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A Dynamic Programming Approach to the Design of Composite Aircraft Wings

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Abstract

A light and reliable aircraft has been the major goal of aircraft designers. It is imperative to design the aircraft wing skins as efficiently as possible since the wing skins comprise more than fifty percent of the structural weight of the aircraft wing. The aircraft wing skin consists of many different types of material and thickness configurations at various locations. Selecting a thickness for each location is perhaps the most significant design task. In this paper, we formulate discrete mathematical programming models to determine the optimal thicknesses for three different criteria: maximize reliability, minimize weight, and achieve a trade-off between maximizing reliability and minimizing weight. These three model formulations are generalized discrete resource-allocation problems, which lend themselves well to the dynamic programming approach. Consequently, we use the dynamic programming method to solve these model formulations. To illustrate our approach, an example is solved in which dynamic programming yields a minimum weight design as well as a trade-off curve for weight versus reliability for an aircraft wing with thirty locations (or panels) and fourteen thickness choices for each location.

Keywords

Aircraft Wing Design, Maximum Reliability Design, Minimum Weight Design, Dynamic Programming, Multiple Objective Optimization, Pareto Optimality

1. Introduction

This paper presents an alternate approach for designing aircraft wings. The outline of the aircraft wing, both in platform and cross-sectional shape, must be suitable for housing a structure capable of performing its job. As currently designed, the basic wing shape is established through aerodynamic analyses, and then a preliminary layout of the wing structure is iteratively improved in a trial-and-error process until a design is obtained with sufficient strength, stiffness, and light-weight structure [1]. We illustrate here the use of optimization in the design process. In particular, with the immense amount of computer RAM now available, we propose that discrete dynamic programming, with its large memory requirements, should be an increasingly useful methodology in the airplane design process. We consider only wings here.

A wing box is composed of skins, spars, and ribs. In general, the skins account for fifty to seventy percent of a wing's structural weight. Consequently, it is crucial to design skins as efficiently as possible. The upper and lower skins play different roles in a wing box. The upper skin is loaded primarily in compression and must be designed to prevent buckling. The lower skin is loaded primarily in tension and requires high tensile strength.

Wing skins made of composite material consist of many different layups and thickness configurations at various locations that are determined typically by stress analysis. Such a location encounters high stress due to lift and drag forces on an aircraft wing. The number of analysis locations of a wing box depends on its geometric and load distribution complexity. There is no established procedure for breaking up the structure into locations for, say, reliability analysis. In general, a probabilistic model location within a structure is chosen to represent an area such that the internal stress and material strength are approximately constant over that area. There can be several hundred locations in a wing probabilistic analysis model with different thicknesses to choose from for each one.

If the thickness is increased at a location, the weight of that location will increase. If the location thickness is decreased while the applied load is kept constant, the internal stress will increase and the reliability of that location will decrease. The choice of thickness for each location is essential for efficient wing box design. Aircraft must balance weight and reliability requirements by the selection of thicknesses at numerous locations. Specifically, the two considerations that need to be addressed by the aircraft designers for choosing optimal thickness for each wing location are: 1) minimizing weight while satisfying reliability requirements and 2) maximizing reliability within a cost or weight limitation.

In this paper, we formulate the mathematical programming models to solve the following problems:

(Problem 1) minimization of aircraft wing weight for a given reliability by selecting a thickness for each location;

(Problem 2) maximization of aircraft wing reliability for a given weight by selecting a thickness for each location;

(Problem 3) investigation of trade-offs between aircraft wing's reliability and weight.

Previous related research includes that of Luo and Grandhi [2], who try to reduce the failure probability in aircrafts resulting from the uncertainty and randomness of the input information used in structural optimization. They illustrate their results with truss and wing structure examples. Pettit and Grandhi [3] achieve minimum weight designs for a truss and a representative aircraft wing subject to a set of reliability constraints. Pettit and Grandhi [4] further present a framework for wing design integrating structural and load analysis, reliability analysis, optimization, and most-probable point estimation. Padmanabhan [5] presents methodologies that facilitate reliability-based optimization (RBO) for multidisciplinary systems enabling concurrent design optimization in each discipline as well as significant reduction in the computational costs. Sobieszczanski-Sobieski and Venter [6] argue for a more exhaustive exploration of the design space to obtain a set of optimal designs including both the Pareto and non-Pareto solutions. They illustrate their method through a numerical example involving the optimization of an aircraft wing structural box design with thousands of degrees of freedom and constraints, and hundreds of design variables. Elham *et al.* [7] present an optimization strategy for wing design that reduces the number of design variables, enables parallel optimization of the airfoils in several spanwise positions, and allows the use of simpler and faster two-dimensional airfoil analysis tools.

