

# Solving Fuzzy Multi-Objective Linear Programming Problem by Applying Statistical Method

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## Abstract

In this paper, the statistical averaging method and the new statistical averaging methods have been used to solve the fuzzy multi-objective linear programming problems. These methods have been applied to form a single objective function from the fuzzy multi-objective linear programming problems. At first, a numerical example of solving fuzzy multi-objective linear programming problem has been provided to validate the maximum risk reduction by the proposed method. The proposed method has been applied to assess the risk of damage due to natural calamities like flood, cyclone, sidor, and storms at the coastal areas in Bangladesh. The proposed method of solving the fuzzy multi-objective linear programming problems by the statistical method has been compared with the Chandra Sen's method. The numerical results show that the proposed method maximizes the risk reduction capacity better than Chandra Sen's method.

## Keywords

Fuzzy Multi-Objective Linear Programming Problem, Fuzzy Linear Programming Problem, Chandra Sen's Method, Statistical Averaging Method, New Statistical Averaging Method

## 1. Introduction

Real world circumstances are not always deterministic. There exist different kinds of uncertainties in social, industrial and economic systems. Different kinds of uncertainties are defined as stochastic uncertainty and fuzziness [1]. A system with a stochastic uncertainty is solved by the stochastic optimization technique

using the probability theory. Also, fuzzy programming technique is widely used to solve problem with uncertainty. A system with uncertainty can be optimized to reduce the risk factors in the system with fuzzy conditions. An optimization of a system with uncertainty using fuzzy conditions is called a fuzzy Optimization. Modelling under a fuzzy environment is called fuzzy modelling. Fuzzy linear programming is one of the most frequently applied fuzzy decision making techniques. We get fuzzy linear programming problem (FLPP) by interchanging the parameters of linear programming problem by fuzzy numbers. Several methods have been proposed in the literature to solve FLPP.

Firstly, Bellman and Zadeh in [2] proposed decision making in fuzzy condition. They explained the use of decision making in fuzzy condition by using a controlled system which is either stochastic or deterministic with multistage decision processes, Zimmermann proposed a formulation of FLPP using theory of fuzzy sets. He used FLPP to the linear vector maximum problem and found that the solutions obtained using FLPP are always efficient. Considering parameters ambiguity, Tanaka and Asai also proposed a formulation of FLPP to find a logical solution [3]. They highlighted that the FLPP with fuzzy numbers may be considered as a model of decision problems in a system with influential human estimations. Not only linear programming problems (LPP), but also multi-objective LPP (MOLPP) and multi-objective nonlinear programming problems were solved using fuzzy approaches which are available in the literature.

Laganathan and Lalitha [4] solved a multi-objective nonlinear programming problem using  $\alpha$ -cut method in fuzzy approach and compared the solution with the solution obtained by Zimmermann [5] who used membership function. Behr *et al.* [6] also used  $\alpha$ -cut method to solve multi-objective linear programming problem (MOLPP) in fuzzy approach and compared the solutions to the solution obtained by Zimmermann [5] who used membership function.

Thakre *et al.* [7] solved an objective function with constraint matrix and cost coefficients which are fuzzy in nature. They used MOLPP with the constraints to solve FLPP and proved that the solutions are independent of weights. Nahar and Alim [8] discussed Chandra Sen's approach, statistical averaging method and new statistical averaging method to solve MOLPP. For MOLFP, a new geometric average technique was proposed by Nahar and Alim [9].

Veeramani and Sumathi [10] solved fuzzy linear fractional programming problem (FLFPP) where the cost, resources and technological coefficients of the objective function were triangular fuzzy numbers. In the solution procedure, they converted the FLFPP into a multi-objective linear fractional programming problem.

There are a lot of methods for solving fully fuzzy linear programming problems in the literature. Ebrahim nejad and Tavana [11] converted the FLPP into an equivalent crisp linear programming problem and solved by simplex method. Here they proposed a new concept in which the coefficients of objective function and the values of the right hand side are represented by trapezoidal fuzzy numbers and other parts are represented by real numbers. A general form of fuzzy

linear fractional programming problem with trapezoidal fuzzy numbers is proposed by Das [12].

Lotfi *et al.* [13] proposed a method to obtain the approximate solution of fully fuzzy linear programming problems. A method to solve fully fuzzy linear programming problems is proposed by Amit Kumar *et al.* [14] using idea of crisp linear programming and ranking function. FLPP is solved by S. Nahar *et al.* [15]. To solve FLPP, Nahar, S. *et al.* [16] used weighted sum method. Here both equal and unequal weight has been used. In this paper there is a discussion for ranking function. Here they used triangular and trapezoidal fuzzy number. Sen, C. [17] developed averaging technique of multi-objective optimization for rural development planning. Akter, M. *et al.* [18] proposed fuzzy synthetic evaluation method for risk assessment on the natural hazard. Nishad, A. [19] used alpha cut fuzzy number for solving fractional programming problem in fuzzy field. Pitam, S., [20] developed goal programming approach for fuzzy multi-objective linear and fuzzy multi-objective linear programming problem.

