1)/6\right]\\&\quad-\left[(100/5+1)/6\right]-\left[(100/5-1)/6\right]\\&\quad-\left[(100/7-1)/6\right]-\left[(100/7-5)/6\right]\\&=34-9\\&=25\end{aligned}$$

2.3.2. Example No. 2

$$\begin{aligned}\pi(200)&=2\left[(200-1)/6\right]+2+\left[(2+1)/6\right]\\&\quad-\left[(200/5+1)/6\right]-\left[(200/5-1)/6\right]\\&\quad-\left[(200/7-1)/6\right]-\left[(200/7-5)/6\right]\\&\quad-\left[(200/11-5)/6\right]-\left[(200/11-7)/6\right]\\&\quad-\left[(200/13-7)/6\right]-\left[(200/13-11)/6\right]\\&\quad+\left[(200/5\times 7+1)/6\right]+\left[(200/5\times 7-1)/6\right]\\&=68-23+1\\&=46\end{aligned}$$

2.3.3. Example No. 3

$$\begin{aligned}\pi(400)&=2\left[(400-1)/6\right]+2+\left[(4+1)/6\right]\\&\quad-\left[(400/5+1)/6\right]-\left[(400/5-1)/6\right]\\&\quad-\left[(400/7-1)/6\right]-\left[(400/7-5)/6\right]\\&\quad-\left[(400/11-5)/6\right]-\left[(400/11-7)/6\right]\\&\quad-\left[(400/13-7)/6\right]-\left[(400/13-11)/6\right]\\&\quad-\left[(400/17-11)/6\right]-\left[(400/17-13)/6\right]\\&\quad-\left[(400/19-13)/6\right]-\left[(400/19-17)/6\right]\\&\quad+\left[(400/5\times 7+1)/6\right]+\left[(400/5\times 7-1)/6\right]\\&\quad+\left[(400/5\times 11+1)/6\right]+\left[(400/5\times 11-1)/6\right]\\&=134-62+6\\&=78\end{aligned}$$

2.3.4. Example No.4

$$
\begin{aligned}
\pi(1000) = {} & 2\left[(1000-1)/6\right]+2+\left[(4+1)/6\right] \\
& -\left[(1000/5+1)/6\right]-\left[(1000/5-1)/6\right] \\
& -\left[(1000/7-1)/6\right]-\left[(1000/7-5)/6\right] \\
& -\left[(1000/11-5)/6\right]-\left[(1000/11-7)/6\right] \\
& -\left[(1000/13-7)/6\right]-\left[(1000/13-11)/6\right] \\
& -\left[(1000/17-11)/6\right]-\left[(1000/17-13)/6\right] \\
& -\left[(1000/19-13)/6\right]-\left[(1000/19-17)/6\right] \\
& -\left[(1000/23-17)/6\right]-\left[(1000/23-19)/6\right] \\
& -\left[(1000/25-19)/6\right]-\left[(1000/25-23)/6\right] \\
& -\left[(1000/29-23)/6\right]-\left[(1000/29-25)/6\right] \\
& -\left[(1000/31-25)/6\right]-\left[(1000/31-29)/6\right] \\
& +\left[(1000/5\times 7+1)/6\right]+\left[(1000/5\times 7-1)/6\right] \\
& +\left[(1000/5\times 11+1)/6\right]+\left[(1000/5\times 11-1)/6\right] \\
& +\left[(1000/5\times 13+1)/6\right]+\left[(1000/5\times 13-1)/6\right] \\
& +\left[(1000/5\times 17+1)/6\right]+\left[(1000/5\times 17-1)/6\right] \\
& +\left[(1000/5\times 19+1)/6\right]+\left[(1000/5\times 19-1)/6\right] \\
& +\left[(1000/5\times 23+1)/6\right]+\left[(1000/5\times 23-1)/6\right] \\
& +\left[(1000/5\times 25+1)/6\right]+\left[(1000/5\times 25-1)/6\right] \\
& +\left[(1000/5\times 29+1)/6\right]+\left[(1000/5\times 29-1)/6\right] \\
& +\left[(1000/5\times 31+1)/6\right]+\left[(1000/5\times 31-1)/6\right] \\
& +\left[(1000/5\times 35+1)/6\right]+\left[(1000/5\times 35-1)/6\right] \\
& +\left[(1000/5\times 37+1)/6\right]+\left[(1000/5\times 37-1)/6\right] \\
& +\left[(1000/7\times 11-1)/6\right]+\left[(1000/7\times 11-5)/6\right] \\
& +\left[(1000/7\times 13-1)/6\right]+\left[(1000/7\times 13-5)/6\right] \\
& +\left[(1000/7\times 17-1)/6\right]+\left[(1000/7\times 17-5)/6\right] \\
& +\left[(1000/7\times 19-1)/6\right]+\left[(1000/7\times 19-5)/6\right] \\
& -\left[(1000/5\times 5\times 7+1)/6\right]-\left[(1000/5\times 5\times 7-1)/6\right] \\
= {} & 334-200+35-1 \\
= {} & 168
\end{aligned}
$$

