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Probability of Matching All Types of Prizes for the First Time Is Maximized at a Surprisingly Early Number of Trials

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Abstract

Suppose that when money or coins are placed in a box containing an arbitrary number of prizes of several different types, one of each type of prize will appear alone each time. Is the probability that all types of prizes will be together for the first time maximized when the majority of prizes are removed from the box? Simulations based on probability theory show that this probability reaches its maximum value in a surprisingly small number of trials, contrary to expectations. This will help us understand not only mathematical phenomena, but also real-world phenomena. Phenomena that do not occur without several substances or conditions seem unlikely to occur, but the results of this study suggest that, contrary to expectations, they are surprisingly likely to occur probabilistically.

Keywords

Probability, Prize, Gacha®, Game

1. Introduction

In the various events that occur in this world, we are sometimes surprised by the unexpected things that seem unlikely to happen at first glance [1] [2]. Such seemingly mysterious events are often explainable when considered from the perspective of probability theory. Probability theory is necessary to understand phenomena that occur with high frequency contrary to expectations, but the theoretical analysis is limited and often requires analysis by numerical simulation [3]-[9]. For example, if you roll the dice 1200 times without cheating, the numbers 1 through 6 will appear almost equally 200 times; if you roll the dice 12 times, each number will not appear evenly twice, and there will be a large bias in the proportion of the number of times each number appears. This example shows that unexpected things can happen in a short span of time. A typical example of this is when the phenomenon follows the Poisson processes. Traffic accidents are generally considered to follow this process. It is customary for the number of traffic accidents to be relatively concentrated in a given month, even when the number of traffic accidents on an annual basis is nearly constant [10]. Therefore, Poisson processes are said to be the flower of probability theory because it yields interesting results in understanding the phenomenon.

The topics that will be discussed in this study are based on experience that happens unexpectedly. Gacha® is a form of purchase of capsules containing prizes and is a registered trademark of T-ARTS Company, Ltd. It is commonly referred to as gacha-gacha in a friendly way. For example, a box contains several capsules of different types, and when a certain amount of money or coins is put in, a capsule comes out. Each capsule contains one prize. Especially in Japan, Gacha and similar machines are very popular, and it is common to see them all lined up at the same time in train stations. This popularity can be attributed to Gacha®'s unique characteristics. The main feature of Gacha® is that you cannot select the contents of the capsule, and you do not know what is in the capsule until you take it out. Therefore, various coined words have been derived. For example, the term "gacha parent" has come to be used. This means that children cannot choose their parents, and their fate depends on whether their parents are "good or bad". In social games, there are also many item-based payment schemes that resemble Gacha®.

It is thought that the most enjoyable part for Gacha® users is collecting all types of prizes. For example, if a box contains four types of famous Disney® characters, users will try to collect all of them. However, since the user does not know which character will appear each time, he/she will have to repeatedly insert money or coins into the box to play the game. Such unique features of Gacha® are versatile features that are not limited to the game world. For example, in order for a certain disease to develop, a set of etiological factors must be present. What is the probability that a certain set of factors will be available? The purpose of this study is to show from probability theory and numerical simulation, using the game Gacha® as an example, that phenomena that do not occur unless several factors are all in line are, contrary to expectations, relatively likely to occur.

2. Methods

The total number of prizes, K, in a box is arbitrary, but for the sake of brevity and without loss of generality, we assume that there are 24 prizes in a box. Twenty-four is a natural number with many divisors, so it is a very useful number as follows. That is, if there are the same number of prizes of the same type, the total number of prizes can be considered without changing the total number of prizes: 12 each for 2 types, 8 each for 3 types, 6 each for 4 types, 4 each for 6 types, 3 each for 8 types, and 2 each for 12 types, and so on. In this study, 3, 4, and 6 types were considered. In other cases, it is considered that no generality would be lost by omitting these results since they could be inferred from these results.

Assumptions are summarized below.

There are N types of prizes in the box. The number of each prize is n_i; $i = 1, 2, \ldots, N$.

$$\sum_{i=1}^{N} n_i = K. (K = 24) \tag{1}$$

The game ends when all the types are matched for the first time by repeatedly drawing one prize from the box each time. In other words, the game ends when $(N - 1)$ types have been aligned just before the end of the game and one of the remaining type is drawn. The R-th attempt is the first time all types of prizes are available. The last prize drawn is the g-th type ($1 \le g \le N$). At that time, there are m_{i_R} prizes of the i-th type in hand: $\{m_{i_R}\}$.

$$\sum_{i=1}^{N} m_{i_R} = R; N \le R \le \left(K - \left(n_g - 1\right)\right). \tag{2}$$

Then, $1 \le m_{i_R} \le n_i$, and $m_{g_R} = 1$.

