American Journal of Computational Mathematics, 2023, 13, 387-397
https://www.scirp.org/journal/ajcm
ISSN Online: 2161-1211
ISSN Print: 2161-1203

On the Application of Adomian Decomposition Method to Special Equations in Physical Sciences

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How to cite this paper: Alsulami, A., Al-Mazmumy, M., Bakodah, H. and Alzaid, N. (2023) On the Application of Adomian Decomposition Method to Special Equations in Physical Sciences. *American Journal of Computational Mathematics*, **13**, 387-397.
https://doi.org/10.4236/ajcm.2023.133022

Received: July 10, 2023
Accepted: September 5, 2023
Published: September 8, 2023

Abstract

The current manuscript makes use of the prominent iterative procedure, called the Adomian Decomposition Method (ADM), to tackle some important special differential equations. The equations of curiosity in this study are the singular equations that arise in many physical science applications. Thus, through the application of the ADM, a generalized recursive scheme was successfully derived and further utilized to obtain closed-form solutions for the models under consideration. The method is, indeed, fascinating as respective exact analytical solutions are accurately acquired with only a small number of iterations.

Keywords

Iterative Scheme, Adomian Decomposition Method, Initial-Value Problems, Singular Ordinary Differential Equations

1. Introduction

Special differential equations of second-order are held with utmost esteem in many fields of physical sciences due to their fascinating properties [1] [2] [3]. Among others, they are characterized by the possession of the well-known special orthogonal functions, which are widely used in science and engineering domains. Additionally, these functions are greatly utilized in numerical methods, and approximation theories, just to mention a few. Besides, solutions of many differential equations are recast to the form of these special functions in mathematical physics [4]-[9].

However, the method of interest in the present study is the celebrated Adomian Decomposition Method (ADM) [10] [11]. This method was devised in the

1980s by George Adomian to treat differential and integral equations. Various reformations and extensions of this method have been reported in the literature; in addition to different reliable modifications of the method among others [12] [13] [14]. Indeed, there exists a huge number of related literature with regard to the development of the ADM that is associated with various forms of Initial-Value Problems (IVPs) of ordinary and partial differential equations [15] [16] [17] [18]. Both the IVPs and boundary-value problems have been effectively treated via the application of the ADM to obtain their resulting closed-form (series) solutions or even the exact analytical solutions in many cases. This is, however, related to the fact that the procedure devised by Adomian tends to rapidly converge to the exact analytical solution, whenever obtainable, see [19] and the references therein.

So far, there have been few works on the solution of special functions. In [9], the series solutions of some special functions are presented. Also, the authors in [20] studied the solution of Legendre differential equation and Chebyshev's differential equation by multiplying them by the singular coefficient and expressed it as based on ADM. In addition, the exact solutions for the Hypergeometric equation and Legendre equation, by writing the equation in the general operator form and finding its inverse, are given in [21]. Next, classical results on the Gauss Hypergeometric equation, Confluent Hypergeometric equation and Bessel equation are discussed in [22]. Additionally, differential transform method for Gauss Hypergeometric equation and Laguerre equations is applied in [23]. As well, the Frobenius method around all regular singular points for κ-Hypergeometric equation is employed in [24], Recently, [25] [26] thoroughly discussed the technique for obtaining an exact solution based on the modified Adomian decomposition method for the Laguerre equations and Chebyshev's differential equation.

What's more, the present study aims to tackle some important special second-order differential equations that arise in many physical models of physical sciences. More specifically, the singular models to be examined in the present study are mentioned as follows: Legendre's differential equation

$$\left(1-x^2\right)v''-2xv'+n\left(n+1\right)v=0,\quad -1<x<1. \tag{1}$$

Chebyshev's differential equation

$$\left(1-x^2\right)v''-xv'+n^2v=0,\quad -1<x<1. \tag{2}$$

Hermite's differential equation

$$v''-2xv'+2nv=0, \tag{3}$$

where in the above equations, n is a non-negative integer. In doing so, we will utilize the aforementioned classical ADM procedure to effectively acquire their respective recursive relations, leading to their closed forms or even exact analytical solutions when the value of n is specified. Additionally, we arrange the paper as follows: Section 2 gives the analysis of the approach of interest—the ADM. Section 3 gives the application of the ADM procedure to the models of concern; while Section 4 gives some concluding remarks.