A good averaging represents maximum characteristics of the data. There are five averages, among them mean, median, mode are called simple averages. Geometric mean and harmonic mean are called special averages. In this work we consider arithmetic mean, geometric mean and harmonic mean. In this paper, we solve fuzzy multi-objective linear programming problem by applying Chandra Sen's method, statistical averaging method and new statistical averaging method. These methods are applied for making single objective from multi-objective linear programming problem. A real life example is given to reduce risk in coastal region. From that data list a FLPP is structured. From that FLPP we convert into MOLPP.

## 2. Concept of a Fuzzy Set

The concept of a fuzzy set is an extension of the concept of a crisp set. A crisp set on a universal set  $U$  is defined by its characteristic function from  $U$  to  $\{0, 1\}$ . A fuzzy set on a domain  $U$  is defined by its membership function from  $U$  to  $[0, 1]$ . Let  $U$  be a nonempty set, to be called the universal set or the universe of discourse or simply a domain. Then by a fuzzy set on  $U$  is meant a function  $A: U \rightarrow [0, 1]$ .  $A$  is called the membership function,  $A(x)$  is called the membership grade of  $x$  in  $(U, A)$ . We also write  $A = \{(A, A(x)): x \in U\}$ .

## 3. Fuzzy Linear Programming Problem

A linear programming problem with fuzzy values is called fuzzy linear programming problem. In this paper, any fuzzy number is denoted by using  $\tilde{\sim}$  above fuzzy number, e.g.,  $\tilde{c}_j$ ,  $\tilde{a}_{ij}$  etc. Consider a fuzzy linear programming problem as in Equation (1).

$$(\tilde{c}, x) = f_i(x_j) = f_i(x) = \max \tilde{z} = \sum_{j=1}^n \tilde{c}_j x_j$$

subject to

$$\sum_{j=1}^n \tilde{a}_{ij} x_j \leq \tilde{b}_i; 1 \leq i \leq m, \exists x_j > 0 \tag{1}$$

The membership functions of  $a_{ij}$  and  $b_i$  have been expressed as in Equation (2) and (3), respectively.

$$\mu_{a_{ij}}(x) = \begin{cases} 1; & x < a_{ij} \\ \frac{a_{ij} + d_{ij} - x}{d_{ij}}; & a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0; & x \geq a_{ij} + d_{ij} \end{cases} \tag{2}$$

$$\mu_{b_i}(x) = \begin{cases} 1; & x \leq b_i \\ \frac{b_i + p_i - x}{p_i}; & b_i \leq x \leq b_i + p_i \\ 0; & b_i + p_i \leq x \end{cases} \tag{3}$$

Let  $\tilde{a}_{ij} = (m_{ij}, l_{ij}, r_{ij})$  and  $\tilde{b}_i = (d_i, e_i, f_i)$  be fuzzy numbers. Therefore, the constraints in Equation (1) can be modified as in Equation (4).

$$\begin{aligned} \max \tilde{z} &= \sum_{j=1}^n \tilde{c}_j x_j \\ \text{subject to} & \\ \sum (m_{ij}, l_{ij}, r_{ij}) x_j &\leq (d_i, e_i, f_i) \quad \forall i = 1 - m \\ x_j &\geq 0, j = 1 - n \end{aligned} \tag{4}$$

Theorem: For any two triangular fuzzy numbers Thakre *et al.* [8]  $A = (s_1, l_1, r_1)$  and  $B = (s_2, l_2, r_2)$

$$\begin{aligned} A \leq B \text{ iff } & s_1 \leq s_2 \\ & s_1 - l_1 \leq s_2 - l_2 \\ & s_1 + r_1 \leq s_2 + r_2 \end{aligned} \tag{5}$$

$$\begin{aligned} \max \tilde{z} &= \sum_{j=1}^n \tilde{c}_j x_j \\ \text{subject to} & \\ \sum m_{ij} x_j &\leq d_i \\ \sum (m_{ij} - l_{ij}) x_j &\leq d_i - e_i \quad \forall i \\ \sum (m_{ij} + r_{ij}) x_j &\leq d_i + f_i \\ x_j &\geq 0 (\forall j) \end{aligned} \tag{6}$$

where membership function of  $\tilde{c}_j(x)$  is

$$\tilde{\mu}_{c_j} = \begin{cases} \frac{x - \alpha_i}{\beta_i - \alpha_i}; & \alpha_i \leq x \leq \beta_i \\ \frac{\gamma_i - x}{\gamma_i - \beta_i}; & \beta_i \leq x \leq \gamma_i \\ 0; & \text{elsewhere} \end{cases} \tag{7}$$

#### 4. Numerical Examples

Consider the fuzzy multi-objective linear programming problem (FMOLPP) as in Equation (8).