The probability, $P(\{m_{i_R}\})$ that this happens is

$$P\left(\left\{m_{i_R}\right\}\right) = \frac{n_g}{\left(K - \left(R - 1\right)\right)\binom{K}{R-1}} \prod\nolimits_{i_R \ne g_R} \binom{n_i}{m_{i_R}} \tag{3}$$

The sum of Equation (3) for all $\{m_{i_R}\}$ combinations that satisfy the conditions of Equation (2) is denoted by P_R. Since $P_R; N \le R \le \left(K - \left(n_g - 1\right)\right)$ are mutually exclusive, the probability that all types of the prizes are aligned for the first time in up to R trials is expressed as the cumulative sum of P_R, CP_R.

We analyze the factors that affect the probability that all types of the prizes will be matched for the first time as follows.

1) The box initially contains a total of 24 prizes.

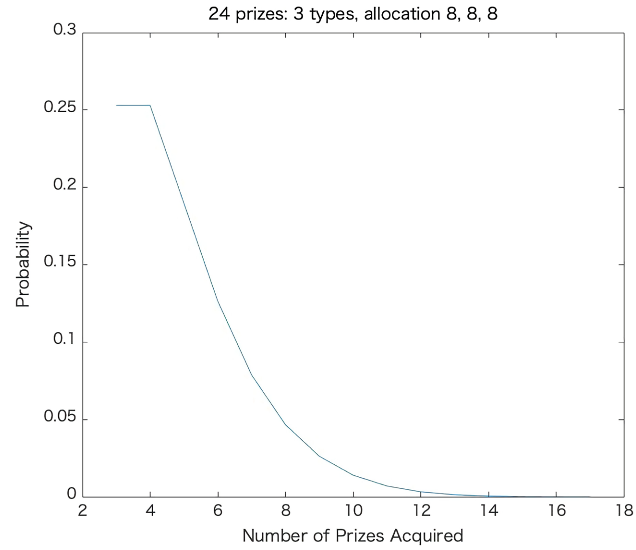
2) The effect of the number of prize types on the probability is studied for 3, 4 and 6 types of prizes.

3) The effect on the probability of the initial allocation of each type of prize is studied.

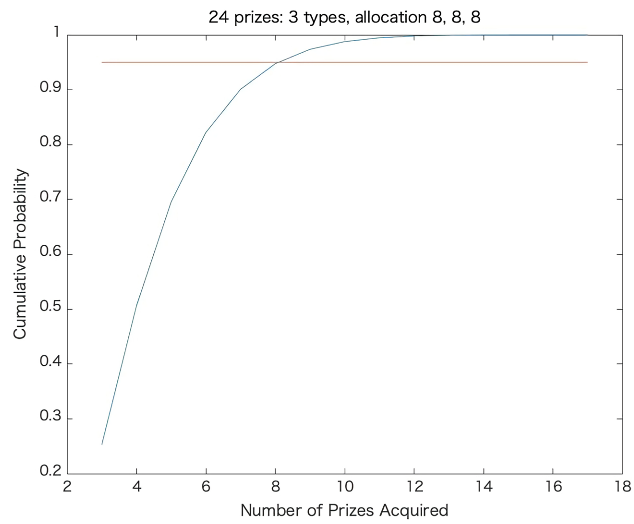
3. Results

As the initial distribution shifts from (8, 8, 8) to (6, 6, 6, 6) to (4, 4, 4, 4, 4, 4), the probability that all types are aligned for the first time shifts toward the higher number of trials, but the probability that all types are aligned is relatively high at the beginning (**Figures 1(a)-3(a)**). As the distribution changed from (8, 8, 8) to (6, 6, 6, 6) to (4, 4, 4, 4, 4, 4), the number of each type decreases, and the number of attempts when CP_R reached 95% or more increases accordingly (**Figures 1(b)-3(b)**).

