Applied Mathematics, 2023, 14, 839-846
https://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

(ρ, τ, σ)-Derivations of Dendriform Algebras

Yousuf A. Alkhezi

Public Authority for Applied Education and Training, College of Basic Education, Mathematics Department, Kuwait City, Kuwait
Email: ya.alkhezi@paaet.edu.kw

How to cite this paper: Alkhezi, Y.A. (2023) (ρ, τ, σ)-Derivations of Dendriform Algebras. *Applied Mathematics*, **14**, 839-846.
https://doi.org/10.4236/am.2023.1412048

Received: October 8, 2023
Accepted: December 10, 2023
Published: December 13, 2023

Abstract

We introduce and investigate the properties of a generalization of the derivation of dendriform algebras. We specify all possible parameter values for the generalized derivations, which depend on parameters. We provide all generalized derivations for complex low-dimensional dendriform algebras.

Keywords

Dendriform Algebras, Derivations, Generalized Derivations

1. Introduction

Generalized derivations of dendriform algebras addresses generalized derivations within the setting of dendriform algebras [1]. Dendriform algebras, as given by Loday [2] are algebraic structures that extend the notion of associative operations by decomposing them into two distinct binary operations. In addition, he proposed dendriform algebras, which are algebraic structures that extend the concept of associative operations by separating them into two different components [3].

Dendriform algebras play a crucial role in the study of extended σ-operators, associative Yang-Baxter equations, infinitesimal bialgebras, and modified Rota-Baxter algebras [3], quantum field theory and renormalization. These applications highlight the relevance and significance of dendriform algebras in theoretical physics.

The purpose of this research is to investigate the connections between dendriform algebras and other mathematical structures, such as Rota-Baxter algebras and post-Lie algebra structures [4] [5]. This study also investigates the embedding of dendriform algebras within Rota-Baxter algebras, offering light on the relationship between these two types of algebras [4]. In addition, they investigate the categorization of post-Lie algebra structures caused by extended derivations and shed light on the connection between these structures and dendriform algebras.

Researchers have already discovered connections between generalized conformal derivations and conformal (ρ,τ,σ)-derivations. In addition, they give a characterization of all conformal (ρ,τ,σ)-derivations of finite simple Lie conformal algebras [6]. This characterization is crucial to comprehending the characteristics and behavior of these derivations. In addition, this demonstrates that there exist no post-Lie algebra structures for semisimple and solvable unimodular Lie algebras [7]. Moreover, they develop the generalized (ρ,τ,σ)-derivations of semisimple Lie algebras [8] [9] [10] [11] [12].

The properties of generalized derivations in dendriform algebras have undergone thorough investigation, offering valuable insights into the algebraic structures, deformations, and cohomology associated with these algebras. This research on generalized derivations further enhances our comprehension of the properties and interconnections within dendriform algebras and their applications across various mathematical contexts.

This study is concerned with describing (ρ,τ,σ)-derivations of dendriform algebras. In this circumstance, generalized derivation can be easily inferred from the definition of derivations of dendriform algebras. In the scenario where $\rho=\tau=\sigma=1$, the dendriform algebra derivations explored in [13] are obtained. We provide a technique for computing derivations in the paper. We utilize the technique in low-dimensional scenarios. Every application's result is displayed in tabular format. We apply the classification result of two-dimensional complex dendriform algebras derived from [14].

2. Preliminaries

This section will start with essential definitions and information needed for further discussions.

Definition 2.1. *Let $\mathcal{E}$ be an algebra over F. $\mathcal{E}$ is said to be dendriform algebra, if it satisfies*

$$(u\prec v)\prec w=u\prec(v\prec w)+u\prec(v\succ w),$$

$$(u\succ v)\prec w=u\succ(v\prec w),$$

$$(u\prec v)\succ w+(u\succ v)\succ w=u\succ(v\succ w).$$

The process $\prec$ and $\succ$ related terms are left product, right product.