2.3.5. Example No. 5

$$
\begin{aligned}
\pi(5000) = {} & 2\left[(5000-1)/6\right]+2+\left[(2+1)/6\right] \\
& -\left[(5000/5+1)/6\right]-\left[(5000/5-1)/6\right] \\
& -\left[(5000/7-1)/6\right]-\left[(5000/7-5)/6\right] \\
& -\left[(5000/11-5)/6\right]-\left[(5000/11-7)/6\right] \\
& -\left[(5000/13-7)/6\right]-\left[(5000/13-11)/6\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\left[(5000/17-11)/6\right]-\left[(5000/17-13)/6\right]\\
&-\left[(5000/19-13)/6\right]-\left[(5000/19-17)/6\right]\\
&-\left[(5000/23-17)/6\right]-\left[(5000/23-19)/6\right]\\
&-\left[(5000/25-19)/6\right]-\left[(5000/25-23)/6\right]\\
&-\left[(5000/29-23)/6\right]-\left[(5000/29-25)/6\right]\\
&-\left[(5000/31-25)/6\right]-\left[(5000/31-29)/6\right]\\
&-\left[(5000/35-29)/6\right]-\left[(5000/35-31)/6\right]\\
&-\left[(5000/37-31)/6\right]-\left[(5000/37-35)/6\right]\\
&-\left[(5000/41-35)/6\right]-\left[(5000/41-37)/6\right]\\
&-\left[(5000/43-37)/6\right]-\left[(5000/43-41)/6\right]\\
&-\left[(5000/47-41)/6\right]-\left[(5000/47-43)/6\right]\\
&-\left[(5000/49-43)/6\right]-\left[(5000/49-47)/6\right]\\
&-\left[(5000/53-47)/6\right]-\left[(5000/53-49)/6\right]\\
&-\left[(5000/55-49)/6\right]-\left[(5000/55-53)/6\right]\\
&-\left[(5000/59-53)/6\right]-\left[(5000/59-55)/6\right]\\
&-\left[(5000/61-55)/6\right]-\left[(5000/61-59)/6\right]\\
&-\left[(5000/65-59)/6\right]-\left[(5000/65-61)/6\right]\\
&-\left[(5000/67-61)/6\right]-\left[(5000/67-65)/6\right]\\
&+\left[(5000/5\times 7+1)/6\right]+\left[(5000/5\times 7-1)/6\right]\\
&+\left[(5000/5\times 11+1)/6\right]+\left[(5000/5\times 11-1)/6\right]\\
&+\left[(5000/5\times 13+1)/6\right]+\left[(5000/5\times 13-1)/6\right]\\
&+\left[(5000/5\times 17+1)/6\right]+\left[(5000/5\times 17-1)/6\right]\\
&+\left[(5000/5\times 19+1)/6\right]+\left[(5000/5\times 19-1)/6\right]\\
&+\left[(5000/5\times 23+1)/6\right]+\left[(5000/5\times 23-1)/6\right]\\
&+\left[(5000/5\times 25+1)/6\right]+\left[(5000/5\times 25-1)/6\right]\\
&+\left[(5000/5\times 29+1)/6\right]+\left[(5000/5\times 29-1)/6\right]\\
&+\left[(5000/5\times 31+1)/6\right]+\left[(5000/5\times 31-1)/6\right]\\
&+\left[(5000/5\times 35+1)/6\right]+\left[(5000/5\times 35-1)/6\right]\\
&+\left[(5000/5\times 37+1)/6\right]+\left[(5000/5\times 37-1)/6\right]\\
&+\left[(5000/5\times 41+1)/6\right]+\left[(5000/5\times 41-1)/6\right]\\
&+\left[(5000/5\times 43+1)/6\right]+\left[(5000/5\times 43-1)/6\right]\\
&+\left[(5000/5\times 47+1)/6\right]+\left[(5000/5\times 47-1)/6\right]\\
&+\left[(5000/5\times 49+1)/6\right]+\left[(5000/5\times 49-1)/6\right]\\
&+\left[(5000/5\times 53+1)/6\right]+\left[(5000/5\times 53-1)/6\right]\\
