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Solving Multi-Objective Linear Programming Problem by Statistical Averaging Method with the Help of Fuzzy Programming Method

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Abstract

A multi-objective linear programming problem is made from fuzzy linear programming problem. It is due the fact that it is used fuzzy programming method during the solution. The Multi objective linear programming problem can be converted into the single objective function by various methods as Chandra Sen's method, weighted sum method, ranking function method, statistical averaging method. In this paper, Chandra Sen's method and statistical averaging method both are used here for making single objective function from multi-objective function. Two multi-objective programming problems are solved to verify the result. One is numerical example and the other is real life example. Then the problems are solved by ordinary simplex method and fuzzy programming method. It can be seen that fuzzy programming method gives better optimal values than the ordinary simplex method.

Keywords

Fuzzy Programming Method, Fuzzy Linear Programming Problem, Multi-Objective Linear Programming Problem, Statistical Averaging Method, New Statistical Averaging Method

1. Introduction

Fuzzy decision-making technique is used repeatedly for fuzzy multi-objective linear programming problem. It has been investigated for more than decades by many researchers. The idea of fuzzy set was first proposed by Zadeh [1].

The idea of fuzzy decision was mentioned by Bellman and Zadeh in 1970.

Multi-Objective Linear Programming problems (MOLPP) with fuzzy goals were taken into account by Zimmerman [2]. The most common approach to solve fuzzy linear programming problem is to shift them to similar deterministic linear program. Zimmerman has introduced fuzzy programming approach to solve crisp multi-objective linear programming problem. Fuzzy linear programming problem (FLPP) in fuzzy environment was introduced by Tanaka and Asai [3]. In study there are several methods for solving MOLPP models by applying fuzzy programming approaches. Thakre *et al.* [4] provided a method to solve FLPP where both the coefficient matrix of the constraints and cost coefficient are fuzzy in nature.

First formulation of fuzzy linear programming (FLP) was proposed by Zimmermann. A new fuzzy primal and dual simplex algorithm is for solving FLP by Mahdavi-Amiri *et al.* [5].

Nasseri and Alizadeh [6] proposed a fuzzy Big-M method. Nehi [7] used ranking function to solve fuzzy MOLPP. A method is proposed by Kiruthiga and Loganathan [8] to find crisp MOLPP from FMOLPP by using ranking function.

In this paper, we solve MOLPP by using fuzzy programming method. For making single objective from multi-objective, we apply Chandra Sen's method and statistical averaging method by Nahar and Alim [9]. It can be seen that when we use fuzzy programming method, we get better result rather than solving ordinary simplex method.

Fuzzy number

Let X be a set of objects for fuzzy statement. The set of ordered pairs $\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$ where $\mu_A : X \to [0,1]$ is a fuzzy set in X. The evaluation function $\mu_A(x)$ is called the membership function.

A fuzzy number $A = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x = b \\ \dfrac{x-b}{b-c}, & b \le x \le c \end{cases}$$

Mathematical model of multi-objective linear programming problem:

$$\max Z_s(x) = \sum_{j=1}^{n} c_j x_j, s = 1, 2, \cdots, s$$

Subject to

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij} x_j (\le, =, \ge) b_i, i = 1, 2, \cdots, m \\ &x_j \ge 0, j = 1, 2, \cdots, n \end{aligned} \tag{1}$$

To verify the result, two examples are shown. One is numerical example (Hypothetical) and the other is real life example (An investigation for the risk reduction along the coastal area).

2. Methods

Chandra Sen's method [10] is very popular method for making single objective LPP from MOLPP. For data analysis statistic and mathematics are related to each other to analyze results. Statistical averaging method is a method using average. For solving optimization problem simplex method is a technique involving single objective function or multi-objective function with some constraints. Fuzzy programming method is an optimization model which is associated with uncertainty. Fuzzy programming method works on the concept of fuzzy logic.

In this paper for making single objective from MOLPP, Chandra Sen's method and statistical averaging methods are used. Later on, the single objective function is solved by the ordinary simplex method and fuzzy programming method and hence the results are compared.

