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An Adaptive Robust Approach to Modeling and Control of Flexible Arm Robots

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Abstract

In this paper, a novel adaptive robust approach to modeling and control of a class of flexible-arm robots subject to actuators unmodeled dynamics is proposed. It is shown how real-time signals measured from a dynamical system can be utilized to improve the accuracy of the mathematical model of flexible robots. Given the elasticity of the robot's arms, flexible manipulators have both passive and active degrees of freedom. A nonlinear robust controller is designed for the active degrees of freedom to enable the robot to follow desired trajectories in the presence of actuators unmodeled dynamics. Furthermore, it is shown that under some feasible conditions, another nonlinear robust controller is designed for the passive degrees of freedom. Moreover, to use the system response for model extraction, two auxiliary signals are proposed to provide sufficient information for improving the accuracy of the dynamics of the system numerically. Additionally, two adaptive laws are proposed in each case to update the two introduced auxiliary signals. As a result, the controller controls the passive degrees of freedom after the active degrees of freedom converge to their desired trajectories. Simultaneously, the information collected from the system to update the auxiliary signals enhances the model accuracy. In the end, simulation results are presented to verify the performance of the proposed controller.

Keywords

Adaptive Control, Robust Control, Vibration Control

1. Introduction

The area of flexible-arm robots has attracted much attention during the last few decades [1] [2]. This interest is due to the advantages that flexible arms offer compared to their rigid counterparts. Weight reduction, lower energy consumption, and faster system response are among several benefits utilized in their numerous applications such as space missions [3].

There has been a great number of studies coping with controlling flexible-arm robots, many of which investigate both theoretical and experimental aspects in this field [4] [5]. On grounds of the flexibility of the arms, along with a trajectory-tracking control problem, vibration control should be also considered for such systems to improve the control system performance [6]. As a result of the vibrations caused by the flexibility of the arms, designing controllers for such systems becomes a challenging task. There exist several research works in the literature addressing the flexibility of the arm. Passive control methods are one way to deal with the vibrations of elastic arms which require modification in physical parameters of the system structure [7]. Due to these structural modifications, absorption properties of the structure can be employed to increase damping properties of the arms. Active control approaches have also been widely used to control flexible systems, in which actuation moments and forces are applied to address the vibrations [8] [9]. However, the mere use of passive control methods to reduce vibrations does not seem to suffice. Merely using active control approaches is not sufficient as well. It is because vibration modes with frequencies near actuators frequencies can lead to instability. Hence, a combination of passive and active controllers can be employed as a suitable solution to vibrations reduction [10] [11]. The boundary feedback scheme is another way to dampen the vibrations of flexible manipulators [12]. Luo *et al.* [13] controlled the vibrations of a class of flexible robots using a shear feedback control method. Lyapunov-based control has also been vastly utilized to cope with the challenges associated with controlling flexible structures [14]. As another example of Lyapunov-based methods, Dadfarnia *et al.* [15] used the Lyapunov stability theory to develop a piezoelectric controller for flexible robots.

Infinite dimensionality is one of the most considerable challenges in modeling flexible robots. The existence of flexibility in the system results in dynamics governed by partial differential equations. Thus, techniques such as modal truncation are employed to express the dynamics by a set of ordinary differential equations [16]. As examples of such methods, Arts *et al.* [17] proposed an adaptive model integration method as a model reduction technique for planar flexible manipulators. Bruls *et al.* [18] used the global modal parameterization technique to reduce model-order of flexible multi-body dynamics. The procedure in both aforementioned methods is to divide the motion into two parts of rigid and elastic.

On account of the elasticity of the arms, there are degrees of freedom on which no actuation acts (Passive degrees of freedom). Consequently, flexible-arm robots fall into the category of underactuated mechanical systems. That is, systems with a lower number of control inputs compared to the number of degrees of freedom. Designing controllers for underactuated systems is an open problem. Control designs for such systems are dynamics-dependent while dynamical characteristics of the system play an essential role in the development of a control strategy. As an example of the existing control methods for underactuated systems, Mahmut Reyhanoglu *et al.* presented a theoretical scheme for modeling and control of underactuated systems. As one of the requirements of their approach, the non-integrable acceleration relations are required to be satisfied [19]. As another example, Zhang and Tarn designed a hybrid switching control strategy for nonlinear and underactuated mechanical systems [20].