In Section 2 we formulate mathematical programming models for solving Problems 1 - 3 above. In Section 3 we discretize the weight minimization problem (W) and formulate it as a dynamic programming problem. A numerical example is solved. In Section 4 we describe how to solve the multiple objective problem (P) and then obtain a trade-off curve for weight versus reliability for the example of Section 3. We offer concluding remarks in Section 5.

2. Formulation of the Mathematical Programming Models

To solve Problems 1 - 3, relationships among *reliability*, *thickness*, and *weight* must be formulated. We develop these relations using Lear Fan 2100 Jet data provided to Northrop Grumman Commercial Aircraft Division (NGCAD) by the Federal Aviation Administration (FAA).

Weight is a product of *area*, *thickness*, and *density*. For each wing location, *weight* increases linearly as *thickness* is increased since the *area* and *density* of a location are constant. Baseline thickness is the standard thickness obtained by deterministic structural analysis, and the baseline thickness is used to find the actual thickness at a particular location of the wing. Actual thickness is the actual measurement of the thickness at a particular location. For example, the baseline thickness of the first location for the Lear Fan 2100 wing is 0.2 inch. Therefore, given the thickness ratio of 0.95, the actual thickness is 0.2 times 0.95, or 0.19 inch.

Figure 1 represents a typical thick box beam wing structure, containing three spars and numerous ribs. This wing box structure usually serves as a fuel tank as well. A wing is divided into three major components upper skin, lower skin and substructure. The wing model for the upper skin is illustrated in **Figure 2**.

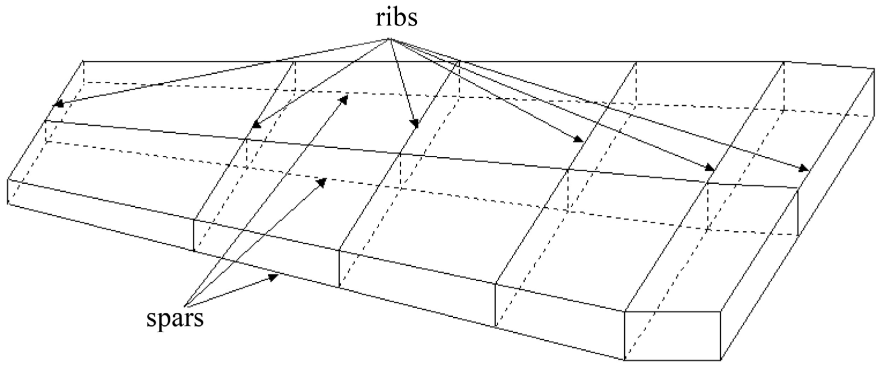

Figure 1. Typical wing structure.

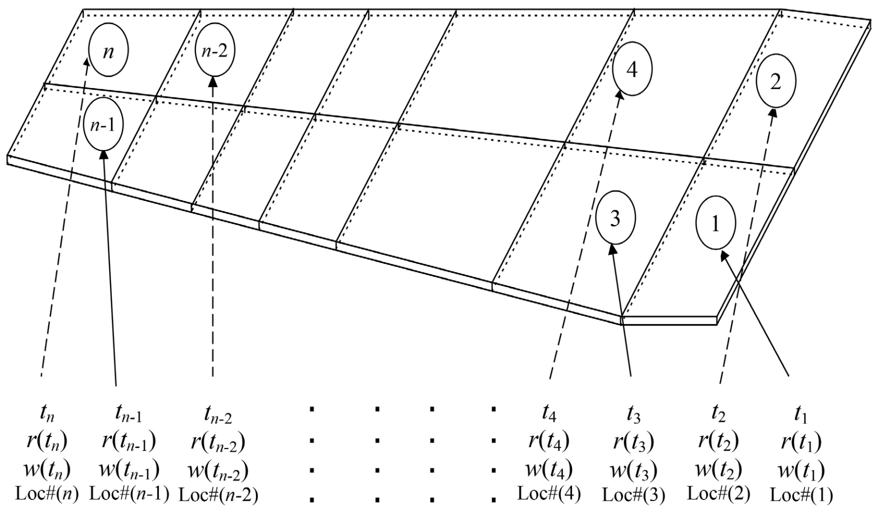

Figure 2. Configuration of a wing skin.