Fuzzy Programming Method

Step 1: Solve MOLPP by using simplex algorithm, considering only one of the objectives at a time and ignoring all others. Repeat the process s times for s different objective functions.

Step 2: Using all solutions in step 1, construct a pay off matrix of size s by s. Then from the pay off matrix estimate the lower bound (L_s) and the upper bound (U_s) for the k^{th} objective function z_s as:

$$L_s \le z_s \le U_s, s = 1, 2, \cdots, s$$

Step 3: Define a fuzzy linear membership function $\mu(z_s(x))$ for the s^{th} objective function $z_s, s = 1, 2, \cdots, s$

$$\mu(z_s(x)) = \begin{cases} 0, & \text{if } z_s \le L_s \\ 1 - \dfrac{U_s - z_s}{U_s - L_s}, & \text{if } L_s \le z_s \le U_s \\ 1, & \text{if } z_s \ge U_s \end{cases} \tag{2}$$

Step 4: Using membership functions we can get a crisp model by introducing an augmented variable λ

$$\max : 1 - \frac{U_s - z_s}{U_s - L_s}, s = 1, 2, \cdots, s$$

$$\min : \frac{U_s - z_s}{U_s - L_s}; s = 1, 2, \cdots, s$$

Subject to (1) can be further written as

Minimize λ

Subject to

$$\begin{aligned} &\sum_{j=1}^{n} c_j x_i + (U_s - L_s)\lambda \ge U_s, s = 1, 2, \cdots, s \\ &\sum_{j=1}^{n} a_{ij} x_j (\le, =, \ge) b_i, i = 1, 2, \cdots, m \\ &\lambda \ge 0, x_j \ge 0, j = 1, 2, \cdots, n \end{aligned} \tag{3}$$

Step 5: Solve the crisp model by simplex algorithm and find optimal solution.

3. Numerical Example (Hypothetical)

$$\begin{aligned} &\max z_1 = 9x_0 + 4x_1 + 5x_2 \\ &\max z_2 = 3x_0 + x_1 + 5x_2 \\ &\max z_3 = x_0 + 2x_1 + 3x_2 \end{aligned} \tag{4}$$

Subject to

$$\begin{aligned} &4x_0 + 2x_1 + 3x_2 \le 5 \\ &5x_0 + 3x_1 + 2x_2 \le 9 \\ &3x_0 + 2x_1 + 7x_2 \le 7 \\ &x_0, x_1, x_2 \ge 0 \end{aligned} \tag{5}$$

For the first objective function in Equation (4) with constraints in equation (5), by applying simplex algorithm we get $z_1 = 11.25$ with (5/4, 0, 0). The value of the objective function is obtained using the steps of simplex method.

Similarly for second objective function in equation (4) with same constraints in equation (5) we get $z_2 = 5.63$ with (14/19, 0, 13/19).

And for last objective function in Equation (4) with same constraints in equation (5) we get $z_3 = 5$ with (0, 7/4, 1/2).

A Pay-off matrix is formulated as in "Table 1".

For the Pay-off matrix, lower and upper bound are established as:

$$\begin{aligned} &9.5 \le z_1 \le 11.25 \\ &3.75 \le z_2 \le 5.67 \\ &1.25 \le z_3 \le 5 \end{aligned} \tag{6}$$

Using Equation (3) we can write as

min λ

Subject to

$$\begin{aligned} &9x_0 + 4x_1 + 5x_2 + 1.75\lambda \ge 11.25 \\ &3x_0 + x_1 + 5x_2 + 1.92\lambda \ge 5.67 \\ &x_0 + 2x_1 + 3x_2 + 3.75\lambda \ge 5 \\ &4x_0 + 2x_1 + 3x_2 \le 5 \\ &5x_0 + 3x_1 + 2x_2 \le 9 \\ &3x_0 + 2x_1 + 7x_2 \le 7 \\ &x_0, x_1, x_2, \lambda \ge 0 \end{aligned} \tag{7}$$

Table 1. Pay off matrix.