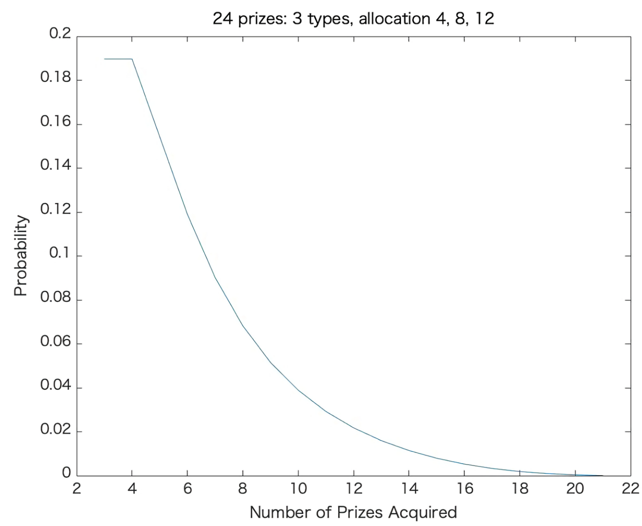
As the distribution changes from (4, 8, 12) to (3, 5, 6, 10) to (1, 2, 3, 4, 5, 9),


24 prizes: 3 types, allocation 8, 8, 8
Probability
Number of Prizes Acquired
0
0.05
0.1
0.15
0.2
0.25
0.3
2
4
6
8
10
12
14
16
18

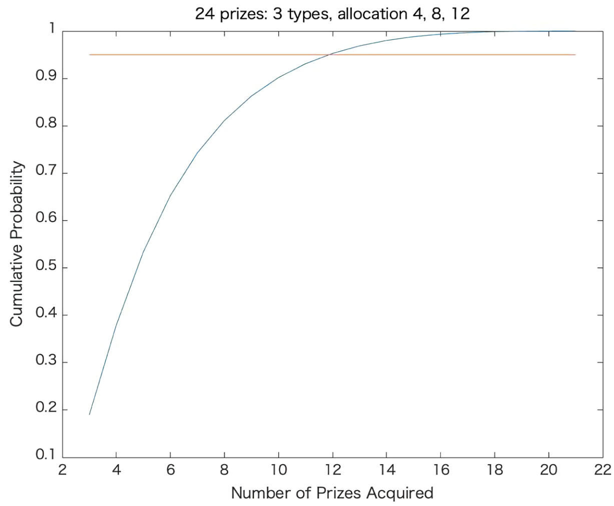
(a)


24 prizes: 3 types, allocation 8, 8, 8
Cumulative Probability
Number of Prizes Acquired
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
4
6
8
10
12
14
16
18

(b)

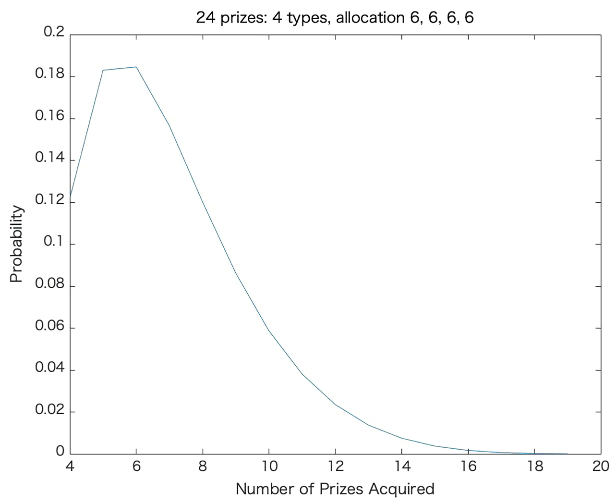

24 prizes: 3 types, allocation 4, 8, 12
Probability
Number of Prizes Acquired
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
2
4
6
8
10
12
14
16
18
20
22

(c)

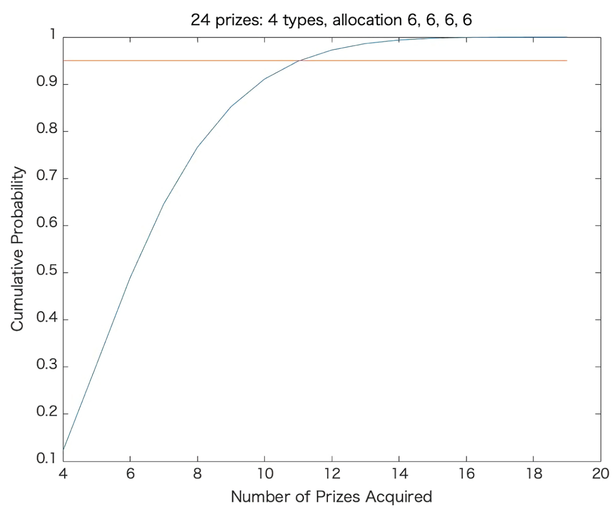


(d)