&+\left[(5000/5\times 55+1)/6\right]+\left[(5000/5\times 55-1)/6\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\left[(5000/5\times59+1)/6\right]+\left[(5000/5\times59-1)/6\right]\\
&+\left[(5000/5\times61+1)/6\right]+\left[(5000/5\times61-1)/6\right]\\
&+\left[(5000/5\times65+1)/6\right]+\left[(5000/5\times65-1)/6\right]\\
&+\left[(5000/5\times67+1)/6\right]+\left[(5000/5\times67-1)/6\right]\\
&+\left[(5000/5\times71+1)/6\right]+\left[(5000/5\times71-1)/6\right]\\
&+\left[(5000/5\times73+1)/6\right]+\left[(5000/5\times73-1)/6\right]\\
&+\left[(5000/5\times77+1)/6\right]+\left[(5000/5\times77-1)/6\right]\\
&+\left[(5000/5\times77+1)/6\right]+\left[(5000/5\times77-1)/6\right]\\
&+\left[(5000/5\times79+1)/6\right]+\left[(5000/5\times79-1)/6\right]\\
&+\left[(5000/5\times83+1)/6\right]+\left[(5000/5\times83-1)/6\right]\\
&+\left[(5000/5\times85+1)/6\right]+\left[(5000/5\times85-1)/6\right]\\
&+\left[(5000/5\times91+1)/6\right]+\left[(5000/5\times91-1)/6\right]\\
&+\left[(5000/5\times95+1)/6\right]+\left[(5000/5\times95-1)/6\right]\\
&+\left[(5000/5\times97+1)/6\right]+\left[(5000/5\times97-1)/6\right]\\
&+\left[(5000/5\times101+1)/6\right]+\left[(5000/5\times101-1)/6\right]\\
&+\left[(5000/5\times103+1)/6\right]+\left[(5000/5\times103-1)/6\right]\\
&+\left[(5000/5\times107+1)/6\right]+\left[(5000/5\times107-1)/6\right]\\
&+\left[(5000/5\times109+1)/6\right]+\left[(5000/5\times109-1)/6\right]\\
&+\left[(5000/5\times103+1)/6\right]+\left[(5000/5\times103-1)/6\right]\\
&+\left[(5000/5\times107+1)/6\right]+\left[(5000/5\times107-1)/6\right]\\
&+\left[(5000/5\times109+1)/6\right]+\left[(5000/5\times109-1)/6\right]\\
&+\left[(5000/5\times113+1)/6\right]+\left[(5000/5\times113-1)/6\right]\\
&+\left[(5000/5\times115+1)/6\right]+\left[(5000/5\times115-1)/6\right]\\
&+\left[(5000/5\times119+1)/6\right]+\left[(5000/5\times119-1)/6\right]\\
&+\left[(5000/5\times121+1)/6\right]+\left[(5000/5\times121-1)/6\right]\\
&+\left[(5000/5\times125+1)/6\right]+\left[(5000/5\times125-1)/6\right]\\
&+\left[(5000/5\times127+1)/6\right]+\left[(5000/5\times127-1)/6\right]\\
&+\left[(5000/5\times131+1)/6\right]+\left[(5000/5\times131-1)/6\right]\\
&+\left[(5000/5\times133+1)/6\right]+\left[(5000/5\times133-1)/6\right]\\
&+\left[(5000/5\times137+1)/6\right]+\left[(5000/5\times137-1)/6\right]\\
&+\left[(5000/5\times139+1)/6\right]+\left[(5000/5\times139-1)/6\right]\\
&+\left[(5000/5\times143+1)/6\right]+\left[(5000/5\times143-1)/6\right]\\
&+\left[(5000/5\times145+1)/6\right]+\left[(5000/5\times145-1)/6\right]\\
&+\left[(5000/5\times149+1)/6\right]+\left[(5000/5\times149-1)/6\right]\\
&+\left[(5000/5\times151+1)/6\right]+\left[(5000/5\times151-1)/6\right]
\end{aligned}
$$