(x_1, x_2, x_3)	Z_1	Z_2	Z_3
(1.25, 0, 0)	11.25	3.75	1.25
(0.74, 0, 0.68)	10.06	5.67	2.78
(0, 1.75, 0.5)	9.5	4.25	5

Using simplex algorithm, we get the result $x_0 = 0.69, x_1 = 0.51, x_2 = 0.41, \lambda = 0.55$.

Finally, the crisp model is solved to get the result $\varphi_1 = 10.3, \varphi_2 = 4.63, \varphi_3 = 2.94$.

Thus, for simplex method we get the optimal values for $z_1 = 11.25, z_2 = 5.63, z_3 = 5$ and for fuzzy programming method we get $\varphi_1 = 10.3, \varphi_2 = 4.63, \varphi_3 = 2.94$.

3.1. Ordinary Simplex Method

For making single objective from multi-objective applying Chandra Sen's method by Sen

$$
\begin{aligned}
z &= \frac{z_1}{\varphi_1} + \frac{z_2}{\varphi_2} + \frac{z_3}{\varphi_3} \\
&= \frac{1}{11.25}(9x_0 + 4x_1 + 5x_2) + \frac{1}{5.63}(3x_0 + x_1 + 5x_2) + \frac{1}{5}(x_0 + 2x_1 + 3x_2) \\
&= x_0(0.8 + 0.53 + 0.2) + x_1(0.35 + 0.18 + 0.4) + x_2(0.44 + 0.89 + 0.6) \\
&= 1.53x_0 + 0.93x_1 + 1.93x_2
\end{aligned}
$$

Thus, the single objective becomes

$$
\max z = 1.53x_0 + 0.93x_1 + 1.93x_2 \tag{8}
$$

From the single objective function in Equation (8) with same constraints (5) by using simplex algorithm we get $z = 2.5925$ with (0, 7/4, 1/2).

For $\varphi_1 = 11.25, \varphi_2 = 5.63, \varphi_3 = 5$ using statistical averaging method such as arithmetic mean ($A.M$), geometric mean ($G.M$) and harmonic mean ($H.M$) we get

$$
\begin{aligned}
A.M &= \frac{11.25 + 5.63 + 5}{3} = 7.29 \\
G.M &= \sqrt[3]{11.25 \times 5.63 \times 5} = 6.687 \\
H.M &= \frac{3}{\frac{1}{11.25} + \frac{1}{5.63} + \frac{1}{5}} = 6.42
\end{aligned}
$$

Applying arithmetic mean formula by Nahar and Alim we can write

$$
\begin{aligned}
\max z &= \frac{1}{A.M}(z_1 + z_2 + z_3) \\
&= \frac{1}{7.29}(9x_0 + 4x_1 + 5x_2 + 3x_0 + x_1 + 5x_2 + x_0 + 2x_1 + 3x_2) \\
&= \frac{1}{7.29}\left[x_0(9 + 3 + 1) + x_1(4 + 1 + 2) + x_2(5 + 5 + 3)\right] \\
&= \frac{1}{7.29}(13x_0 + 7x_1 + 13x_2) \\
&= 1.783x_0 + 0.96x_1 + 1.783x_2
\end{aligned}
$$

Thus, the single objective becomes

$$
\max z = 1.783x_0 + 0.96x_1 + 1.783x_2 \tag{9}
$$

From the single objective function in Equation (9) with same constraints (5) by using simplex algorithm we get

$z = 2.5715$ with (0, 7/4, 1/2)

Applying geometric mean formula by Nahar and Alim we can write

$$\begin{aligned} \max z &= \frac{1}{G.M}\left(z_1 + z_2 + z_3\right) \\ &= \frac{1}{6.687}\left(13x_0 + 7x_1 + 13x_2\right) \\ &= 1.94x_0 + 1.05x_1 + 1.94x_2 \end{aligned}$$

Thus, the single objective is

$$\max z = 1.94x_0 + 1.05x_1 + 1.94x_2 \tag{10}$$

From the single objective function in equation (10) with same constraints (5) by using simplex algorithm we get $z = 2.8075$ with (0, 7/4, 1/2).