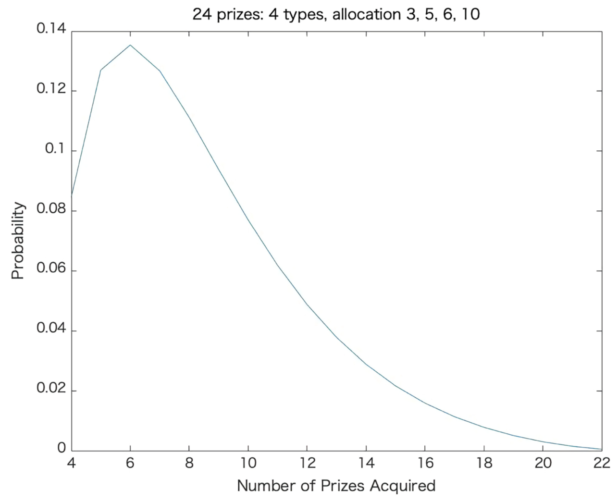
Figure 1. (a)-(b) Distribution of types of prizes in the box, (8, 8, 8); (c)-(d) Distribution of types of prizes in the box, (4, 8, 12).



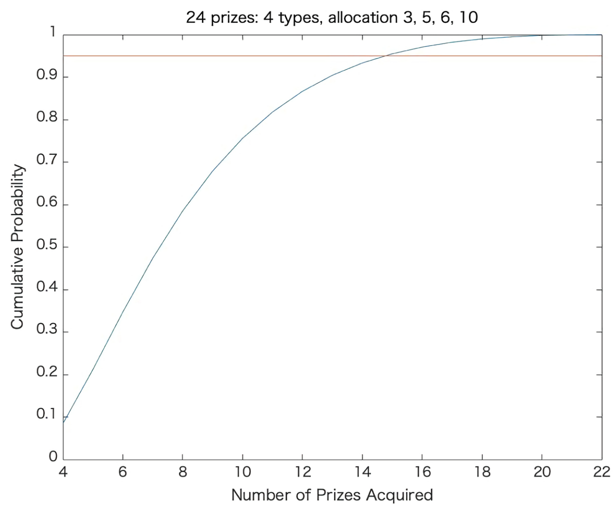
(a)



(b)

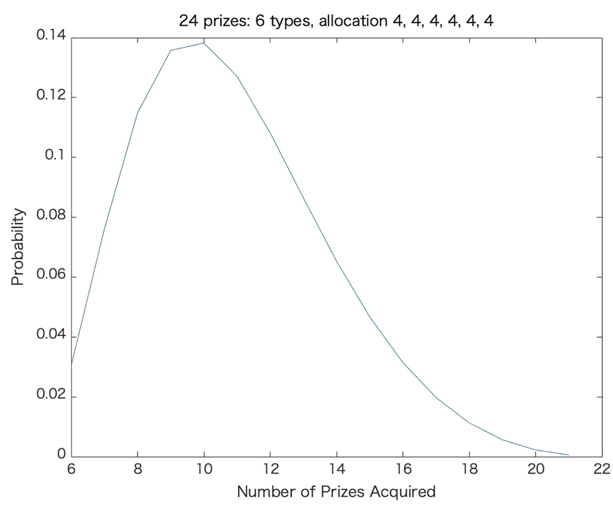


(c)



(d)

Figure 2. (a)-(b) Distribution of types of prizes in the box, (6, 6, 6, 6); (c)-(d) Distribution of types of prizes in the box, (3, 5, 6, 10).



(a)