Feedback linearization methods are wildly used for nonlinear systems. Nevertheless, As a result of the underactuated nature of flexible arms, the exact feedback linearization method [21] cannot be employed for such mechanical systems. Hence, the partial feedback linearization method [22] is considered as one the most suitable way to cope with underactuated systems. That is, the entire system can be either linearized with respect to the active degrees of freedom (collocated problem) or with respect to the passive degrees of freedom (non-collocated problem). Be that as it may, such a control strategy fails to handle uncertainties as control inputs to feedback-linearized systems depend on the governing equations.

In this paper, a novel adaptive robust nonlinear controller is designed for a class of multi-link flexible arms subject to uncertainties and unmodeled dynamics of the actuators. The contribution of the current research work is to utilize the system's real-time responses to improve the accuracy of the available mathematical model. Additionally, despite uncertainties and underactuated nature of the system, the proposed controller is able to track desired trajectories asymptotically. That is, despite the presence of uncertainties and disturbance sources, the tracking error converges to zero. To do such, first, an adaptive robust controller is designed for the active degrees of freedom. Meanwhile, adaptive laws are proposed to estimate the system uncertainties as well as approximating actuators uncertain dynamics. Furthermore, another adaptive robust controller is designed to control the passive degrees of freedom where the stabilities of the overall closed-loop system, in both cases, in the presence of uncertainties are established. In addition, due to the flexibility of the arms, the extraction of the system dynamics is demanding and, for the most part, should be done analytically. In the present approach, two adaptive signals are introduced and synthesized such that the cumbersome analytical part of the model extraction can be done numerically.

2. Dynamic Equations

In the current study, a multi-link flexible-arm robot is considered for modeling and control purposes. The investigated system is assumed to include m links containing l elastic modes. Therefore, the total degrees of freedom come to $n = m + l$.

Equations governing a flexible arm using the Euler-Bernoulli beam can be written as follows

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + C(q,\dot{q}) + G(q) = \begin{pmatrix} 0 \\ T \end{pmatrix} \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ is the positive definite inertial matrix; $C(q,\dot{q}) \in \mathbb{R}^n$ includes Coriolis and centrifugal terms; $G(q) \in \mathbb{R}^n$ contains gravitational terms; $q^{\mathrm{T}} = [q_1\ q_2]^{\mathrm{T}}$ is the generalized coordinates vector where $q_1 \in \mathbb{R}^l$ denotes the passive degrees of freedom, and $q_2 \in \mathbb{R}^m$ represents the active degrees of freedom. $T \in \mathbb{R}^m$ is the torque generated by the actuators acting on the active degrees of freedom. $C(q,\dot{q})$ and $G(q)$ can be calculated as [23]

$$C(q,\dot{q}) = \dot{M}(q)\dot{q} - \frac{1}{2}\frac{\partial\left(\dot{q}^{\mathrm{T}} M(q)\dot{q}\right)}{\partial q} \tag{2}$$

$$G(q) = \frac{\partial U}{\partial q} \tag{3}$$

where U denotes potential energy of the system. Extracting the dynamic equations of flexible robot arms is time consuming and can be analytically complicated. Due to its analytical nature, there exists a high chance of computational errors as the equations are derived. The extraction of the dynamic equations can be divided into two phases: 1) Extracting the portions of the dynamics that are straightforward and demand less calculations such as kinetic and potential energy. It is to be noted that the inertia matrix $M(q)$ can be arrived at directly from the rearrangement of the kinetic energy of the system. 2) Obtaining the portions of the dynamics which require lengthy analytical calculations such as $G(q)$ and $C(q,\dot{q})$. It is worth noting that calculating $G(q)$ and $C(q,\dot{q})$ require calculus of variations which can be cumbersome for systems with a large degrees of freedom. The idea of this paper is to propose a way to procure $G(q)$ and $C(q,\dot{q})$ numerically using the system responses. For this purpose, (1) is rewritten as