Definition 2.2. *If $(\mathcal{E}_1,\prec_1,\succ_1)$, $(\mathcal{E},\prec_2,\succ_2)$ are dendriform algebra. Then, a function from $\mathcal{E}_1\to\mathcal{E}_2$ is a homomorphism: if*

$$\pi(a\prec_1 b)=\pi(a)\prec_2\pi(b)$$

and

$$\pi(a\succ_1 b)=\pi(a)\succ_2\pi(b)$$

$$\forall a,b\in\mathcal{E}_1.$$

Definition 2.3. *Let $\mathcal{E}$ be a dendriform algebra over a field F and linear transformation $d:\mathcal{E}\to\mathcal{E}$ satisfying*

$$d(a*b)=d(a)*b+a*d(b)$$
$$\forall a,b\in\mathcal{E},$$

where $*:=\prec,\succ$.

The collection of all derived dendriform algebras $\mathcal{E}$ is a subspace of $End_F(\mathcal{E})$. This subspace equipped with the bracket $[d_1,d_2]=d_1\circ d_2-d_2\circ d_1$ is a Lie algebra denoted by $\mathrm{Der}(\mathcal{E})$.

3. (ρ,τ,σ)-Derivations

The spaces of (ρ,τ,σ)-derivations of dendriform algebras would be employed in special cases to define some invariant functions, which are very important tools for geometrical representations of dendriform algebras. The definition of derivations can be generalized in several non-equivalent ways. We bring forward another type of generalization of the derivations. The notion of the derivation of dendriform algebras is generalized; (ρ,τ,σ)-derivations of $\mathcal{E}$ are introduced and their relevant properties are shown. All possible intersections of spaces containing these derivations are investigated. Examples of spaces of (ρ,τ,σ)-derivations of low dimensional dendriform algebras are presented. In special cases, the spaces of (ρ,τ,σ)-derivation are from operator dendriform algebras.

[13] provides various different generalization strategies regarding the definition (2.3) of derivations. We present another form of generalization regarding the derivations.

A mapping $d\in End(\mathcal{E})$ is said to be (ρ,τ,σ)-derivation of $\mathcal{E}$ (ρ,τ,σ) are fixed elements of *F*) if for all $a,b\in\mathcal{E}$,

$$\rho d(a*b)=\tau d(a)*b+\sigma a*d(b),$$

where $*:=\prec,\succ$. The set that (ρ,τ,σ)-derivations we denote by $\mathrm{Der}_{(\rho,\tau,\sigma)}(\mathcal{E})$. It is clear that $\mathrm{Der}_{(\rho,\tau,\sigma)}(\mathcal{E})$ is a linear subspace of $End(\mathcal{E})$.

Lemma 3.1. *Let* $\mathcal{E}$ *be a dendriform algebra. Then,* ρ,τ,σ *in* $\mathrm{Der}_{(\rho,\tau,\sigma)}(\mathcal{E})$ *in the following manner*:

$$\mathrm{Der}_{(1,1,1)}(\mathcal{E})=\{d\in End(E)\,|\,d(a\star b)=d(a)\star b+a\star d(b)\};$$

$$\mathrm{Der}_{(1,1,0)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,d(a\star b)=d(a)\star b\};$$

$$\mathrm{Der}_{(1,0,1)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,d(a\star b)=a\star d(b)\};$$

$$\mathrm{Der}_{(1,0,0)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,d(x\star y)=0\};$$

$$\mathrm{Der}_{(0,1,1)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,d(a)\star b=a\star d(b)\};$$

$$\mathrm{Der}_{(0,0,1)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,a\star d(b)=0\};$$

$$\mathrm{Der}_{(0,1,0)}(\mathcal{E})=\{d\in End(\mathcal{E})\,|\,d(a)\star b=0\};$$

where $\star=\prec,\succ$.

Proof. Suppose $\rho\neq 0$. By applying the operator *d* to the dendriform algebra identities, we obtain the system of equations.

$$\beta(\beta-\alpha)=0 \text{ and } \gamma(\gamma-\alpha)=0.$$

Using one by one approach, there are determined the various values of

$$(\rho,\tau,\sigma):(\rho,\rho,\rho),(\rho,\rho,0),(\rho,0,\rho) \text{ and } (\rho,0,0).$$

Considering the circumstance that

$$\mathrm{Der}_{(\rho,\tau,\sigma)}(\mathcal{E})=\mathrm{Der}_{(1,\tau/\rho,\sigma)}(\mathcal{E}).$$

In 1 - 4, we obtain the necessary inequality.