$$\begin{aligned}
 \max \tilde{z}^1 &= (7, 10, 14)x_1 + (20, 25, 35)x_2 \\
 \max \tilde{z}^2 &= (10, 14, 25)x_1 + (25, 35, 40)x_2 \\
 \text{Subject to :} & \\
 (3, 2, 1)x_1 + (6, 4, 1)x_2 &\leq (13, 5, 2) \\
 (4, 1, 2)x_1 + (6, 5, 4)x_2 &\leq (7, 4, 2)
 \end{aligned} \tag{8}$$

where the membership function of  $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{c}_4$  are

$$\mu_{\tilde{c}_1}(x) = \begin{cases} \frac{x-7}{3}; & 7 < x \leq 10 \\ \frac{14-x}{4}; & 10 < x \leq 14 \\ 0; & \text{elsewhere} \end{cases} \tag{9}$$

$$\mu_{\tilde{c}_2}(x) = \begin{cases} \frac{x-20}{5}; & 20 < x \leq 25 \\ \frac{35-x}{10}; & 25 < x \leq 35 \\ 0; & \text{elsewhere} \end{cases} \tag{10}$$

$$\mu_{\tilde{c}_3}(x) = \begin{cases} \frac{x-10}{4}; & 10 < x \leq 14 \\ \frac{25-x}{9}; & 14 < x \leq 25 \\ 0; & \text{elsewhere} \end{cases} \tag{11}$$

$$\mu_{\tilde{c}_4}(x) = \begin{cases} \frac{x-25}{5}; & 25 < x \leq 35 \\ \frac{40-x}{5}; & 35 < x \leq 40 \\ 0; & \text{elsewhere} \end{cases} \tag{12}$$

From Equation (8) we get,

$$\begin{aligned}
 \max z_1 &= 7x_1 + 20x_2 \\
 \max z_2 &= 10x_1 + 25x_2 \\
 \max z_3 &= 14x_1 + 35x_2 \\
 \max z_4 &= 25x_1 + 40x_2
 \end{aligned} \tag{13}$$

Subject to

$$\begin{aligned}
 3x_1 + 6x_2 &\leq 13 \\
 x_1 + 2x_2 &\leq 8 \\
 4x_1 + 7x_2 &\leq 15 \\
 4x_1 + 6x_2 &\leq 7 \\
 3x_1 + x_2 &\leq 3 \\
 x_1 + 10x_2 &\leq 9 \\
 x_1, x_2 &\geq 0
 \end{aligned} \tag{14}$$

For the first objective function in Equation (13) with constraints in Equation (14), by applying simplex algorithm we get

$$\phi_1 = 20.3529 \text{ with } (0.4706, 0.8529)$$

Similarly, for the second objective function in Equation (13) with constraints in Equation (14), we get

$$\phi_2 = 26.0294 \text{ with } (0.4706, 0.8529)$$

Similarly for third objective function in Equation (13) with constraints in Equation (14), we get

$$\phi_3 = 36.4412 \text{ with } (0.4706, 0.8529)$$

And for last objective function in Equation (13) with constraints in Equation (14), we get

$$\phi_4 = 45.8824 \text{ with } (0.4706, 0.8529)$$

#### 4.1. Chandra Sen's Method

Chandra Sen's method is a multi-objective optimization technique which is used for making single objective from multi-objectives. In the last three decades several new multi-objective optimization techniques have been developed.

Applying Chandra Sen's method [17] for making single objective function from multi objective function

$$\begin{aligned} \max z &= \frac{z_1}{\phi_1} + \frac{z_2}{\phi_2} + \frac{z_3}{\phi_3} + \frac{z_4}{\phi_4} \\ &= \frac{1}{20.3529}(7x_1 + 20x_2) + \frac{1}{26.0294}(10x_1 + 25x_2) \\ &\quad + \frac{1}{36.4412}(14x_1 + 35x_2) + \frac{1}{45.8824}(25x_1 + 40x_2) \\ &= x_1(0.344 + 0.384 + 0.384 + 0.545) + x_2(0.983 + 0.96 + 0.96 + 0.872) \\ &= 1.657x_1 + 3.775x_2 \end{aligned}$$

Thus the single objective function becomes

$$\max z = 1.657x_1 + 3.775x_2 \quad (15)$$

For this objective function in Equation (15) with constraints in Equation (14), we get

$$z = 3.9996 \text{ with } (0.4706, 0.8529)$$

#### 4.2. Statistical Averaging Method

There are three mean known as arithmetic mean, geometric mean and harmonic mean. For ungrouped raw data, mean is defined as the sum of the objectives divided by the number of observations. It is easy to understand and easy to calculate. If the number of items is sufficiently large, it is more accurate and more reliable. Geometric mean of a series containing  $n$  observations is the  $n$ th root of the product of the values. Harmonic mean of a set of observations is defined as the reciprocal of the arithmetic average of the reciprocal of the given values.