We now assume realistically that internal stress decreases with an increase in thickness. It follows that reliability increases with a decrease in internal stress from an increase in thickness and weight. In particular, wing reliability can approach 1.0 when a wing panel is extremely thick and heavy. However, reliability grows nonlinearly with increasing thickness as depicted in **Figure 3** based on the data in **Table 1** for panel 1 of the Lear Fan 2100 Jet. Both axes in **Figure 3** have been formatted appropriately in order to clearly display the nonlinear relationship between reliability and thickness. This nonlinear relationship can be attributed to two factors: 1) the reliability is calculated using the joint probability of nonlinear functions load and resistance [8] and 2) the failure at this location occurs due to buckling, which is a nonlinear function of thickness [9]. Regardless, for each panel, reliability is a strictly increasing function of the panel's thickness.

For panel $i = 1, 2, 3, \cdots, n$, let t_i denote its thickness, with a resulting reliability $r(t_i)$ and weight $w(t_i)$, which are assumed continuous functions of t_i. Then for a sufficiently large n as determined by a preliminary structural analysis, the total reliability and weight for the upper skin can be estimated by the following expressions.

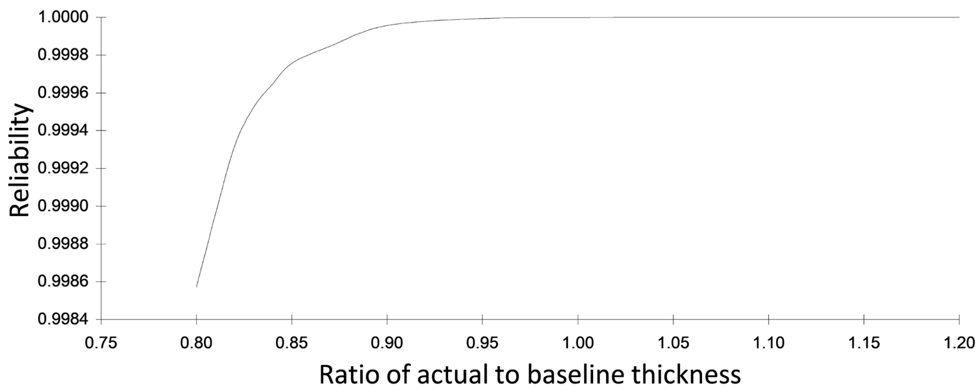

Figure 3. Relationship between reliability and thickness.

Table 1. Reliability and weight of panel 1 for 14 different thicknesses.

Thickness Number	Ratio of Actual Thickness to Baseline Thickness	Reliability	Weight (pounds)
1	1.20	0.999999995672	5.06
2	1.15	0.999999984471	4.85
3	1.10	0.999999937039	4.64
4	1.05	0.999999759895	4.43
5	1.00	0.999998691860	4.22
6	0.95	0.999992878710	4.01
7	0.90	0.999956177500	3.80
8	0.87	0.999847181000	3.67
9	0.85	0.999757110000	3.59
10	0.84	0.999648429000	3.54
11	0.83	0.999522932000	3.50
12	0.82	0.999318171000	3.46
13	0.81	0.998962640000	3.41
14	0.80	0.998571180000	3.38

$$\text{Total Weight} = \sum_{i=1}^{n} w(t_i) \tag{1}$$

$$\text{Total Reliability} = \prod_{i=1}^{n} r(t_i) \tag{2}$$

We now get the following mathematical programming model formulations.