Applying Harmonic mean formula by Nahar and Alim (2017) we can write

$$\begin{aligned} \max z &= \frac{1}{H.M}\left(z_1 + z_2 + z_3\right) \\ &= \frac{1}{6.42}\left(13x_0 + 7x_1 + 13x_2\right) \\ &= 2.02x_0 + 1.09x_1 + 2.024x_2 \end{aligned}$$

Thus, the single objective is

$$\max z = 2.02x_0 + 1.09x_1 + 2.024x_2 \tag{11}$$

From the single objective function in Equation (11) with same constraints (5) by using simplex algorithm we get $z = 2.9175$ with (0, 7/4, 1/2).

3.2. Fuzzy Programming Method

Using the value of fuzzy programming method applying Chanda Sen's method by Sen

$$\begin{aligned} \max z &= \frac{z_1}{\varphi_1} + \frac{z_2}{\varphi_2} + \frac{z_3}{\varphi_3} \\ &= \frac{z_1}{10.3} + \frac{z_2}{4.63} + \frac{z_3}{2.94} \\ &= \frac{1}{10.3}\left(9x_0 + 4x_1 + 5x_2\right) + \frac{1}{4.63}\left(3x_0 + x_1 + 5x_2\right) + \frac{1}{2.94}\left(x_0 + 2x_1 + 3x_2\right) \\ &= x_0\left(0.874 + 0.65 + 0.34\right) + x_1\left(0.38 + 0.216 + 0.68\right) + x_2\left(0.48 + 1.08 + 1.02\right) \\ &= 1.864x_0 + 1.276x_1 + 2.58x_2 \end{aligned}$$

Thus, the single objective is

$$\max z = 1.864x_0 + 1.276x_1 + 2.58x_2 \tag{12}$$

From the single objective function in Equation (12) with same constraints (5) by using simplex algorithm we get $z = 3.523$ with (0, 7/4, 1/2).

For $\varphi_1 = 10.3, \varphi_2 = 4.63, \varphi_3 = 2.94$ using statistical averaging method such as arithmetic mean (*A.M*), geometric mean (*G.M*) and harmonic mean (*H.M*) we get

$$A.M = \frac{11.25 + 5.63 + 5}{3} = 7.29$$
$$G.M = \sqrt[3]{10.3 \times 4.63 \times 2.94} = 5.11$$
$$H.M = \frac{3}{\frac{1}{10.3} + \frac{1}{4.63} + \frac{1}{2.94}} = 4.594$$

Applying arithmetic mean formula by Nahar and Alim (2017) we can write

$$\begin{aligned} \max z &= \frac{1}{A.M}(z_1 + z_2 + z_3) \\ &= \frac{1}{5.957}(9x_0 + 4x_1 + 5x_2 + 3x_0 + x_1 + 5x_2 + x_0 + 2x_1 + 3x_2) \\ &= \frac{1}{5.957}\left[x_0(9+3+1) + x_1(4+1+2) + x_2(5+5+3)\right] \\ &= \frac{1}{5.957}(13x_0 + 7x_1 + 13x_2) \\ &= 2.182x_0 + 1.175x_1 + 2.182x_2 \end{aligned}$$

$$\max z = 2.182x_0 + 1.175x_1 + 2.182x_2 \tag{13}$$

From the single objective function in Equation (13) with same constraints (5) by using simplex algorithm we get $z = 3.1472$ with (0, 7/4, 1/2).

Applying geometric mean formula by Nahar and Alim (2017) we can write

$$\begin{aligned} \max z &= \frac{1}{G.M}(z_1 + z_2 + z_3) \\ &= \frac{1}{5.11}(13x_0 + 7x_1 + 13x_2) \\ &= 2.544x_0 + 1.369x_1 + 2.544x_2 \end{aligned}$$

Thus, single objective is

$$\max z = 2.544x_0 + 1.369x_1 + 2.544x_2 \tag{14}$$

From the single objective function in Equation (14) with same constraints (5) by using simplex algorithm we get $z = 3.65$ with (0, 7/4, 1/2).

