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The Mean Deviation from the Median of the Dagum Distribution

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Abstract

The Dagum model is particularly suitable for the analysis of the distributions of economic quantities, such as income, assets and consumption. The purpose of this note is to derive the expression of the mean deviation from the median of the Dagum distribution to study the behavior of the scale and shape parameters in terms of absolute variability and in terms of relative variability.

Keywords

Mean Deviation from the Median, Dagum Distribution, Scale and Shape Parameters

1. Introduction

Camilo Dagum in 1977 introduced a new distribution model particularly suited to describe the personal distribution of income. This model, in fact, thanks to the presence of a greater number of parameters (4 in the more general version) than other models, proves to be particularly flexible and adaptable to describe even deeply dissimilar income distributions (Dagum, 1977) [1]. For this model Dagum had obtained the main characteristic values (mean, mode, median, variance, moments, Lorenz curve and concentration ratio). The Dagum distribution has been studied by several authors that have proposed several variations to increase the flexibility of the Dagum distribution in modeling lifetime data. Some recent modifications concern log-Dagum distribution (Domma and Perri, 2009) [2], Mc-Dagum distribution (Oluyede and Rajasoorya, 2013) [3], beta-Dagum distribution (Domma and Condino, 2013) [4], gamma-Dagum distribution (Oluyede *et al.*, 2014) [5], weighted Dagum distribution (Oluyede and Ye, 2014) [6], exponentiated Kumaraswamy-Dagum distribution (Huang and

Oloyede, 2014) [7], transmuted Dagum distribution (Elbatal and Aryal, 2015) [8], extended Dagum distribution (Silva *et al.*, 2015) [9] and Dagum-Poisson distribution (Oluyede *et al.*, 2016) [10], exponentiated generalized exponential Dagum distribution (Nasiru *et al.*, 2019) [11], moreover, regarding properties and methods of estimation of the parameters of the Dagum distribution. Domma *et al.* (2011a [12], 2011b [13]) determined the observed information matrix in right censored samples and debated aspects of the maximum likelihood estimation for censored data. In the 2013 Shahzad and Asghar [14] obtained the L-moments and TL-moments in closed form to estimate the parameters of the Dagum distribution. Al-Zahrani (2016) [15] proposed a reliability test plan under the assumption that the life of a product follows a Dagum distribution. Dey *et al.* (2017) [16] studied the properties and different methods of estimating the parameters of the Dagum distribution.

2. Dagum Distribution

Girone and Viola (2009) [17] and Girone (2010) [18] obtained the expression of the mean difference and the mean deviation. It is very important to underline that the mean deviation from the median is invariant with respect to translations and it is homogeneous to the variable. Therefore, without losing generality, we can consider the density function with only one shape parameters

$$f(x) = \beta\delta x^{-(\delta+1)}\left(1+x^{-\delta}\right)^{-(\beta+1)}, \ 0 < x < \infty,$$

and the distribution function

$$F(x) = \left(1+x^{-\delta}\right)^{-\beta}.$$

The mean value and the median of this distribution are:

$$\mu = \beta B\left(\beta + 1/\delta, 1 - 1/\delta\right),$$

$$Me = \left(2^{1/\beta} - 1\right)^{-1/\delta}.$$

3. The Mean Deviation from the Median

The formula of the mean deviation from the median is:

$$S_{Me} = \int_{-\infty}^{\infty} |x - Me| f(x)\mathrm{d}x.$$

A formula that avoids the absolute value and that splits the calculation into two parts is:

$$S_{Me} = \int_{-\infty}^{Me} (Me - x) f(x)\mathrm{d}x + \int_{Me}^{\infty} (x - Me) f(x)\mathrm{d}x;$$

the above formula represents the first attempt in simplifying the calculations. After simple steps, the formula becomes

$$S_{Me} = Me\int_{-\infty}^{Me} f(x)\mathrm{d}x - \int_{-\infty}^{Me} xf(x)\mathrm{d}x + \int_{Me}^{\infty} xf(x)\mathrm{d}x - Me\int_{Me}^{\infty} f(x)\mathrm{d}x,$$

considering that the first and last terms offset each other, we arrive at the formula

$$S_{Me} = \int_{Me}^{\infty} xf(x)\mathrm{d}x - \int_{-\infty}^{Me} xf(x)\mathrm{d}x,$$

formula that can be simplified taking into account that

$$\int_{Me}^{\infty} xf(x)\mathrm{d}x - \int_{-\infty}^{Me} xf(x)\mathrm{d}x = \int_{-\infty}^{\infty} xf(x)\mathrm{d}x = \mu,$$

and that allows to obtain

$$S_{Me} = \mu - 2\int_{-\infty}^{Me} xf(x)\mathrm{d}x.$$

Then we have to calculate the only integral present in the formula of the mean deviation from the median considering that, in our case, the Dagum density function starts from 0. With the aid of the Mathematica software we obtain a very heavy expression of the integral that, however, after a few steps can be simplified into the following formula:

$$\int_0^{Me} xf(x)\mathrm{d}x = \frac{\beta\delta {}_2F_1\left[\beta + 1/\delta, 1+\beta, 1+\beta, 1+\beta+1/\delta, -\left(2^{1/\beta}-1\right)^{-1}\right]}{(1+\beta\delta)\left(2^{1/\beta}-1\right)^{\beta+1/\delta}}$$

and then the formula of the mean deviation from the median in the Dagum model results:

$$S_{Me} = \beta B(\beta + 1/\delta, 1-1/\delta) - 2^{\frac{\beta\delta {}_2F_1\left[\beta + 1/\delta, 1+\beta, 1+\beta, 1+\beta+1/\delta, -\left(2^{1/\beta}-1\right)^{-1}\right]}{(1+\beta\delta)\left(2^{1/\beta}-1\right)^{\beta+1/\delta}}}$$

an expression that cannot be simplified but that, for some values of β and δ, gives more compact results.