Applying arithmetic mean, geometric mean and harmonic mean among  $\phi_1, \phi_2, \phi_3, \phi_4$

$$A.M = \frac{20.3529 + 26.0294 + 36.412 + 45.8824}{4} = 32.176$$

$$G.M = \sqrt[4]{20.3529 \times 26.0294 \times 36.412 \times 45.8824} = 30.68$$

$$H.M = \frac{4}{\frac{1}{20.3529} + \frac{1}{26.0294} + \frac{1}{36.412} + \frac{1}{45.8824}}$$

$$= \frac{4}{0.049 + 0.038 + 0.027 + 0.0217} = 29.477$$

*Arithmetic averaging method:*

$$\begin{aligned} \max z &= \frac{1}{A.M} (z_1 + z_2 + z_3 + z_4) \\ &= \frac{1}{32.176} (7x_1 + 20x_2 + 10x_1 + 25x_2 + 14x_1 + 35x_2 + 25x_1 + 40x_2) \\ &= \frac{1}{32.176} [56x_1 + 120x_2] \\ &= 1.740x_1 + 3.729x_2 \end{aligned}$$

Thus the single objective function becomes

$$\max z = 1.740x_1 + 3.729x_2 \quad (16)$$

For this objective function in Equation (16) with constraints in Equation (14), we get

$$z = 3.9994 \text{ with } (0.4706, 0.8529)$$

*Geometric averaging method:*

$$\begin{aligned} \max z &= \frac{1}{G.M} (z_1 + z_2 + z_3 + z_4) = \frac{1}{G.M} (56x_1 + 120x_2) \\ &= \frac{1}{30.68} (56x_1 + 120x_2) = 1.825x_1 + 3.911x_2 \end{aligned}$$

Thus the single objective function becomes

$$\max z = 1.825x_1 + 3.911x_2 \quad (17)$$

For this objective function in Equation (17) with constraints in Equation (14), we get

$$z = 4.1947 \text{ with } (0.4706, 0.8529)$$

*Harmonic averaging method:*

$$\begin{aligned} \max z &= \frac{1}{H.M} (z_1 + z_2 + z_3 + z_4) = \frac{1}{H.M} (56x_1 + 120x_2) \\ &= \frac{1}{29.477} (56x_1 + 120x_2) = 1.899x_1 + 4.071x_2 \end{aligned}$$

Thus the single objective becomes

$$\max z = 1.899x_1 + 4.071x_2 \quad (18)$$

For this objective function in Equation (18) with constraints in Equation (14), we get

$$z = 4.3660 \text{ with } (0.4706, 0.8529)$$

**Table 1** shows the comparison between the Chandra Sen’s method and the statistical averaging methods. Here statistical averaging method consists of arithmetic mean, geometric mean and harmonic mean. The statistical averaging method gives better result than the Chandra Sen’s method. These methods are used for obtaining a single objective function from multi-objective functions. We can see the comparison graphically “as shown in **Figure 1**”.

### 4.3. New Statistical Averaging Method

Choosing minimum from the optimal values of maximum type in Chandra Sen’s method we get

$$\begin{aligned} \max z &= \frac{1}{20.3529}(z_1 + z_2 + z_3 + z_4) \\ &= 0.04913(56x_1 + 120x_2) \\ &= 2.75128x_1 + 5.8956x_2 \end{aligned}$$

Thus the single objective becomes

$$\max z = 2.75128x_1 + 5.8956x_2 \tag{19}$$

For this objective function in Equation (19) with constraints in Equation (14), we get

$$z = 6.3233 \text{ with } (0.4706, 0.8529)$$

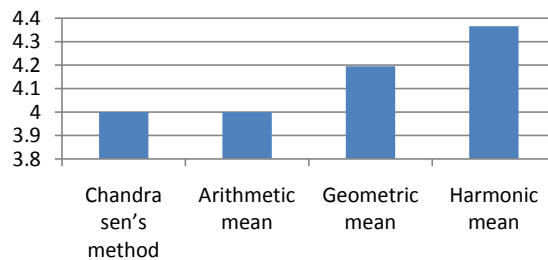
We can find a single objective function from multi objective functions by using any method among Chandra Sen’s method, statistical averaging method, and new statistical averaging method. The **Table 2** shows that the statistical averaging method and the new statistical averaging method gives better optimization than the Chandra Sen’s method.

## 5. Reduce Damage and Loses at the Coastal Area in Bangladesh

**Table 3** shows secondary data of four parameters such as cropping intensity,

**Table 1.** Comparison between Chandra Sen’s method and statistical averaging method.

Chandra Sen’s method	Arithmetic mean	Geometric mean	Harmonic mean
3.9996	3.9994	4.1947	4.3660



**Figure 1.** Optimal values of FLPP in Chandra Sen’s method and statistical averaging method.



**Table 2.** Comparison among Chandra Sen's method, statistical averaging method, and new statistical averaging method.

Chandra Sen's Method	Statistical Averaging Method (SAM)	New Statistical Averaging Method (NSAM)
$Z_{\max} = 3.9996$ with (0.4706, 0.8529)	$Z_{\max} = 4.3660$ with (0.4706, 0.8529)	$Z_{\max} = 6.3233$ with (0.4706, 0.8529)

**Table 3.** Data based on the data of four coastal areas.

Coastal areas	Cropping intensity	Shelter	Erosion	Population density
kutubdia	13	100	0	10
Mahekl	2	37	0	18
Pekua	6	47	0	27
Manpura	5	96	28	0
sum	26	280	28	55 = 389
	$26/389 = 0.067$	$280/389 = 0.72$	$28/389 = 0.072$	$55/389 = 0.1413$

shelter, erosion, and population density at four coastal areas in Bangladesh. These data set have been collected from IWFm (BUET). The objective in this paper is to reduce the risk and hazard of the coastal area during natural disasters by solving FMOLPP. The targets set in this paper are to maximize cropping intensity and shelter and to minimize erosion and population density. For these purposes, those four parameters have been defined using four variables which are defined as decision variables.