Applying harmonic mean formula by Nahar and Alim (2017) we can write

$$\begin{aligned} \max z &= \frac{1}{H.M}(z_1 + z_2 + z_3) \\ &= \frac{1}{4.594}(13x_0 + 7x_1 + 13x_2) \\ &= 2.83x_0 + 1.525x_1 + 2.83x_2 \end{aligned}$$

Thus, the single objective is

$$\max z = 2.83x_0 + 1.525x_1 + 2.83x_2 \tag{15}$$

From the single objective function in Equation (15) with same constraints (5) by using simplex algorithm we get $z = 4.083$ with (0, 7/4, 1/2). All these results can be shown in "**Table 2**".

"**Table 2**" can be described as in "**Figure 1**".

Table 2. Comparison between Chandra Sen's method and statistical averaging method in both method ordinary simplex and fuzzy programming method.

Methods	Chandra Sen's method	Arithmetic mean	Geometric mean	Harmonic mean
Ordinary Simplex Method	2.5925	2.5715	2.8075	2.9175
Fuzzy programming method	3.523	3.1472	3.65	4.083

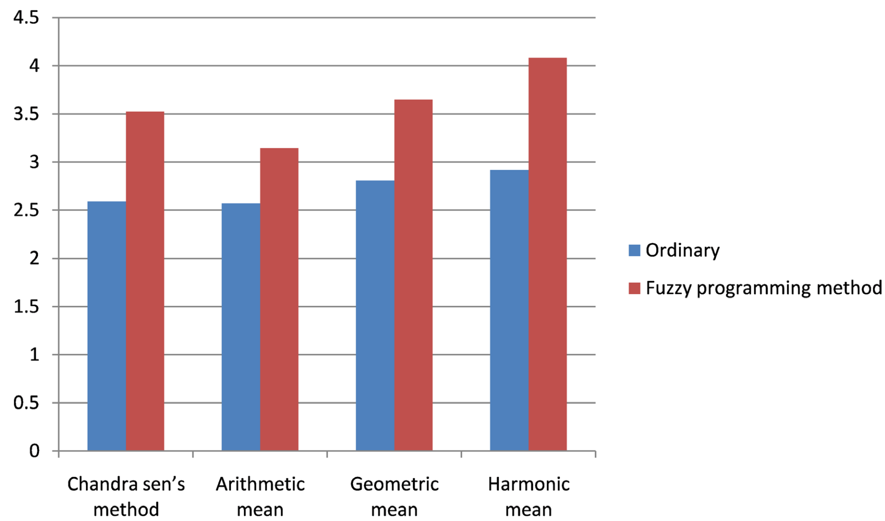

Figure 1. Optimal values of MOLPP by using ordinary and fuzzy programming method.

4. Real Life Example

For the real life example, an example is taken from Nahar, *et al.* [11] in 2022. In Nahar, et al., a Fuzzy multi-objective linear programming problem is formulated from a data list collected from IWFM (BUET) which is a secondary data. The objective was to reduce the risk and hazard of the coastal area during natural disasters. They consider four parameters and they maximize cropping intensity and shelter and minimize erosion and population density. They selected four parameters defined as decision variables, x_1, x_2, x_3, x_4, which are risk and vulnerability indicators. x_1 for cropping intensity, x_2 for shelter, x_3 for erosion and x_4 for population density.

The objective function of fuzzy linear programming problem is by Nahar, *et al.*

$$\max Z = (0, 0.00335, 0.067)x_1 + (0, 0.36, 0.72)x_2 + (-0.072, -0.036, 0)x_3 + (-0.1413, -0.07065, 0)x_4$$

With fuzzy constraints

$$(6.0877, 26, 45.9123)x_1 + (41.7423, 70, 98.2577)x_2 + (-5.1243, 7, 19.1243)x_3 + (3.7907, 13.75, 23.7093)x_4 \le (90.8769, 95, 99.1231)$$