$$+\left[(5000/5\times155+1)/6\right]+\left[(5000/5\times155-1)/6\right]$$
$$+\left[(5000/5\times157+1)/6\right]+\left[(5000/5\times157-1)/6\right]$$
$$+\left[(5000/5\times161+1)/6\right]+\left[(5000/5\times161-1)/6\right]$$
$$+\left[(5000/5\times163+1)/6\right]+\left[(5000/5\times163-1)/6\right]$$
$$+\left[(5000/5\times167+1)/6\right]+\left[(5000/5\times167-1)/6\right]$$
$$+\left[(5000/5\times169+1)/6\right]+\left[(5000/5\times169-1)/6\right]$$
$$+\left[(5000/5\times173+1)/6\right]+\left[(5000/5\times173-1)/6\right]$$
$$+\left[(5000/5\times175+1)/6\right]+\left[(5000/5\times175-1)/6\right]$$
$$+\left[(5000/5\times179+1)/6\right]+\left[(5000/5\times179-1)/6\right]$$
$$+\left[(5000/5\times181+1)/6\right]+\left[(5000/5\times181-1)/6\right]$$
$$+\left[(5000/5\times185+1)/6\right]+\left[(5000/5\times185-1)/6\right]$$
$$+\left[(5000/5\times187+1)/6\right]+\left[(5000/5\times187-1)/6\right]$$
$$+\left[(5000/5\times191+1)/6\right]+\left[(5000/5\times191-1)/6\right]$$
$$+\left[(5000/5\times193+1)/6\right]+\left[(5000/5\times193-1)/6\right]$$
$$+\left[(5000/5\times197+1)/6\right]+\left[(5000/5\times197-1)/6\right]$$
$$+\left[(5000/5\times199+1)/6\right]+\left[(5000/5\times199-1)/6\right]$$
$$+\left[(5000/7\times11-1)/6\right]+\left[(5000/7\times11-5)/6\right]$$
$$+\left[(5000/7\times13-1)/6\right]+\left[(5000/7\times13-5)/6\right]$$
$$+\left[(5000/7\times17-1)/6\right]+\left[(5000/7\times17-5)/6\right]$$
$$+\left[(5000/7\times19-1)/6\right]+\left[(5000/7\times19-5)/6\right]$$
$$+\left[(5000/7\times23-1)/6\right]+\left[(5000/7\times23-5)/6\right]$$
$$+\left[(5000/7\times25-1)/6\right]+\left[(5000/7\times25-5)/6\right]$$
$$+\left[(5000/7\times29-1)/6\right]+\left[(5000/7\times29-5)/6\right]$$
$$+\left[(5000/7\times31-1)/6\right]+\left[(5000/7\times31-5)/6\right]$$
$$+\left[(5000/7\times35-1)/6\right]+\left[(5000/7\times35-5)/6\right]$$
$$+\left[(5000/7\times37-1)/6\right]+\left[(5000/7\times37-5)/6\right]$$
$$+\left[(5000/7\times41-1)/6\right]+\left[(5000/7\times41-5)/6\right]$$
$$+\left[(5000/7\times43-1)/6\right]+\left[(5000/7\times43-5)/6\right]$$
$$+\left[(5000/7\times47-1)/6\right]+\left[(5000/7\times47-5)/6\right]$$
$$+\left[(5000/7\times49-1)/6\right]+\left[(5000/7\times49-5)/6\right]$$
$$+\left[(5000/7\times53-1)/6\right]+\left[(5000/7\times53-5)/6\right]$$
$$+\left[(5000/7\times55-1)/6\right]+\left[(5000/7\times55-5)/6\right]$$
$$+\left[(5000/7\times59-1)/6\right]+\left[(5000/7\times59-5)/6\right]$$
$$+\left[(5000/7\times61-1)/6\right]+\left[(5000/7\times61-5)/6\right]$$
$$+\left[(5000/7\times65-1)/6\right]+\left[(5000/7\times65-5)/6\right]$$