Our decision variables are  $x_1, x_2, x_3, x_4$  which are risk and vulnerability indicators.  $x_1$  for cropping intensity,  $x_2$  for shelter,  $x_3$  for erosion and  $x_4$  for population density. Constant vector is terminated value for risk reduction capacity (on expert opinion). Decision variables are in different scales. For this reason, these variables have been normalized. Constraints are established based on the data of four coastal areas. The risk reduction coefficients of the objective functions are calculated using weighted method.

$$\begin{aligned} \max Z &= 0.067x_1 + 0.72x_2 \\ \min Z &= 0.072x_3 + 0.1413x_4 \end{aligned} \quad (20)$$

Rearranging Equation (20), we get

$$\begin{aligned} \max Z_1 &= 0.067x_1 + 0.72x_2 + 0x_3 + 0x_4 \\ \max Z_2 &= 0x_1 + 0x_2 - 0.072x_3 - 0.1413x_4 \end{aligned} \quad (21)$$

### 5.1. Finding Fuzzy Objective Function for Fuzzy Linear Programming Problem

Writing Equation (21) in matrix form, we get

$$\max \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 0.067 & 0.72 & 0 & 0 \\ 0 & 0 & -0.072 & -0.1413 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (22)$$

Triangular fuzzy number for the coefficient of  $x_1$  is found from **Table 4**.

Standard deviation,

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{0.001122 + 0.001122}{2}} = \sqrt{0.001122} = 0.0335$$

$$(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (0.0335 - 0.0335, 0.0335, 0.0335 + 0.0335) = (0, 0.0335, 0.067), \text{ triangular fuzzy}$$

number.

Triangular fuzzy number for the coefficient of  $x_2$  is found from **Table 5**.

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{0.1296 + 0.1296}{2}} = \sqrt{0.1296} = 0.36$$

$$(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (0.36 - 0.36, 0.36, 0.36 + 0.36) = (0, 0.36, 0.72), \text{ triangular fuzzy}$$

number.

Triangular fuzzy number for the coefficient of  $x_3$  is found from **Table 6**.

Standard deviation,

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{0.001296 + 0.001296}{2}} = \sqrt{0.001296} = 0.036$$

$$(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (-0.036 - 0.036, -0.036, -0.036 + 0.036) = (-0.072, -0.036, 0), \text{ triangular fuzzy}$$

number.

Triangular fuzzy number for the coefficient of  $x_4$  is found from **Table 7**.

Standard deviation,

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{0.00499 + 0.00499}{2}} = \sqrt{0.00499} = 0.07065$$

$$(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (-0.07065 - 0.07065, -0.07065, -0.07065 + 0.07065) = (-0.1413, -0.07065, 0), \text{ triangular}$$

number.

The objective function of fuzzy linear programming problem is

$$\begin{aligned} \max Z = & (0, 0.00335, 0.067)x_1 + (0, 0.36, 0.72)x_2 \\ & + (-0.072, -0.036, 0)x_3 + (-0.1413, -0.07065, 0)x_4 \end{aligned}$$

Thus the multiple objective functions become as like Equation (13)

$$\begin{aligned} \max Z_1 &= 0x_1 + 0x_2 - 0.072x_3 - 0.1413x_4 \\ \max Z_2 &= 0.00335x_1 + 0.36x_2 - 0.036x_3 - 0.07065x_4 \\ \max Z_3 &= 0.067x_1 + 0.72x_2 + 0x_3 + 0x_4 \end{aligned} \quad (23)$$

## 5.2. Finding Constraints of Fuzzy Linear Programming Problem

Triangular fuzzy number for the coefficient of  $x_1$  is found from **Table 8**.

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{1586}{4}} = \sqrt{396.5} = 19.9123$$

$$(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (26 - 19.9123, 26, 26 + 19.9123) = (6.0877, 26, 45.9123), \text{ triangular}$$

number.

Triangular fuzzy number for the coefficient of  $x_2$  is found from **Table 9**.

**Table 4.** Coefficient of  $x_1$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0.067	0.0335	0.0335	0.001122
0		-0.0335	0.001122

**Table 5.** Coefficient of  $x_2$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0.72	0.36	0.36	0.1296
0		-0.36	0.1296

**Table 6.** Coefficient of  $x_3$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0	-0.036	0.036	0.001296
-0.072		-0.036	0.001296

**Table 7.** Coefficient of  $x_4$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0	-0.0706536	0.07065	0.00499
-0.1413		-0.07065	0.00499

**Table 8.** Coefficient of  $x_1$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
13	26	-13	169
2		-24	576
6		-20	400
5		-21	441

**Table 9.** Coefficient of  $x_2$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
100	70	30	900
37		-33	1089
47		-23	529
96		26	676

Standard deviation,  $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{2}} = \sqrt{\frac{3194}{4}} = \sqrt{798.5} = 28.2577$   
 $(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (70 - 28.2577, 70, 70 + 28.2577) = (41.7423, 70, 98.2577)$ ,  
 triangular fuzzy number.