Thus, the multiple objective functions become as like by Nahar, *et al.*

$$
\begin{aligned}
\max Z_1 &= 0\,x_1 + 0\,x_2 - 0.072\,x_3 - 0.1413\,x_4 \\
\max Z_2 &= 0.00335\,x_1 + 0.36\,x_2 - 0.036\,x_3 - 0.07065\,x_4 \\
\max Z_3 &= 0.067\,x_1 + 0.72\,x_2 + 0\,x_3 + 0\,x_4
\end{aligned} \tag{16}
$$

subject to

$$
\begin{aligned}
&6.0877\,x_1 + 41.7423\,x_2 + (-5.1243)\,x_3 + 3.7907\,x_4 \le 90.8769 \\
&(-19.9123)\,x_1 + (-28.2577)\,x_2 + (-12.1243)\,x_3 + (-9.9593)\,x_4 \le -4.1231 \\
&52\,x_1 + 140\,x_2 + 14\,x_3 + 27.5\,x_4 \le 190
\end{aligned} \tag{17}
$$

For the first objective function in Equation (16) with same constraints in Equation (17), by applying simplex algorithm we get

$$\phi_1 = 0 \quad \text{with } (0, 0.1459, 0, 0)$$

For the second objective function in Equation (16) with same constraints in Equation (17), by applying simplex algorithm we get

$$\phi_2 = 0.4886 \quad \text{with } (0, 1.3571, 0, 0)$$

Similarly for third objective function, in equation (16) with same constraints in Equation (17), we get

$$\phi_3 = 0.9771 \quad \text{with } (0, 1.3571, 0, 0)$$

4.1. Chandra Sen's Method

Applying Chandra Sen's method for making single objective function from multi objective functions

$$
\begin{aligned}
\max z &= \frac{z_1}{\phi_1} + \frac{z_2}{\phi_2} + \frac{z_3}{\phi_3} \\
&= 0.0754178x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4
\end{aligned}
$$

Thus, the single objective function becomes

$$\max Z = 0.0754178x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4 \tag{18}$$

For this objective function in Equation (18) with same constraints in Equation (17) we get the result 2 with (0, 1.3571, 0, 0).

4.2. Statistical Averaging Method

Applying arithmetic mean, geometric mean and harmonic mean among ϕ_1, ϕ_2, ϕ_3

$$
\begin{aligned}
A.M &= 0.48857 \\
G.M &= 0 \\
H.M &= 0.97716
\end{aligned}
$$

Arithmetic averaging method:

$$
\begin{aligned}
\max z &= \frac{1}{A.M}\left(z_1 + z_2 + z_3\right) \\
&= 0.154364x_1 + 3.016272x_2 - 0.150809x_3 - 0.295963x_4
\end{aligned}
$$

Thus, the single objective function becomes

$$\max Z = 0.154364x_1 + 3.016272x_2 - 0.150809x_3 - 0.295963x_4 \quad (19)$$

For this objective function in Equation (19) with same constraints in equation (17) we get the result 4.0935 with (0, 1.3571, 0, 0).

Harmonic averaging method:

$$\max z = \frac{1}{H.M}(z_1 + z_2 + z_3)$$
$$= 0.07718x_1 + 1.508099x_2 - 0.0754031x_3 - 0.147978x_4$$

Thus, the single objective becomes

$$\max Z = 0.07718x_1 + 1.508099x_2 - 0.0754031x_3 - 0.147978x_4 \quad (20)$$

For this objective function in Equation (20) with same constraints in equation (17) we get the result 2.0467 with (0, 1.3571, 0, 0).

It can be seen from "Table 3" that in statistical averaging method, arithmetic averaging and harmonic averaging gives better result than those Chandra Sen's method.

4.3. New Statistical Averaging Method

Choosing minimum from the optimal values of maximum type in Chandra Sen's method we get m = 0.4886

$$\max z = \frac{1}{0.4886}(z_1 + z_2 + z_3)$$
$$= 2.0467(0.075417x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4)$$
$$= 0.15435x_1 + 3.01614x_2 - 0.1508x_3 - 0..29595x_4$$

Thus, the single objective becomes

$$\max Z = 0.15435x_1 + 3.01614x_2 - 0.1508x_3 - 0.29595x_4 \quad (21)$$

For this objective function in Equation (21) with same constraints in equation (17) we get the result 4.0933 with (0, 1.3571, 0, 0).

It can be seen from "Table 4" that statistical averaging method and new statistical averaging method give better result than Chandra Sen's method.

Table 3. Comparison between Chandra Sen's method and statistical averaging method.