$$+\left[(5000/7\times 67-1)/6\right]+\left[(5000/7\times 67-5)/6\right]$$
$$+\left[(5000/7\times 71-1)/6\right]+\left[(5000/7\times 71-5)/6\right]$$
$$+\left[(5000/7\times 73-1)/6\right]+\left[(5000/7\times 73-5)/6\right]$$
$$+\left[(5000/7\times 77-1)/6\right]+\left[(5000/7\times 77-5)/6\right]$$
$$+\left[(5000/7\times 79-1)/6\right]+\left[(5000/7\times 79-5)/6\right]$$
$$+\left[(5000/7\times 83-1)/6\right]+\left[(5000/7\times 83-5)/6\right]$$
$$+\left[(5000/7\times 85-1)/6\right]+\left[(5000/7\times 85-5)/6\right]$$
$$+\left[(5000/7\times 89-1)/6\right]+\left[(5000/7\times 89-5)/6\right]$$
$$+\left[(5000/7\times 91-1)/6\right]+\left[(5000/7\times 91-5)/6\right]$$
$$+\left[(5000/7\times 95-1)/6\right]+\left[(5000/7\times 95-5)/6\right]$$
$$+\left[(5000/7\times 97-1)/6\right]+\left[(5000/7\times 97-5)/6\right]$$
$$+\left[(5000/7\times 101-1)/6\right]+\left[(5000/7\times 101-5)/6\right]$$
$$+\left[(5000/11\times 13-5)/6\right]+\left[(5000/11\times 13-7)/6\right]$$
$$+\left[(5000/11\times 17-5)/6\right]+\left[(5000/11\times 17-7)/6\right]$$
$$+\left[(5000/11\times 19-5)/6\right]+\left[(5000/11\times 19-7)/6\right]$$
$$+\left[(5000/11\times 23-5)/6\right]+\left[(5000/11\times 23-7)/6\right]$$
$$+\left[(5000/11\times 25-5)/6\right]+\left[(5000/11\times 25-7)/6\right]$$
$$+\left[(5000/11\times 29-5)/6\right]+\left[(5000/11\times 29-7)/6\right]$$
$$+\left[(5000/11\times 31-5)/6\right]+\left[(5000/11\times 31-7)/6\right]$$
$$+\left[(5000/11\times 35-5)/6\right]+\left[(5000/11\times 35-7)/6\right]$$
$$+\left[(5000/11\times 37-5)/6\right]+\left[(5000/11\times 37-7)/6\right]$$
$$+\left[(5000/11\times 41-5)/6\right]+\left[(5000/11\times 41-7)/6\right]$$
$$+\left[(5000/13\times 17-7)/6\right]+\left[(5000/13\times 17-11)/6\right]$$
$$+\left[(5000/13\times 19-7)/6\right]+\left[(5000/13\times 19-11)/6\right]$$
$$+\left[(5000/13\times 23-7)/6\right]+\left[(5000/13\times 23-11)/6\right]$$
$$+\left[(5000/13\times 25-7)/6\right]+\left[(5000/13\times 25-11)/6\right]$$
$$+\left[(5000/13\times 29-7)/6\right]+\left[(5000/13\times 29-11)/6\right]$$
$$-\left[(5000/5\times 5\times 7+1)/6\right]-\left[(5000/5\times 5\times 7-1)/6\right]$$
$$-\left[(5000/5\times 5\times 11+1)/6\right]-\left[(5000/5\times 5\times 11-1)/6\right]$$
$$-\left[(5000/5\times 5\times 13+1)/6\right]-\left[(5000/5\times 5\times 13-1)/6\right]$$
$$-\left[(5000/5\times 5\times 17+1)/6\right]-\left[(5000/5\times 5\times 17-1)/6\right]$$
$$-\left[(5000/5\times 5\times 19+1)/6\right]-\left[(5000/5\times 5\times 19-1)/6\right]$$
$$-\left[(5000/5\times 5\times 23+1)/6\right]-\left[(5000/5\times 5\times 23-1)/6\right]$$
$$-\left[(5000/5\times 5\times 25+1)/6\right]-\left[(5000/5\times 5\times 25-1)/6\right]$$
$$-\left[(5000/5\times 5\times 29+1)/6\right]-\left[(5000/5\times 5\times 29-1)/6\right]$$