2.1. Weight Minimization

Problem 1 can now be formulated as an optimization problem to minimize the total weight of a wing within a specified minimum reliability level r_0 $(0 < r_0 < 1)$ by selecting a thickness for each wing panel. From Equations (1) and (2), the weight minimization model (W) can be formulated as

$$(W) \ \underset{t_i \geq 0}{\text{minimize}} \ \sum_{i=1}^{n} w(t_i)$$
$$\text{subject to}$$
$$\prod_{i=1}^{n} r(t_i) \geq r_0,$$

where the t_i are the decision variables and r_0 is the required overall reliability.

2.2. Reliability Maximization

Problem 2 similarly becomes the optimization problem of maximizing total reliability of a wing for a specified upper weight limit $w_0 > 0$ by selecting a thickness for each wing panel. From Equations (1) and (2), the reliability maximization model (*R*) can be formulated as

$$(R) \ \underset{t_i \geq 0}{\text{maximize}} \ \prod_{i=1}^{n} r(t_i)$$
$$\text{subject to}$$
$$\sum_{i=1}^{n} w(t_i) \leq w_0,$$

where the t_i are the decision variables and w_0 is the specified weight limit.

2.3. Trade-Offs between the Weight and Reliability Objectives

Problem 3 involves a trade-off between reliability and weight requirements since designers want both a reliable and light aircraft. Unfortunately, the objectives of maximizing reliability and minimizing weight conflict with each other. We formulate a multiple objective model (*P*) that yields a trade-off known as a Pareto optimal solution, which is a nondominated feasible point $(t_1^*, t_2^*, \cdots, t_n^*)$ [10] [11] [12]. More precisely, a feasible point $(t_1^*, t_2^*, \cdots, t_n^*)$ to (*P*) is nondominated if and only if there does not exist another feasible point $(\bar{t}_1, \bar{t}_2, \cdots, \bar{t}_n)$ such that $(\bar{t}_1, \bar{t}_2, \cdots, \bar{t}_n)$ is at least as good as $(t_1^*, t_2^*, \cdots, t_n^*)$ for every objective function in (*P*) and is strictly better than $(t_1^*, t_2^*, \cdots, t_n^*)$ for at least one objective function in (*P*). This multiple objective optimization model (*P*) can be formulated as

$$(P) \begin{cases} \underset{t_i \geq 0}{\text{minimize}} \ \sum_{i=1}^{n} w(t_i) \\ \underset{t_i \geq 0}{\text{maximize}} \ \prod_{i=1}^{n} r(t_i) \end{cases}$$
$$\text{subject to}$$
$$\prod_{i=1}^{n} r(t_i) \geq r_0$$
$$\sum_{i=1}^{n} w(t_i) \leq w_0$$

for the decision variables t_i, with r_0 and w_0 being the reliability and weight limits, respectively.

2.4. Results Relating the Weight and Reliability

From previous discussion, as well as Figure 3, we have the following two results useful in finding a trade-off curve in Section 4.

Result 1. *The weight* $w(t_i)$ *and reliability* $r(t_i)$ *are strictly increasing functions of the thickness* t_i *and strictly increasing functions of each other.*

Result 2. *Let* $t_1^*, t_2^*, \cdots, t_n^*$ *solve* (W) *for fixed* r_0 *to give* $\sum_{i=1}^{n} w(t_i^*)$ *and let* $\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_n$ *solve* (W) *for fixed* $r_1 > r_0$ *to give* $\sum_{i=1}^{n} w(\hat{t}_i)$. *Then* $\sum_{i=1}^{n} w(\hat{t}_i) > \sum_{i=1}^{n} w(t_i^*)$.

In the next section, we describe our solution approach using dynamic programming and present an example.

3. Weight Minimization by Dynamic Programming

We now consider the problems (W), (R), and (P) where each of the major aircraft wing components *upper skin*, *lower skin*, and *substructure* has ten different panels. Hence there will be thirty panels across the wing structure as illustrated in Figures 4-6.

We discretize the problem since letting the t_i be continuous presents mathematical complications. Thus for each panel, fourteen different thicknesses will be considered that adequately represent an entire panel for design purposes. Material strength, operational damage, manufacturing defects, moisture absorption, and gust were incorporated into the NGCAD probabilistic design program to obtain a predicted structural reliability of the wing box for each thickness in each panel. Table 1 shows the resulting reliability and weight associated with different thicknesses for panel 1 with baseline thickness of 0.2 inch. The reliability $r(t_i)$ in Table 1 is simply the dimensionless probability in (0, 1) that panel *i* will not fail.