Chandra Sen's method	Arithmetic averaging method	Harmonic averaging method
Z_{max} = 2 with (0, 1.3571, 0, 0)	Z_{max} = 4.0935 with (0, 1.3571, 0, 0)	Z_{max} = 2.0467 with (0, 1.3571, 0, 0)

Table 4. Comparison among Chandra Sen's method, statistical averaging method and new statistical averaging method.

Chandra Sen's Method	Statistical Averaging Method (SAM)	New Statistical Averaging Method (NSAM)
Z_{max} = 2 with (0, 1.3571, 0, 0)	Z_{max} = 4.0935 with (0, 1.3571, 0, 0)	Z_{max} = 4.0933 with (0, 1.3571, 0, 0)

4.4. Solving Multi-Objective Linear Programming Problem by Fuzzy Programming Method

$$\begin{aligned} &\max Z_1 = 0x_1 + 0x_2 - 0.072x_3 - 0.1413x_4 \\ &\max Z_2 = 0.00335x_1 + 0.36x_2 - 0.036x_3 - 0.07065x_4 \\ &\max Z_3 = 0.067x_1 + 0.72x_2 + 0x_3 + 0x_4 \end{aligned} \tag{22}$$

subject to

$$\begin{aligned} &6.0877x_1 + 41.7423x_2 + (-5.1243)x_3 + 3.7907x_4 \le 90.8769 \\ &(-19.9123)x_1 + (-28.2577)x_2 + (-12.1243)x_3 + (-9.9593)x_4 \le -4.1231 \\ &52x_1 + 140x_2 + 14x_3 + 27.5x_4 \le 190 \end{aligned} \tag{23}$$

For the first objective function in Equation (22) with constraints in equation (23) by using simplex algorithm we get $z_1 = 0$ with (0, 0.1459, 0, 0).

Similarly for second objective function in equation (22) with same constraints in Equation (23) by using simplex algorithm we get $z_2 = 1.224$ with (3.6538, 0, 0, 0).

And for the last objective function in Equation (22) with same constraints in Equation (23) by using simplex algorithm we get $z_3 = 0.9771$ with (0, 1.3571, 0, 0).

A pay-off matrix is formulated as in "**Table 5**".

For the Pay-off matrix, lower and upper bound are established as:

$$\begin{aligned} &0 \le z_1 \le 0 \\ &0.0525 \le z_2 \le 1.224 \\ &0.10505 \le z_3 \le 0.9771 \end{aligned} \tag{24}$$

Using the membership functions as defined and introducing augmented variable λ a crisp model is formulated as

min λ

Subject to

$$\begin{aligned} &0x_1 + 0x_2 + (-0.072)x_3 - 0.1413x_4 \ge 0 \\ &0.00335x_1 + 0.36x_2 + (-0.036)x_3 + (-0.07065)x_4 + 1.1715\lambda \ge 1.224 \\ &52x_1 + 140x_2 + 14x_3 + 27.5x_4 + 0.872\lambda \ge 0.9771190 \end{aligned}$$

$$\begin{aligned} &6.0877x_1 + 41.7423x_2 + (-5.1243)x_3 + 3.7907x_4 \le 90.8769 \\ &(-19.9123)x_1 + (-28.2577)x_2 + (-12.1243)x_3 + (-9.9593)x_4 \le -4.1231 \\ &52x_1 + 140x_2 + 14x_3 + 27.5x_4 \le 190 \\ &x_1, x_2, x_3, x_4, \lambda \ge 0 \end{aligned} \tag{25}$$

Table 5. Pay-off matrix.

(x_1, x_2, x_3)	Z_1	Z_2	Z_3
(0, 0.1459, 0, 0)	0	0.0525	0.10505
(3.6538, 0, 0, 0)	0	1.224	0.245
(0, 1.3571, 0, 0)	0	0.488	0.9771

Using simplex algorithm, we get the result

$x_1 = 0, x_2 = 1.3571, x_3 = 0, x_4 = 0, \lambda = 0.6278$

Finally, the crisp model is solved to get the result

$\phi_1 = 0, \phi_2 = 0.4885, \phi_3 = 0.977$

Using the value of Fuzzy Programming method applying Chanda Sen's method

$$\max z = 0.0753x_1 + 1.47382x_2 - 0.0737x_3 - 0.1446x_4 \tag{26}$$

For the single objective function in Equation (26) with constraints in Equation (23), by using simplex algorithm we get $z = 2.002$ with (0, 1.3571, 0, 0).