Let equal $\rho=0$ now. Then, we possess

$$(0,\tau,\tau),(0,\tau,0) \text{ if } \tau\neq 0,$$

and

$$(0,0,\sigma) \text{ if } \tau=0.$$

Thus, we obtain

$$\mathrm{Der}_{(0,\tau,\sigma)}(\mathcal{E})=\mathrm{Der}_{(0,1,\sigma/\tau)}(\mathcal{E}) \text{ if } \tau\neq 0,$$

and

$$\mathrm{Der}_{(0,0,\sigma)}(\mathcal{E})=\mathrm{Der}_{(0,0,1)}(\mathcal{E}) \text{ if } \tau=0 \text{ and } \tau\neq 0.$$

If σ equals zero. Then, $\mathrm{Der}_{(0,0,0)}(\mathcal{E})=End\mathcal{E}$. □

Lemma 3.2. *Let $\mathcal{E}$ be dendriform algebra and d_1,d_2 be (ρ,τ,σ)-derivation on $\mathcal{E}$. Then, $[d_1,d_2]=d_1\circ d_2-d_2\circ d_1$ is a (ρ^2,τ^2,σ^2)-derivation of $\mathcal{E}$.*

Proof. Consider the two equations shown below:

$$\alpha d_1(a\star b)=\tau d_1(a)\star b+\gamma a\star d_1(b) \tag{1}$$

and

$$\rho d_2(a\star b)=\tau d_1(a)\star b+\sigma a\star d_2(b). \tag{2}$$

First occurrence $\rho\neq 0$.

$$\begin{aligned}\rho^2[d_1,d_2](a\star b)&=\rho^2(d_1\circ d_2)(a\star b)-\rho^2(d_2\circ d_1)(x\star y)\\&=\rho d_1(\rho d_2(a\star b))-\rho d_2(\rho d_1(a\star b))\\&=\rho d_1(\tau d_2(a)\star b+\sigma a\star d_2(b))\\&\quad-\rho d_2(\beta d_1(a)\star b+\sigma a\star d_1(b))\\&=\tau(\rho d_1(d_2(a)\star b))+\sigma(\rho d_1(ad_2(b)))\\&\quad-\tau(\rho d_2(d_1(a)\star b)+\sigma\rho d_2(x\star d_1(b))).\end{aligned}$$

Eventually we get;

$$\rho^2[d_1,d_2](a\star b)=\tau^2[d_1,d_2](a)\star b+\sigma^2 a\star[d_1,d_2](b). \tag{3}$$

Case two for $\rho=0$. Then, from the Equations (1) and (2) we get;

$$\tau d_1(a)\star b=-\sigma a\star d_1 b, \tag{4}$$

$$\tau d_2(a)\star b=-\sigma a\star d_2 b. \tag{5}$$

Table 1. The description of (ρ,τ,σ)-derivations of two-dimensional dendriform algebras.

IC	(ρ,τ,σ)	$Der_{(\rho,\tau,\sigma)}$	Dim	IC	(ρ,τ,σ)	$Der_{(\rho,\tau,\sigma)}$	Dim
	(1, 1, 1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 2d_{11} \end{pmatrix}$	2		(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
	(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$	2		(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
$Dend_2^1$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1	$Dend_2^2$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1, 0, 0)	trivial	0		(1, 0, 0)	trivial	0
	(0, 1, 1)	trivial	0		(0, 1, 1)	trivial	0
	(0, 0, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1		(0, 0, 1)	trivial	0
	(0, 1, 0)	trivial	0		(0, 1, 0)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
	(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1		(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & 0 \end{pmatrix}$	1
$Dend_2^3$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2	$Dend_2^4$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1, 0, 0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 0 \end{pmatrix}$	2		(1, 0, 0)	trivial	0
	(0, 1, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 1)	trivial	0
	(0, 0, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 0, 1)	trivial	0
	(0, 1, 0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 0)	trivial	0
	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1		(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$	2
$Dend_2^5$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1	$Dend_2^6$	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$	2
	(1, 0, 0)	trivial	0		(1, 0, 0)	trivial	0