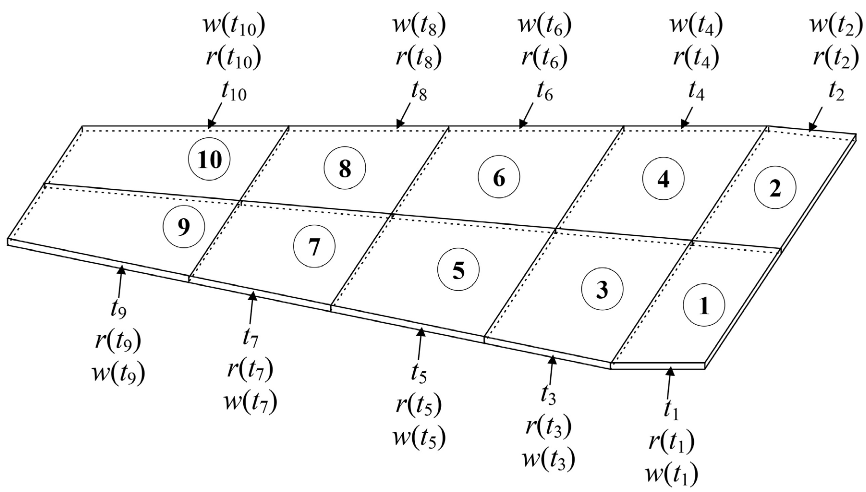

Figure 4. Upper skin.

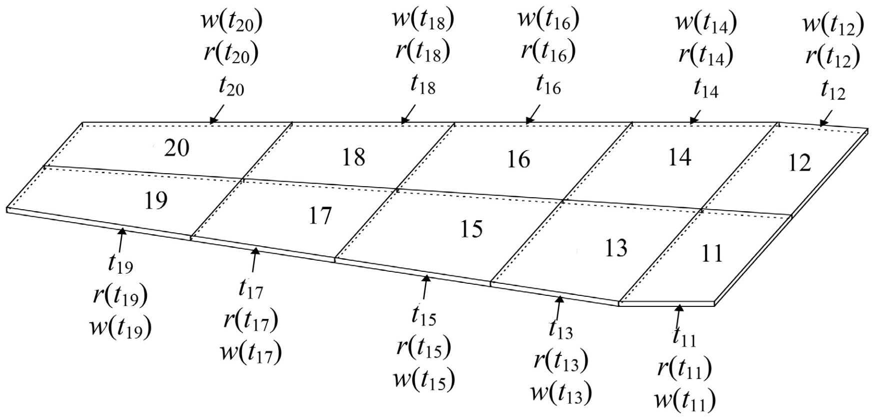

Figure 5. Lower skin.