Applying arithmetic mean formula by Nahar and Alim (2017) we get the single objective function

$$\max z = 0.096x_1 + 1.474x_2 - 0.1474x_3 - 0.289x_4 \tag{27}$$

For the single objective function in Equation (27) with constraints in equation (23), by using simplex algorithm we get $z = 2.004$ with (0, 1.3571, 0, 0).

Applying harmonic mean formula by Nahar and Alim (2017) we get the single objective function

$$\max z = 0.108x_1 + 1.658x_2 - 0.166x_3 - 0.325x_4 \tag{28}$$

For the single objective function in Equation (28) with constraints in Equation (23), by using simplex algorithm we get $z = 2.2501$ with (0, 1.3571, 0, 0).

4.5. New Statistical Averaging Method

Choosing minimum from the optimal values of maximum type in Chandra Sen's method we get $m = 0.4885$

$$\begin{aligned} \max z &= \frac{1}{0.2442}(z_1 + z_2 + z_3) \\ &= 4.094(0.07035x_1 + 1.08x_2 - 0.108x_3 - 0.21195x_4) \\ &= 0.288x_1 + 4.4215x_2 - 0.442x_3 - 0.8677x_4 \end{aligned} \tag{29}$$

For this objective function in Equation (29) with same constraints in equation (23) we get the result 6.006 with (0, 1.3571, 0, 0).

It can be seen from "**Table 6**" that statistical averaging method and new statistical averaging method give better result than Chandra Sen's method.

This table can be described in the following bar diagram in "**Figure 2**".

Table 6. Comparison among Chandra Sen's method, statistical averaging method and new statistical averaging method.

Methods	Chandra Sen's Method	Statistical Averaging Method (SAM)	New Statistical Averaging Method (NSAM)
Ordinary simplex method	$Z_{max} = 2$ with (0, 1.3571, 0, 0)	$Z_{max} = 4.0935$ with (0, 1.3571, 0, 0)	$Z_{max} = 4.0933$with (0, 1.3571, 0, 0)
Fuzzy programming method	$Z_{max} = 2$ with (0, 1.3571, 0, 0)	$Z_{max} = 2.2501$ with (0, 1.3571, 0, 0)	$Z_{max} = 6.006$ with (0, 1.3571, 0, 0)

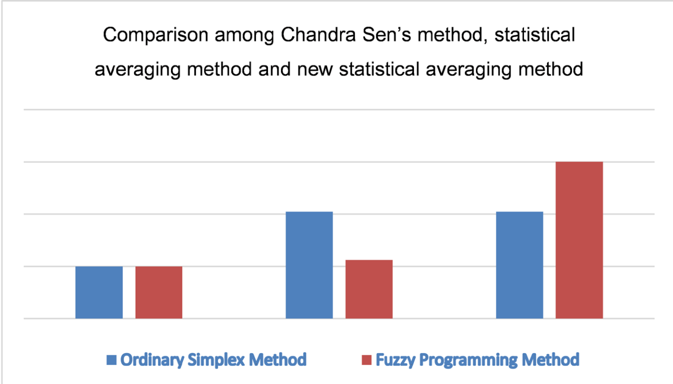

Figure 2. Optimal values of MOLPP by using ordinary and fuzzy programming method.

5. Conclusion

For finding optimal values here we apply simplex method and fuzzy programming method. Fuzzy programming method gives better optimal values than simplex method. Our next stage is making single objective function from multi-objective functions. For this we apply statistical averaging method and new statistical method. It can be seen that statistical averaging method gives better result than Chandra Sen's method in both ordinary simplex method and in fuzzy programming method. To reduce risk in the coastal region fuzzy programming method is used here. It can be seen that risk reduction capacity is maximized in fuzzy programming method.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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