Triangular fuzzy number for the coefficient of  $x_3$  is found from **Table 10**.

Standard deviation,  $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{588}{4}} = \sqrt{147} = 12.1243$   
 $(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (7 - 12.1243, 7, 7 + 12.1243) = (-5.1243, 7, 19.1243)$ , triangular fuzzy number.

Triangular fuzzy number for the coefficient of  $x_4$  is found from **Table 11**.

Standard deviation,  $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{396.75}{4}} = \sqrt{99.1875} = 9.9593$   
 $(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (13.75 - 9.9593, 13.75, 13.75 + 9.9593) = (3.7907, 13.75, 23.7093)$ , triangular fuzzy number.

Triangular fuzzy number for the constant term is found from **Table 12**.

Standard deviation,  $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{2}} = \sqrt{\frac{68}{4}} = \sqrt{17} = 4.1231$   
 $(\bar{x} - \sigma, \bar{x}, \bar{x} + \sigma) = (95 - 4.1231, 95, 95 + 4.1231) = (90.8769, 95, 99.1231)$ , triangular fuzzy number.

Fuzzy constraints become

$$\begin{aligned} &(6.0877, 26, 45.9123)x_1 + (41.7423, 70, 98.2577)x_2 \\ &+ (-5.1243, 7, 19.1243)x_3 + (3.7907, 13.75, 23.7093)x_4 \\ &\leq (90.8769, 95, 99.1231) \end{aligned} \tag{24}$$

Equation (24) is as like (25)

$$(a, b, c)x_1 + (d, e, f)x_2 + (g, h, i)x_3 + (j, k, l)x_4 \leq (m, n, o) \tag{25}$$

which becomes with the help of Equation (6)

$$\begin{aligned} &ax_1 + dx_2 + gx_3 + jx_4 \leq m \\ &(a - b)x_1 + (d - e)x_2 + (g - h)x_3 + (j - k)x_4 \leq m - n \\ &(a + c)x_1 + (d + f)x_2 + (g + i)x_3 + (j + l)x_4 \leq m + o \end{aligned} \tag{26}$$

**Table 10.** Coefficient of  $x_3$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0	7	-7	49
0		-7	49
0		-7	49
28		21	441

**Table 11.** Coefficient of  $x_4$ .

$x_i$	mean, $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
10	13.75	-3.75	14.0625
18		4.25	18.0625
27		13.25	175.5625
0		-13.75	189.0625

**Table 12.** Constant term.

$b_i$	mean, $\bar{b}$	$b_i - \bar{b}$	$(b_i - \bar{b})^2$
100		5	25
90	95	-5	25
98		3	9
92		-3	9

First equation of (26) becomes

$$ax_1 + dx_2 + gx_3 + jx_4 \leq m$$

$$6.0877x_1 + 41.7423x_2 + (-5.1243)x_3 + 3.7907x_4 \leq 90.8769$$

Second equation of (26) becomes

$$(a-b)x_1 + (d-e)x_2 + (g-h)x_3 + (j-k)x_4 \leq m-n$$

$$(6.0877-26)x_1 + (41.7423-70)x_2 + (-5.1243-7)x_3$$

$$+(3.7907-13.75)x_4 \leq 90.8769-95$$

$$(-19.9123)x_1 + (-28.2577)x_2 + (-12.1243)x_3 + (-9.9593)x_4 \leq -4.1231$$

Third equation of (26) becomes

$$(a+c)x_1 + (d+f)x_2 + (g+i)x_3 + (j+l)x_4 \leq m+o$$

$$(6.0877+45.9123)x_1 + (41.7423+98.2577)x_2 + (-5.1243+19.1243)x_3$$

$$+(3.7907+23.7093)x_4 \leq 90.8769+99.1231$$

$$52x_1 + 140x_2 + 14x_3 + 27.5x_4 \leq 190$$

The Equation (18) is written as the following Fuzzy MOLPP

$$\max Z_1 = 0x_1 + 0x_2 - 0.072x_3 - 0.1413x_4$$

$$\max Z_2 = 0.00335x_1 + 0.36x_2 - 0.036x_3 - 0.07065x_4 \quad (27)$$

$$\max Z_3 = 0.067x_1 + 0.72x_2 + 0x_3 + 0x_4$$

subject to

$$6.0877x_1 + 41.7423x_2 + (-5.1243)x_3 + 3.7907x_4 \leq 90.8769$$

$$(-19.9123)x_1 + (-28.2577)x_2 + (-12.1243)x_3 + (-9.9593)x_4 \leq -4.1231 \quad (28)$$