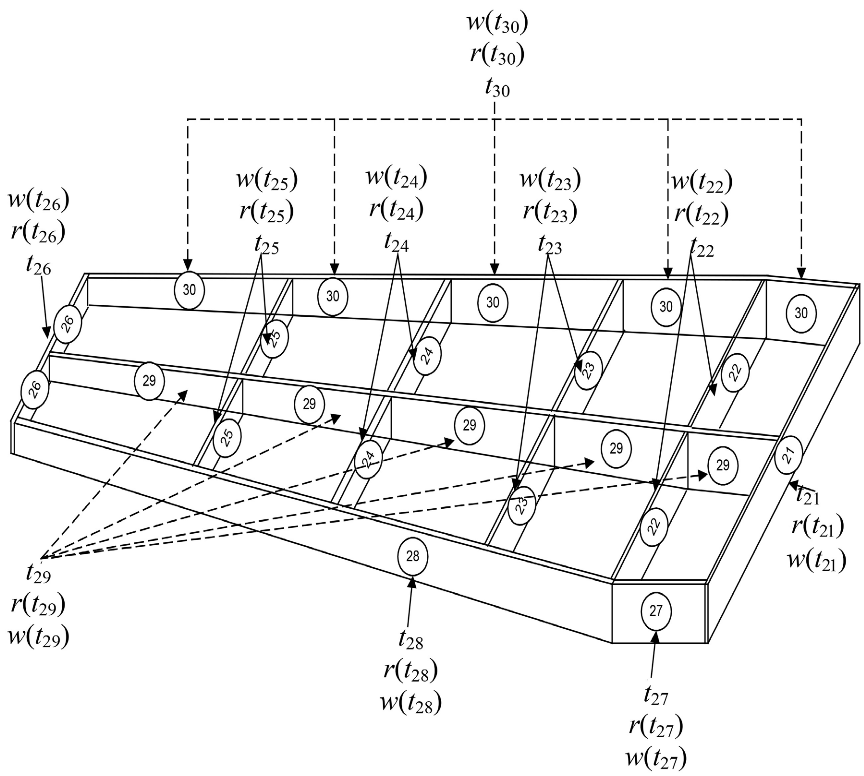

Figure 6. Substructure.

We next note that the mathematical programming models (W) and (R) represent generalized resource allocation problems with a single constraint, which are known to be efficiently solved using the optimization technique called dynamic programming [13]. Dynamic programming has been extensively used for inventory analysis, allocation problems, discrete control theory, and chemical engineering applications. We use it to find approximate solutions to (W) by limiting ourselves to a finite number of thickness ratios for each panel as illustrated in **Table 1** for panel 1. The use of dynamic programming requires defining the associated stages i, state variables s_i, decision variables t_i, return functions $g(s_i, t_i)$ dependent upon both state and decision variables, and recursive equations. For the weight minimization problem (W), these items are shown in Table 2.

Dynamic programming involves working backwards in the following sense. At stage 30 we minimize $g(s_{30}, t_{30})$ over the finite preselected values for t_{30} for each possible value of s_{30}. At stages $i = 29, 28, \cdots, 1$ in that order, for each possible value of s_i we solve $f_i(s_i) = \min[g(s_i, t_i) + f_{i+1}(s_{i+1})], i = 1, 2, \cdots, 29$ over the fourteen preselected values for t_i, where $s_{i+1} = s_i / r(t_i)$ from the stage transformations. There are only a finite possible number of values of each s_i at stage i because there are only fourteen possible t_i at each stage. However, finding and solving the optimal t_i for each s_i is memory intensive. Finally, at stage 1, $s_1 = r_0$. At that point, we proceed forward. For $s_1 = r_0$, the optimal t_1 is obtained by minimizing $[g(s_1, t_1) + f_2(s_1/r(t_1))]$ over the fourteen values of t_1, where each $g(s_1, t_1)$ is computed and each $f_2(s_1/r(t_1))$ is known from previously obtaining $f_2(s_2)$ for all s_2. With the optimal t_1 known, $s_2 = s_1/r(t_1)$ is computed and the optimal t_2 is determined at stage 2. Then the optimal $t_3, t_4, \cdots, t_{30}$ are similarly obtained. Figure 7 illustrates the dynamic programming

Table 2. Definitions for dynamic programming formulation of problem (W).

stage number = i	There are 30 stages (panel locations).
decision variables = t_i	Thickness for panel i
state variable at stage $i = s_i$	Overall reliability ($\geq r_0$) required for the remaining stages $i, i+1, \ldots, 30$ presenting restrictions on future decisions
return function at stage $i = g(s_i, t_i)$	Weight of panel i for a fixed remaining reliability s_i and thickness for this s_i
state transformations	$s_{i+1} = \dfrac{s_i}{r(t_i)}, i = 1, 2, \cdots, 29$
recursive equations	$f_i(s_i) = \min_{t_i}[g(s_i, t_i) + f_{i+1}(s_i/r(t_i))], i = 1, 2, \cdots, 29$ $f_{30}(s_{30}) = \min_{t_{30}} g(s_{30}, t_{30})$