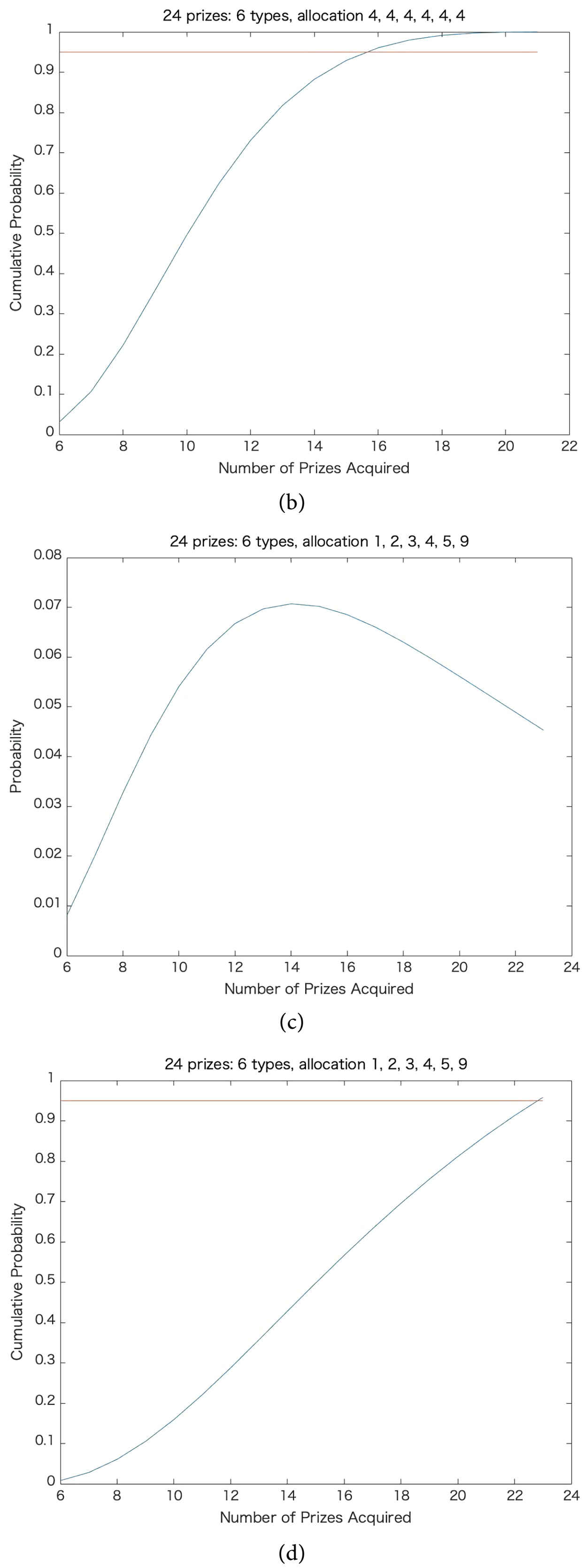


Figure 3. (a)-(b) Distribution of types of prizes in the box, (4, 4, 4, 4, 4, 4); (c)-(d) Distribution of types of prizes in the box, (1, 2, 3, 4, 5, 9).

the bias in the proportion of each type increases, but the probability of having all types for the first time is greatest when the number of trials is relatively small (**Figures 1(c)-3(c)**). Thus, as the percentage bias of each type increases, the number of attempts when the CP_R reached 95% increases (**Figures 1(d)-3(d)**).

4. Discussion

Although the examples examined in this study are few, it is generally clear that the probability of getting all types of prizes for the first time is maximized rather early, even if the box contains an arbitrary number of different types of prizes in an arbitrary proportion. Even when the number of a certain type is as extremely small as one as in (1, 2, 3, 4, 5, 9), the probability of having all types of prizes for the first time is maximized when about half of the box is removed. It is never the case that the maximum probability is reached when most of prizes in the box are taken out. This is a seemingly unexpected result. However, the probability of almost certainly getting all the types for the first time, *i.e.* reaching $CP_R \geq 0.95$, depends greatly on the distribution of the prizes. The implication of this result is that if there is only one box, the majority of prizes will still have to be removed to ensure that all types of the prizes are available for the first time. Hence, it can be seen that the vendor who manages the boxes should set a large bias in the distribution of the prizes in order to get more sales. An unscrupulous vendor might intentionally not put one type of prize in a box, but this is an out-of-bounds practice. From the user's point of view, it is better to try similar boxes instead of sticking to one box and trying to get all types of prizes, so as not to waste money or coins. However, when there are many similar boxes, there is a high probability that someone will get all types of prizes surprisingly early if several users start the game at once. For example, in the case of (1, 2, 3, 4, 5, 9), the probability of collecting all types of prizes for the first time is at most 0.07. Then 14 prizes are collected. If there are 15 similar boxes and 15 people start this game at once, there is a very high probability that one of them will be the first to get all the types on the 14th attempt.

The unique nature of Gacha® will help us to understand various phenomena beyond just taking out a few prizes from a box. For example, if we assume that some organic substances are taken into a certain cell in a living organism and a certain combination is generated and transformed into a toxic substance, a toxic substance will be generated by one of the cells rather easily because there are so many cells similar to this cell. Furthermore, the unique properties of Gacha® will be useful for understanding not only material phenomena but also social phenomena. Let us assume that a conflict occurs when several conditions are met. Since there are many communities in the world, it is not unlikely that some of them will have a conflict under certain conditions.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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