$$52x_1 + 140x_2 + 14x_3 + 27.5x_4 \leq 190$$

For the first objective function in Equation (27) with same constraints in Equation (28), by applying simplex algorithm we get

$$\phi_1 = 0 \quad \text{with } (0, 0.1459, 0, 0)$$

For the second objective function in Equation (27) with same constraints in Equation (28), by applying simplex algorithm we get

$$\phi_2 = 0.4886 \quad \text{with } (0, 1.3571, 0, 0)$$

Similarly for third objective function, in Equation (27) with same constraints in Equation (28), we get

$$\phi_3 = 0.9771 \quad \text{with } (0, 1.3571, 0, 0)$$

### 5.3. Chandra Sen's Method

Applying Chandra Sen's method for making single objective function from multi objective functions

$$\begin{aligned}\max z &= \frac{z_1}{\phi_1} + \frac{z_2}{\phi_2} + \frac{z_3}{\phi_3} \\ &= 0.0754178x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4\end{aligned}$$

Thus the single objective function becomes

$$\max Z = 0.0754178x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4 \quad (29)$$

For this objective function in Equation (29) with same constraints in Equation (28) we get the result 2 with (0, 1.3571, 0, 0).

### 5.4. Statistical Averaging Method

Applying arithmetic mean, geometric mean and harmonic mean among  $\phi_1, \phi_2, \phi_3$

$$A.M = 0.48857$$

$$G.M = 0$$

$$H.M = 0.97716$$

*Arithmetic averaging method:*

$$\begin{aligned}\max z &= \frac{1}{A.M}(z_1 + z_2 + z_3) \\ &= 0.154364x_1 + 3.016272x_2 - 0.150809x_3 - 0.295963x_4\end{aligned}$$

Thus the single objective function becomes

$$\max Z = 0.154364x_1 + 3.016272x_2 - 0.150809x_3 - 0.295963x_4 \quad (30)$$

For this objective function in Equation (30) with same constraints in Equation (28) we get the result 4.0935 with (0, 1.3571, 0, 0)

*Harmonic averaging method:*

$$\begin{aligned}\max z &= \frac{1}{H.M}(z_1 + z_2 + z_3) \\ &= 0.07718x_1 + 1.508099x_2 - 0.0754031x_3 - 0.147978x_4\end{aligned}$$

Thus the single objective becomes

$$\max Z = 0.07718x_1 + 1.508099x_2 - 0.0754031x_3 - 0.147978x_4 \quad (31)$$

For this objective function in Equation (26) with same constraints in Equation (23) we get the result 2.0467 with (0, 1.3571, 0, 0).

**Table 13** shows that the statistical averaging method, the arithmetic averaging, and the harmonic averaging gives better result than the Chandra Sen's method.

**Table 13.** Comparison between Chandra Sen's method and statistical averaging method.

Chandra Sen's method	Arithmetic averaging method	Harmonic averaging method
$Z_{\max} = 2$ with (0, 1.3571, 0, 0)	$Z_{\max} = 4.0935$ with (0, 1.3571, 0, 0)	$Z_{\max} = 2.0467$ with (0, 1.3571, 0, 0)

**Table 14.** Comparison among Chandra Sen's method, statistical averaging method, and new statistical averaging method.

Chandra Sen's method	Statistical averaging method (SAM)	New statistical averaging method (NSAM)
$Z_{\max} = 2$ with (0, 1.3571, 0, 0)	$Z_{\max} = 4.0935$ with (0, 1.3571, 0, 0)	$Z_{\max} = 4.0933$ with (0, 1.3571, 0, 0)

## 5.5. New Statistical Averaging Method

Choosing minimum from the optimal values of maximum type in Chandra Sen's method we get  $m = 0.4886$

$$\begin{aligned} \max z &= \frac{1}{0.4886}(z_1 + z_2 + z_3) \\ &= 2.0467(0.075417x_1 + 1.47366x_2 - 0.0736812x_3 - 0.144599x_4) \\ &= 0.15435x_1 + 3.01614x_2 - 0.1508x_3 - 0.29595x_4 \end{aligned}$$

Thus the single objective becomes

$$\max Z = 0.15435x_1 + 3.01614x_2 - 0.1508x_3 - 0.29595x_4 \quad (32)$$

For this objective function in Equation (27) with same constraints in Equation (23) we get the result 4.0933 with (0, 1.3571, 0, 0).

**Table 14** shows that the statistical averaging method and the new statistical averaging method give better result than the Chandra Sen's method.