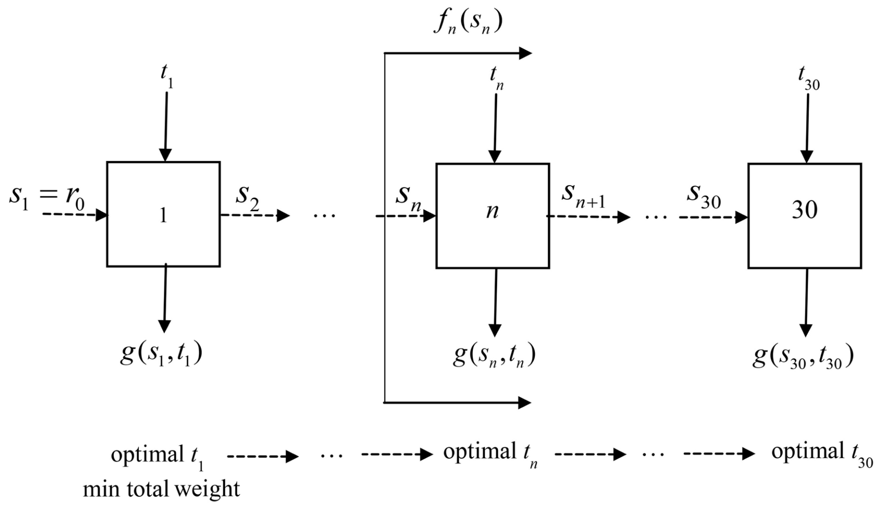

Figure 7. Flow chart for solving problem (W) using dynamic programming.

approach of working backwards. The top part of **Figure 7** indicates working backward for all possible s_i. The bottom part of **Figure 7** indicates working forward to obtain the actual values of the optimal t_i and the minimum total weight.

As an example, we now solve (W) for the 30 panel locations and $r_0 = 0.99999$, using the 14 thicknesses, reliabilities, and weights illustrated in **Table 1**. The resulting minimal weight for the aircraft wing is 249.93 pounds for the thickness choices for each panel shown in **Table 3**. The actual reliability for the minimum weight design is 0.9999905, which is slightly higher than required. Further examples are found in [14].

Table 3. Choices for each panel in the example.

Panel	Ratio of Actual Thickness to Baseline Thickness as in Table 1
1	1.05
2	1.05
3	0.95
4	0.90
5	0.95
6	0.87
7	0.87
8	0.87
9	0.85
10	0.87
11	0.80
12	0.80
13	0.80
14	0.80
15	0.80
16	0.80
17	0.80
18	0.80
19	0.80
20	0.80
21	0.80
22	0.80
23	0.80
24	0.80
25	0.80
26	0.80
27	0.80
28	0.80
29	0.80
30	0.80

4. Trade-Offs between Weight and Reliability

In this section, we determine Pareto optimal solutions to the multiple objective problem (*P*) from section 2. These Pareto optimal solutions are the nondominated feasible solutions to (*P*), which are then plotted to yield the trade-off curve between total weight and total reliability. Winston [12] has examples of the trade-off curve approach for two linear objective functions where dynamic programming is not used. Here we use the following procedure to obtain the trade-off curve of Figure 8.

Step 1. Choose a set of closely spaced values for r_0 in problem (*P*) designated as $r_0^k, k = 1, \cdots, p$ in order of increasing value with $r_0^1 = 0.99990$ being the smallest acceptable total reliability. See **Table 4**. Let $w_0 = 270$ pounds be the largest acceptable weight for the wing structure.

Table 4. Pareto optimal solutions to the multiple objective problem (*P*).

k	Reliability r_0^k	Weight (pounds) w^k
1	0.99990	243.68
2	0.99991	244.04
3	0.99992	244.35
4	0.99993	244.80
5	0.99994	245.29
6	0.99995	245.90
7	0.99996	246.61
8	0.99997	247.85
9	0.99998	249.84
10	0.99999	250.18
11	0.999991	250.48
12	0.999992	250.92
13	0.999993	251.58
14	0.999994	252.29
15	0.999995	252.89
16	0.999996	253.72
17	0.999997	255.21
18	0.999998	257.44
19	0.999999	257.66
20	0.9999991	257.96
21	0.9999992	258.23
22	0.9999993	258.64
23	0.9999994	258.85
24	0.9999995	259.27
25	0.9999996	260.36

Continued

26	0.9999997	261.68
27	0.9999998	263.34
28	0.9999999	263.44
29	0.99999991	263.65
30	0.99999992	264.27
31	0.99999993	264.48
32	0.99999994	265.21
33	0.99999995	266.05
34	0.99999996	267.02
35	0.99999997	268.11