2. Analysis of the Method

The present section gives a generalized derivation procedure for tackling nonlinear Initial-Value Problems (IVPs) based on the ADM. To do so, let us consider the following differential equation [16]

$$G\left(v(x)\right)=g(x), \tag{4}$$

with G representing a generalized ordinary (or partial) differential operator. This operator being general, it can equally be expressed to involve both linear and nonlinear operators. Thus, we decompose the operator further, and rewrite the above equation as follows

$$Lv+Rv+Nv=g, \tag{5}$$

where L is the highest linear operator that is invertible, with $R<L$; while N is specifically the nonlinear operator. More so, we rewrite the latter equation as follows

$$Lv=g-Rv-Nv,$$

such that applying the inverse operator L^{-1} to both sides of the above equation yields

$$v=\phi(x)+L^{-1}g-L^{-1}Rv-L^{-1}Nv. \tag{6}$$

where $\phi(x)$ is the function emanating from the prescribed initial data.

Further, the iterative procedure by the name ADM decomposes the solution $v(x)$ using an infinite series of the following form

$$v(x)=\sum_{m=0}^{\infty}v_m(x), \tag{7}$$

while the nonlinear component Nv is equally decomposed using the following infinite series

$$N(v)=\sum_{m=0}^{\infty}B_m, \tag{8}$$

where B_m's are polynomials devised by Adomian, and recursively determined using the following scheme [18]

$$B_m=\frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m}\left[N\left(\sum_{j=0}^{m}\lambda^j v_j\right)\right]_{\lambda=0}, \quad m=0,1,2,\cdots \tag{9}$$

Therefore, upon substituting Equations (7) and (8) into Equation (6), one gets

$$\sum_{m=0}^{\infty}v_m(x)=\phi(x)+L^{-1}g(x)-L^{-1}R\sum_{m=0}^{\infty}v_m(x)-L^{-1}\sum_{m=0}^{\infty}B_m.$$

Furthermore, the ADM procedure swiftly reveals the generalized recursive solution for the problem from the above equation as follows

$$\begin{cases} v_0=\phi(x)+L^{-1}g(x), \\ v_1=-L^{-1}Rv_0-L^{-1}B_0, \\ v_2=-L^{-1}Rv_1-L^{-1}B_1, \\ v_3=-L^{-1}Rv_2-L^{-1}B_2, \\ \vdots \\ v_{m+1}=-L^{-1}Rv_m-L^{-1}B_m, \quad m\geq 0, \end{cases} \tag{10}$$

where B_m's are the Adomian polynomials computed from Equation (9). Expressing few of these terms, we get

$$\begin{aligned}
B_0 &= N(v_0), \\
B_1 &= \frac{\mathrm{d}N(v_0)}{dv_0} v_1, \\
B_2 &= \frac{\mathrm{d}N(v_0)}{\mathrm{d}v_0} v_2 + \frac{1}{2}\frac{\mathrm{d}^2N(v_0)}{\mathrm{d}v_0^2} v_1^2, \\
B_3 &= \frac{\mathrm{d}N(v_0)}{\mathrm{d}v_0} v_3 + \frac{\mathrm{d}^2N(v_0)}{\mathrm{d}v_0^2} v_1 v_2 + \frac{1}{3!}\frac{\mathrm{d}^3N(v_0)}{\mathrm{d}v_0^3} v_1^3, \\
&\vdots
\end{aligned} \tag{11}$$

Remarkable, it is obvious that the Adomian polynomials B_m's depend on the solution components v_m. For instance, B_0 relies merely on v_0; B_1 relies merely on v_0 and v_1; B_2 relies merely on v_0, v_1 and v_2, and so on.