## 6. Conclusion

In this paper, a fuzzy multi-objective linear programming problem has been solved using the Chandra Sen's method, the statistical averaging method, and the new statistical averaging method. The fuzzy multi-objective linear programming problem with constraints has been established from the data of cropping intensity, shelter, erosion, and population density at four coastal areas in Bangladesh. To maximize the risk reduction capacity, those data of cropping intensity, shelter, erosion, and population density have been set as fuzzy parameters which are triangular fuzzy numbers. The solutions obtained using the statistical averaging method and the new statistical averaging method are better than that of the Chandra Sen's method.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

- [1] Zimmermann, H.J. (1978) Fuzzy Programming and Linear Programming with Several Objective Functions. *Fuzzy Sets and System*, **1**, 45-55.  
[https://doi.org/10.1016/0165-0114\(78\)90031-3](https://doi.org/10.1016/0165-0114(78)90031-3)
- [2] Bellman, R.E. and Zadeh, L.A. (1970) Decision-Making in Fuzzy Environment. *Man-*

- agement Science, **17**, 141-164. <https://doi.org/10.1287/mnsc.17.4.B141>
- [3] Tanaka, H. and Asai, K. (1984) Fuzzy Linear Programming Problems with Fuzzy Numbers. *Fuzzy Sets and Systems*, **13**, 1-10. [https://doi.org/10.1016/0165-0114\(84\)90022-8](https://doi.org/10.1016/0165-0114(84)90022-8)
- [4] Loganathan, C. and Lalitha, M. (2016) Solving Multi-Objective Mathematical Programming Problems in Fuzzy Approach. *Introduction Journal of Mathematics and Its Applications*, **3**, 21-25.
- [5] Zimmermann, H.J. (1983) Fuzzy Sets in Operational Research. *EJOR*, **13**, 201-216. [https://doi.org/10.1016/0377-2217\(83\)90048-6](https://doi.org/10.1016/0377-2217(83)90048-6)
- [6] Behera, S.K., et al. () Mathematical Programming Problem in Fuzzy Approach. *International Journal on CSE*, 3790-3799.
- [7] Behera, et al. (2021) Solution of Multi-Objective Mathematical Programming Problems in Fuzzy Approach. *International Journal on Computer Science and Engineering*, **3**, 3790-3799.
- [8] Thakre, P.A., Shelar, D.S. and Thakre, S.P. (2009) Solving Fuzzy Linear Programming Problem as MOLPP. *World Congress on Engineering*, London, 1-3 July 2009, 978-988.
- [9] Nahar, S. and Alim, M.A. (2017) A New Geometric Average Technique to Solve Multi-Objective Linear Fractional Programming Problem and Comparison with New Arithmetic Average Technique. *IOSR Journal of Mathematics*, **13**, 39-52. <https://doi.org/10.9790/5728-1303013952>
- [10] Nahar, S. and Alim, M.A. (2017) A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem. *International Journal of Science and Research*, **6**, 623-629.
- [11] Veeramani, C. and Sumathi, M. (2014) Fuzzy Mathematical Programming Approach for Solving Fuzzy Linear Fractional Programming Problem. *RAIRO-Operations Research*, **48**, 109-122. <https://doi.org/10.1051/ro/2013056>
- [12] Ebrahimnejad, A. and Tavana, M. (2014) A Novel Method for Solving Linear Programming Problems with Symmetric Trapezoidal Fuzzy Numbers. *Applied Mathematical Modelling*, **38**, 4388-4395. <https://doi.org/10.1016/j.apm.2014.02.024>
- [13] Das, S.K., Mandal, T. and Edalatpanah, S.A. (2017) A New Approach for Solving Fully Fuzzy Linear Fractional Programming Problems Using the Multi-Objective Linear Programming. *RAIRO-Operations Research*, **51**, 285-297. <https://doi.org/10.1051/ro/2016022>
- [14] Lotfi, F.H., Allahviranloo, T., Jondabeh, M.A. and Alizadeh, L. (2009) Solving a Fully Fuzzy Linear Programming Using Lexicography Method and Fuzzy Approximate Solution. *Applied Mathematical Modelling*, **33**, 3151-3156. <https://doi.org/10.1016/j.apm.2008.10.020>
- [15] Kumar, A., Kaur, J. and Singh, P. (2011) A New Method for Solving Fully Fuzzy Linear Programming Problems. *Applied Mathematical Modelling*, **35**, 817-823. <https://doi.org/10.1016/j.apm.2010.07.037>
- [16] Nahar, S., et al. (2019) Solving Fuzzy LPP Using Weighted Sum and Comparisons with Ranking Function. *International Journal of Scientific & Engineering Research*, **10**, 480-484.
- [17] Sen, C. (1983) A New Approach for Multi Objective Rural Development Planning. *The Indian Economic Journal*, **30**, 91-96.
- [18] Akter, M., Jahan, M., Kabir, R., et al. (2019) Risk Assessment Based on Fuzzy Synthetic Evaluation Method. *Science of the Total Environment*, **658**, 818-829.



<https://doi.org/10.1016/j.scitotenv.2018.12.204>

- [19] Nishad, A. and Singh, S. (2015) Goal Programming for Solving Fractional Programming Problem in Fuzzy Environment. *Applied Mathematics*, **6**, 2360-2374.  
<https://doi.org/10.4236/am.2015.614208>
- [20] Pitam, S., Kumar, S.D. and Singh, R.K. (2011) Fuzzy Multi-Objective Linear plus Linear Fractional Programming Problem: Approximation and Goal Programming Approach. *International Journal of Mathematics and Computers in Simulation*, **5**, 395-404.