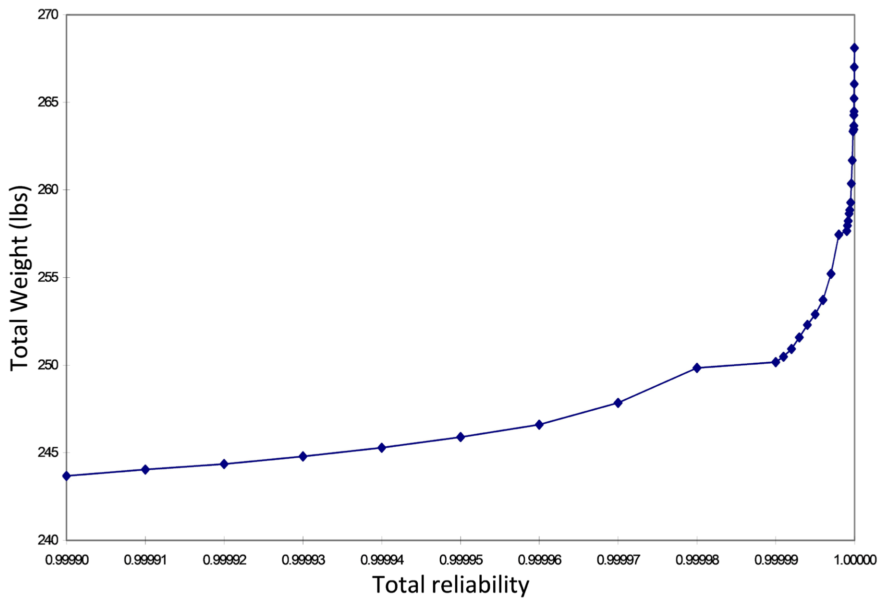

Figure 8. Trade-off curve for the multiple objective problem (*P*).

Step 2. Solve (*W*) for a fixed reliability r_0^1 to obtain the minimum weight w^1. Then $\left(r_0^1, w^1\right)$ is a Pareto optimum for the set of feasible (total reliability, total weight) for problem (*P*) since the total weight increases as the total reliability increases from Results 1 and 2.

Step 3. For $k = 2,\cdots,p$, solve (*W*) for r_0^k to obtain the minimum weight w^k. If $w^k > 270$, then stop with the Pareto optima obtained so far. Otherwise, each $\left(r_0^k, w^k\right)$ is a Pareto optimum for problem (*P*) for the reason of Step 2.

Step 4. Plot the Pareto optima obtained in Steps 2 and 3 to give the trade-off curve for $r_0 = 0.99990$ and $w_0 = 270$.

Any point (total reliability, total weight) on the curve of Figure 8 is nondominated for (*P*). Among these Pareto optima, a designer could select a particular one according to some further relevant criterion besides weight minimization and reliability maximization.

5. Conclusions and Future Work

A discrete dynamic programming approach was presented for obtaining optimal aircraft wing designs. The criteria were: 1) minimizing an aircraft wing's weight while satisfying reliability requirements by selecting thicknesses for wing box components, 2) maximizing an aircraft wing's reliability within weight limitations by choosing thicknesses for various wing locations, and 3) determining trade-off designs between criteria (1) and (2). A numerical example was presented to illustrate our approach. A principal advantage of dynamic programming is that, except at stage 1, the optimization at any stage is performed for all possible incoming states. Hence, it is a simple matter to change s_1 and obtain a new design. In other words, sensitivity analysis is not difficult with our approach. We reiterate that with the immense amount of computer RAM now available for computations, dynamic programming should be a desirable methodology for use in the airplane design process.

There are at least three principal directions for further research. First, the wing configuration model itself could be modified. Second, a continuous version of the model (*i.e.*, not considering only a fixed number of thickness values) could be developed and possibly solved by continuous dynamic programming or other approaches. Third, problems (W) and (R) shared some special relationships (*e.g.*, Results 1 and 2) that simplified the solution of (P) via a trade-off curve. Perhaps a class of optimization problems having similar relationships could be studied and have their general properties established.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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