Finally, a realistic solution is obtained by considering the following m-term approximations as

$$\Psi_m = \sum_{j=0}^{m-1} v_j, \tag{12}$$

where

$$v(x) = \lim_{m\to\infty} \Psi_m(x) = \sum_{j=0}^{\infty} v_j(x). \tag{13}$$

3. Application to Standard Equations

This section presents the application of the ADM procedure presented earlier in the above section to some selected singular special differential equations' IVPs. More specifically, we make consideration to Legendre's differential equation, Chebyshev's differential equation and Hermite's differential equation.

3.1. Legendre's Differential Equation

Consider the Legendre's differential equation

$$(1-x^2)v'' - 2xv' + n(n+1)v = 0, \quad -1 < x < 1, \tag{14}$$

or alternatively rewritten as follows

$$v'' = x^2v'' + 2xv' - n(n+1)v, \quad -1 < x < 1. \tag{15}$$

So, we consider the right-hand side of the equation as a normal nonhomogeneous term, where the differential operator L is defined by $\frac{\mathrm{d}^2}{\mathrm{d}x^2}$.

More so, we consider the inverse operator L^{-1} as a two-fold integral operator defined by

$$L^{-1}(.) = \int_0^x \int_0^x (\,)\,\mathrm{d}x\mathrm{d}x.$$

Thus, applying the inverse operator L^{-1} to both side of Equation (15), one gets

$$v=\phi(x)+L^{-1}\left[x^2v''+2xv'-n(n+1)v\right],$$

such that

$$L\phi(x)=0.$$

Thus, based on the ADM procedure, the solution $v(x)$ is introduced through an infinite summation of components $v_m(x)$ earlier discussed in the methodology. Hence, the recursive solution of Legendre's differential equation is as follows

$$\begin{aligned} &v_0(x)=\phi(x),\\ &v_{k+1}(x)=L^{-1}\left[x^2v_k''+2xv_k'-n(n+1)v_k\right], k\ge 0 \end{aligned}$$

where the overall solution $v(x)$ follows immediately by summing the above components as follows

$$v(x)=\lim_{m\to\infty}\Psi_m(x)=\sum_{j=0}^{\infty}v_j(x). \tag{16}$$

Case 1: $n=1$

Let us consider an IVP featuring Legendre's differential equation with $n=1$ as follows

$$\begin{aligned} &\left(1-x^2\right)v''-2xv'+2v=0,\\ &v(0)=0,\ v'(0)=1. \end{aligned} \tag{17}$$

Therefore, we re-express the governing differential equation using operator notation as follows

$$Lv=x^2v''+2xv'-2v. \tag{18}$$

What's more, we apply the inverse operator L^{-1} to the both sides of the above equation to obtain

$$v=x+L^{-1}\left[x^2v''+2xv'-2v\right].$$

Lastly, through the application of the ADM, the overall recursive relation takes the following form

$$\begin{aligned} &v_0(x)=x,\\ &v_1(x)=L^{-1}\left[x^2v_0''+2xv_0'-2v_0\right]=0,\\ &v_{k+1}(x)=L^{-1}\left[x^2v_k''+2xv_k'-2v_k\right]=0,\quad k\ge 1, \end{aligned}$$

and it implies that

$$v(x)=x. \tag{19}$$

In fact, this is a well-known exact analytical solution for Legendre's differential equation when $n=1$.