4. Expressions of the Mean Deviation from the Median for Some Values of δ and β

In this paragraph the expressions of the mean deviation from the median are given for some values of δ and β.

For $\delta = 2$ and $\beta = 1$ $S_{Me} = 1$,

for $\delta = 2$ and $\beta = 2$ $S_{Me} = \sqrt{\frac{1}{2} + \frac{5}{\sqrt{2}}} + \frac{3\pi}{4} - 3\operatorname{arccot}\sqrt{-1+\sqrt{2}}$,

for $\delta = 3$ and $\beta = 1$ $S_{Me} = 1 - \frac{2\log 2}{3}$,

for $\delta = 4$ and $\beta = 1$ $S_{Me} = \frac{1}{4}\left(4 + \sqrt{2}\log\left[2-\sqrt{2}\right] - \sqrt{2}\log\left[2+\sqrt{2}\right]\right)$.

Table 1 shows the mean deviation from the median values for some values of δ and β. The same and other values are shown in **Figure 1**.

The values shown in **Table 1** represent the single values assumed by the mean deviation from the median, obtained by crossing some values assumed by the δ parameter and the β parameter.

With δ is equal to 1.5, the mean deviation from the median is 1.21 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median

Table 1. Values of the mean deviation from the median for some values of δ and β.

δ	β									
	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
1.5	1.21	1.92	2.51	3.02	3.50	3.94	4.36	4.76	5.14	5.51
2.0	0.73	1.00	1.20	1.37	1.52	1.65	1.78	1.89	2.00	2.11
2.5	0.56	0.69	0.79	0.87	0.94	1.00	1.06	1.11	1.16	1.21
3.0	0.47	0.54	0.59	0.63	0.67	0.71	0.74	0.77	0.80	0.82
3.5	0.41	0.44	0.47	0.50	0.52	0.54	0.56	0.58	0.60	0.61
4.0	0.36	0.38	0.39	0.41	0.43	0.44	0.45	0.46	0.48	0.49
4.5	0.33	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.39	0.40
5.0	0.30	0.29	0.30	0.30	0.31	0.32	0.32	0.33	0.33	0.34

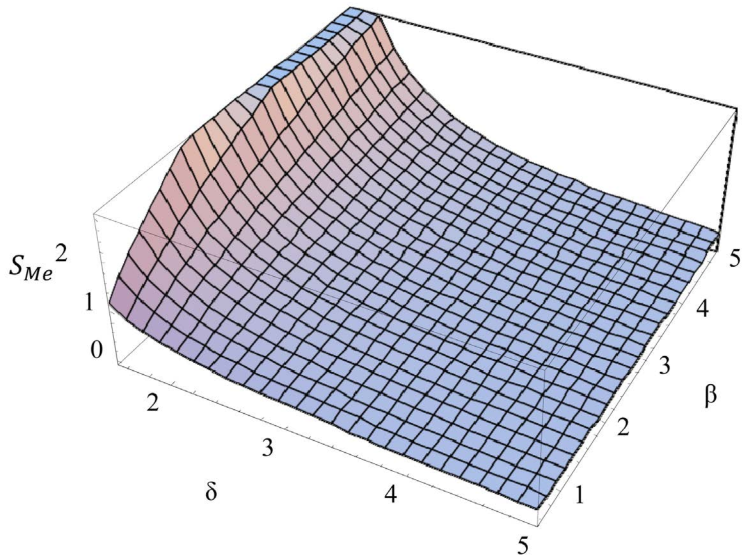

Figure 1. Graphical representation of the mean deviation from the median.

increases until 5.51. With δ is equal to 2.0, the mean deviation from the median is 0.73 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 2.11. With δ is equal to 2.5, the mean deviation from the median is 0.56 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 1.21. With δ is equal to 3.0, the mean deviation from the median is 0.47 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 0.82. With δ is equal to 3.5, the mean deviation from the median is 0.41 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 0.61. With δ is equal to 4.0, the mean deviation from the median is 0.36 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 0.49. With δ is equal to 4.5, the mean deviation from the median is 0.33 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 0.40. Finally with δ is equal to 5.0, the mean deviation from the median is 0.30 when β is 0.5 and, as β increases until it reaches 5.0, the mean deviation from the median increases until 0.34.

So we can state that the mean deviation from the median decreases as δ increases, but increases as β increases.

The values shown in **Table 1** are displayed in **Figure 1**. And this graph allows us to have a visual perception of the trend of the mean deviation from the median at the values of δ and β.

As it can be seen, the mean deviation from the median seems to increase as β increases and decrease as δ increases; moreover, it increases as the scale parameter λ increases.

5. Conclusion

In this paper an explicit and compact expression of the mean deviation from the median for the distribution of Dagum was obtained. This expression allows us to examine, with great evidence, the behavior of the scale and shape parameters in terms of absolute variability and in terms of relative variability.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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