$$
\begin{aligned}
&-\left[(5000/5\times5\times31+1)/6\right]-\left[(5000/5\times5\times31-1)/6\right]\\
&-\left[(5000/5\times5\times35+1)/6\right]-\left[(5000/5\times5\times35-1)/6\right]\\
&-\left[(5000/5\times5\times37+1)/6\right]-\left[(5000/5\times5\times37-1)/6\right]\\
&-\left[(5000/5\times5\times41+1)/6\right]-\left[(5000/5\times5\times41-1)/6\right]\\
&-\left[(5000/5\times7\times7+1)/6\right]-\left[(5000/5\times7\times7-1)/6\right]\\
&-\left[(5000/5\times7\times11+1)/6\right]-\left[(5000/5\times7\times11-1)/6\right]\\
&-\left[(5000/5\times7\times13+1)/6\right]-\left[(5000/5\times7\times13-1)/6\right]\\
&-\left[(5000/5\times7\times17+1)/6\right]-\left[(5000/5\times7\times17-1)/6\right]\\
&-\left[(5000/5\times7\times19+1)/6\right]-\left[(5000/5\times7\times19-1)/6\right]\\
&-\left[(5000/5\times7\times23+1)/6\right]-\left[(5000/5\times7\times23-1)/6\right]\\
&-\left[(5000/5\times7\times25+1)/6\right]-\left[(5000/5\times7\times25-1)/6\right]\\
&-\left[(5000/5\times11\times11+1)/6\right]-\left[(5000/5\times11\times11-1)/6\right]\\
&-\left[(5000/5\times11\times13+1)/6\right]-\left[(5000/5\times11\times13-1)/6\right]\\
&-\left[(5000/5\times11\times17+1)/6\right]-\left[(5000/5\times11\times17-1)/6\right]\\
&-\left[(5000/5\times11\times19+1)/6\right]-\left[(5000/5\times11\times19-1)/6\right]\\
&-\left[(5000/5\times13\times13+1)/6\right]-\left[(5000/5\times13\times13-1)/6\right]\\
&-\left[(5000/7\times7\times11+1)/6\right]-\left[(5000/7\times7\times11-1)/6\right]\\
&-\left[(5000/7\times7\times13+1)/6\right]-\left[(5000/7\times7\times13-1)/6\right]\\
&-\left[(5000/5\times5\times5\times7+1)/6\right]-\left[(5000/5\times5\times5\times7-1)/6\right]\\
&-\left[(5000/5\times5\times5\times11+1)/6\right]-\left[(5000/5\times5\times5\times11-1)/6\right]\\
&-\left[(5000/5\times5\times7\times7+1)/6\right]-\left[(5000/5\times5\times7\times7-1)/6\right]\\
&=1668-1441+495-54+1\\
&=699
\end{aligned}
$$

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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