Continued

	(0, 1, 1)	trivial	0		(0, 1, 1)	trivial	0
	(0, 0, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 0, 1)	trivial	0
	(0, 1, 0)	trivial	0		(0, 1, 0)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
Dend_2^7	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1	Dend_2^8	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2		(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2		(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1, 0, 0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 0 \end{pmatrix}$	2		(1, 0, 0)	trivial	0
	(0, 1, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 1)	trivial	0
	(0, 0, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 0, 1)	trivial	0
	(0, 1, 0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 0)	trivial	0
Dend_2^9	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0	Dend_2^{10}	(1, 1, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
	(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2		(1, 1, 0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1, 0, 1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1		(1, 0, 1)	$\begin{pmatrix} 0 & 0 \\ 0 & d_{22} \end{pmatrix}$	1
	(1, 0, 0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 0 \end{pmatrix}$	2		(1, 0, 0)	trivial	0
	(0, 1, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 1)	trivial	0
	(0, 0, 1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 0, 1)	trivial	0
	(0, 1, 0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2		(0, 1, 0)	trivial	0

Hence,

$$\tau^2[d_1,d_2](x\star y)=\tau^2(d_1\circ d_2)(a)\star b-\rho^2(d_2\circ d_1)(a)\star b$$
$$=\tau^2(d_1d_2)(a)\star b-\tau^2(d_2d_1)(a)\star b$$
$$=\tau(\tau d_1(d_2(a)\star b-\tau(\tau d_2(d_1(a)\star b.$$

The result of replacing the equations from (4) and (5) is

$$\tau^2[d_1,d_2](a)\star b=\tau(-\sigma d_2)(a)d_1(b)+\tau(\sigma d_1)(a)d_2b$$
$$=-\sigma(\tau d_2(x)d_1(b)+\sigma(\tau d_1(a)d_2(b).$$

Again, we obtain because of (4) and (5):

$$\sigma^2a\star d_2\left(d_1\left(b\right)\right)-\sigma^2x\star d_1\left(d_2\left(y\right)\right)=\sigma^2a\left[d_2\left(d_1\left(b\right)\right)-d_1\left(d_2\left(b\right)\right)\right]$$
$$=\sigma^2a\star\left[d_1,d_2\right]\left(b\right).$$

□

Remark 3.1. *In any* $\rho,\tau,\sigma\in F$ *the dimension of the vector space* $\mathrm{Der}_{(\rho,\tau,\sigma)}\mathcal{E}$ *is an isomorphism invariant of dendriform algebras.*

(ρ,τ,σ)-Derivations of Low-Dimensional Dendriform Algebras

This section provides a discussion of the generalized derivation of two-dimensional complex dendriform algebras. A matrix representation of the element d_{ij} of the generalized derivation it transforms the vector space linearly $\mathcal{E}$ *i.e* $d\left(e_i\right)=\sum_{j=1}^{n}d_{ji}e_j$, $i=1,2,\cdots,n$. The entries in the generalized derivation according to its definition, η_{ij}, $i,j=1,2,\cdots,n$, of the matrix $\left[d_{ij}\right]_{i,j=1,2,\cdots,n}$ must satisfy the following systems of equations:

$$\sum_{t=1}^{n}\left(\rho\sigma_{ij}^{t}d_{st}-\tau d_{ti}\sigma_{tj}^{s}-\sigma d_{tj}\sigma_{it}^{s}\right)=0$$
$$\forall i,j,s=1,2,\cdots,n$$

$$\sum_{l=1}^{n}\left(\rho\delta_{st}^{l}d_{ml}-\tau d_{ls}\delta_{lt}^{m}-\sigma d_{lt}\delta_{sl}^{m}\right)=0$$
$$\forall s,t,m=1,2,3,\cdots,n.$$

Let's use this strategy to determine the generalized derivations of complex dendriform algebras of dimension two [14] (**Table 1**).

4. Conclusion

This study enabled us to calculate the generalized derivation of two-dimensional dendriform algebras and determine the dimension of the generalized derivation for each representative class, ranging from 0 to 2.

Acknowledgements

We would like to express our appreciation to the referees for their valuable comments and suggestions. Additionally, we acknowledge the generous financial support from the Public Authority for Applied Education and Training for funding this research (Project No. BE-23-01).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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