Case 2: $n=2$

Let us consider an IVP featuring Legendre's differential equation with $n=2$ as follows

$$
\begin{aligned}
&\left(1-x^2\right)v''-2xv'+6v=0,\\
&v(0)=-\frac{1}{2},\ v'(0)=0.
\end{aligned} \tag{20}
$$

Thus, upon expressing the governing differential equation using operator notation as follows

$$Lv=x^2v''+2xv'-6v, \tag{21}$$

we further apply the inverse operator L^{-1} to both sides of the latter equation to obtain

$$v=-\frac{1}{2}+L^{-1}\left[x^2v''+2xv'-6v\right].$$

Then, the application of the ADM yields the following overall recursive relation

$$
\begin{aligned}
&v_0(x)=-\frac{1}{2},\\
&v_1(x)=L^{-1}\left[x^2v_0''+2xv_0'-6v_0\right]=\frac{3}{2}x^2,\\
&v_2(x)=L^{-1}\left[x^2v_1''+2xv_1'-6v_1\right]=0,\\
&v_{k+1}(x)=L^{-1}\left[x^2v_k''+2xv_k'-6v_k\right]=0,\quad k\ge 2,
\end{aligned}
$$

and it implies,

$$v(x)=-\frac{1}{2}+\frac{3}{2}x^2, \tag{22}$$

which is a known exact analytical solution for Legendre's differential equation when $n=2$.

3.2. Chebyshev's Differential Equation

Consider the Chebyshev's differential equation

$$\left(1-x^2\right)v''-xv'+n^2v=0,\quad -1<x<1. \tag{23}$$

Equally, using the ADM procedure described above, the corresponding recursive scheme is obtained by

$$
\begin{aligned}
&v_0(x)=\phi(x),\\
&v_{k+1}(x)=L^{-1}\left[x^2v_k''+xv_k'-n^2v_k\right],k\ge 0.
\end{aligned}
$$

Case 1: $n=1$

Let us consider an IVP featuring Chebyshev's differential equation with $n=1$ as follows

$$
\begin{aligned}
&\left(1-x^2\right)v''-xv'+v=0,\\
&v(0)=0,\ v'(0)=1.
\end{aligned} \tag{24}
$$

Now, expressing the governing differential equation using operator notation becomes

$$Lv = x^2 v'' + xv' - v, \tag{25}$$

such that upon applying L^{-1} to both sides of Equation (25) reveals

$$v = x + L^{-1}\left[x^2 v'' + xv' - v\right].$$

What's more, without loss of generality, the resulting recursive scheme for the IVP is found as follows

$$\begin{aligned} &v_0(x) = x, \\ &v_{k+1}(x) = L^{-1}\left[x^2 v_k'' + xv_k' - v_k\right] = 0, \quad k \geq 0, \end{aligned}$$

implying

$$v(x) = x, \tag{26}$$

which is indeed the exact analytical solution for Chebyshev's differential equation when $n = 1$.

Case 2: $n = 2$

Let us consider an IVP featuring Chebyshev's differential equation with $n = 2$ as follows

$$\begin{aligned} &\left(1 - x^2\right)v'' - xv' + 4v = 0, \\ &v(0) = -1, v'(0) = 0. \end{aligned} \tag{27}$$

In the same manner, we express the governing differential equation above using operator notation as follows

$$Lv = \left[x^2 v'' + xv' - 4v\right], \tag{28}$$

such that after applying the inverse linear differential operator L^{-1} to both sides of the latter equation reveals

$$v = -1 + L^{-1}\left[x^2 v'' + xv' - 4v\right].$$

As in the preceding problem, the resulting recursive relation is thus obtained as follows

$$\begin{aligned} &v_0(x) = -1, \\ &v_1(x) = L^{-1}\left[x^2 v_0'' + xv_0' - 4v_0\right] = 2x^2, \\ &v_2(x) = L^{-1}\left[x^2 v_1'' + xv_1' - 4v_1\right] = 0, \\ &v_{k+1}(x) = L^{-1}\left[x^2 v_k'' + xv_k' - 4v_k\right] = 0, \quad k \geq 2, \end{aligned}$$

which upon taking the sum gives

$$v(x) = -1 + 2x^2, \tag{29}$$

that is also the exact analytical solution for Chebyshev's differential equation when $n = 2$.

3.3. Hermite's Differential Equation

Consider the Hermite's differential equation

$$v'' - 2xv' + 2nv = 0. \tag{30}$$

Equally, using the ADM procedure described above, the corresponding recursive scheme is obtained by

$$v_0(x) = \phi(x),$$
$$v_{k+1}(x) = L^{-1}[2xv_k' - 2nv_k], k \ge 0.$$

Case 1: $n = 1$

Let us consider an IVP of Hermite's differential equation with $n = 1$ as follows

$$\begin{aligned} &v'' - 2xv' + 2v = 0, \\ &v(0) = 0, v'(0) = 2. \end{aligned} \tag{31}$$

In a similar way, we express the governing differential equation above via an operator notation as follows

$$Lv = 2xv' - 2v, \tag{32}$$

such that after applying the inverse linear differential operator L^{-1} to both sides of the latter equation reveals

$$v = 2x + L^{-1}[2xv' - 2v].$$

Then, as in the preceding problem, the resulting recursive relation is thus obtained as follows

$$v_0(x) = 2x,$$
$$v_{k+1}(x) = L^{-1}[2xv_k' - 2v_k] = 0, \quad k \ge 0,$$

which upon summing the components v_n gives

$$v(x) = 2x, \tag{33}$$

which is a well-known exact analytical solution for Hermite's differential equation when $n = 1$.

Case 2: $n = 2$

Let us consider an IVP of Hermite's differential equation with $n = 2$ as follows

$$\begin{aligned} &v'' - 2xv' + 4v = 0, \\ &v(0) = -2, v'(0) = 0. \end{aligned} \tag{34}$$

Therefore, expressing the governing differential equation above via an operator notation, one gets

$$Lv = [2xv' - 4v], \tag{35}$$

such that after applying the inverse linear differential operator L^{-1} to both sides of the latter equation reveals

$$v = -2 + L^{-1}[2xv' - 4v].$$

Lastly, the resulting recursive relation is thus obtained via the application of the ADM procedure as follows

$$
\begin{aligned}
&v_0 = -2,\\
&v_1(x) = L^{-1}\left[2xv_0' - 4v_0\right] = 4x^2,\\
&v_2(x) = L^{-1}\left[2xv_1' - 4v_1\right] = 0,\\
&v_{k+1}(x) = L^{-1}\left[2xv_k' - 4v_k\right] = 0, \quad k \geq 2,
\end{aligned}
$$

that sums to,

$$v(x) = -2 + 4x^2, \tag{36}$$

which is the exact analytical solution for Hermite's differential equation when $n = 2$.

Notably, it is worth mentioning here that, as the present study considers only the Legendre's, Chebyshev's, and Hermite's differential equations for $n = 1$ and $n = 2$, the same ADM procedure can equally be extended to other classes of special differential equations in mathematical physics for any value of *n*, in general. Equations like Laguerre, Gegenbauer, Jacobi differential equations, and others (both singular and nonsingular) could, in the same way, be treated via the ADM for any given *n*. More so, other special equations that have no orthogonal functions like Bessel, modified Bessel, Riccati, and Euler to state a few could also be examined by the method; in addition to the higher-order differential equations that are also evenly covered by the technique.

4. Conclusion

In conclusion, the present study makes use of the prominent Adomian iterative procedure to recurrently tackle some important special differential equations in physical sciences. As a particular interest, the known singular equations including Legendre's equation, Chebyshev's equation, and Hermite's equation have been successfully treated as test problems. Respective exact closed-form solutions for the models have been fruitfully acquired when $n = 1$ and $n = 2$. In fact, the acquired solutions happen to be precisely the same as the available results in the literature. Lastly, the used methodology is highly recommended for the study of both singular and nonsingular functional equations.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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