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ T + d(t) \end{pmatrix} \tag{4}$$

where

$$h_1(q,\dot{q}) = \dot{M}_{11}(q)\dot{q}_1 + \dot{M}_{12}(q)\dot{q}_2 - \frac{1}{2}\frac{\partial\left(\dot{q}^{\mathrm{T}} M(q)\dot{q}\right)}{\partial q_1} + \frac{\partial U}{\partial q_1} \tag{5}$$

$$h_2(q,\dot{q}) = \dot{M}_{21}(q)\dot{q}_1 + \dot{M}_{22}(q)\dot{q}_2 - \frac{1}{2}\frac{\partial\left(\dot{q}^{\mathrm{T}} M(q)\dot{q}\right)}{\partial q_2} + \frac{\partial U}{\partial q_2} \tag{6}$$

and $d(t)$ is an unknown vector added to the equations to account for uncertain dynamics of the actuators. It is also assumed that $\|d(t)\| \le \gamma$ where γ is an unknown constant.

In order to complete the dynamic equations given in (4), $h_1(q,\dot{q})$ and $h_2(q,\dot{q})$ should be computed. As it follows from (5) and (6), $h_1(q,\dot{q})$ and

$h_2(q,\dot{q})$ are expressed in terms of the matrix *M*, system states, and its potential energy. Therefore, one can use the measurement of the states along with the kinetic and potential energy of the system and use numerical computations to obtain h_1 and h_2. Hence, the system dynamics is computed as it runs without going through analytical calculations. However, such dynamic equations are prone to numerical and measurement errors. Thus, a special measure should be taken to improve the accuracy of such computations. To compensate for such errors, two unknown auxiliary signals P and S are employed as follows

$$M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + H_1 P = 0 \tag{7}$$

$$M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + H_2 S = T \tag{8}$$

where $P \in \mathbb{R}^l$ and $S \in \mathbb{R}^m$ are two unknown auxiliary signals introduced to account for numerical and measurement errors; $H_1(q,\dot{q}) \in \mathbb{R}^{l\times l}, H_2(q,\dot{q}) \in \mathbb{R}^{m\times m}$ are two diagonal matrices constructed as $H_1(q,\dot{q}) = diag\left[h_1^2, h_1^2, \cdots, h_1^n\right]$ and $H_2(q,\dot{q}) = diag\left[h_2^2, h_2^2, \cdots, h_2^n\right]$; h_i^j denotes the j^{th} element of the vector h_i where $i = 1,2$. Given the physical and geometric properties of flexible arm robots, it is not restrictive to assume that both H_1 and H_2, as defined above, have full ranks. Therefore, their column space can be utilized as a basis for the vectors h_1 and h_2 at any given time. Thus, for any exact value of h_1 and h_2 at any given time, there exist two unique vectors P and S such that $h_1 = H_1 P$ and $h_2 = H_2 S$. It is clear that the vectors P and S change in values over time. Hence, their values need to be updated according to the system response. Therefore, an adaptive procedure is developed for each of these vectors along with designing a controller in Section 3. Consequently, as the system runs, its response is used to update the values of P and S.

3. Control Design

In this section, an adaptive controller is designed for the considered system in the presence of uncertainties. Alongside the adaptive controller, two adaptive laws are derived for updating the values of P and S which are used to complete the dynamic equations. As it was shown in the previous section, (7) and (8) exhibit the dynamic equations governing an n-degree of freedom flexible-arm, with m active and l passive degrees of freedom. The first objective is to design a controller for the active degrees of freedom.

3.1. Control Design for Active Degrees of Freedom

Employing (7), (8) can be rewritten as follows

$$M_{21}\left(M_{11}^{-1}\left(-M_{12}\ddot{q}_2 - H_1 P\right)\right) + M_{22}\ddot{q}_2 + H_2 S = T + d \tag{9}$$

Two matrices N and R are defined as $R = M_{21}M_{11}^{-1}$ and $N = M_{22} - RM_{12}$. Consequently, (9) comes to

$$N\ddot{q}_2 + H_2 S - RH_1 P = T + d \tag{10}$$

As N is positive definite, (10) can be expressed as follows

$$\ddot{q}_2 = -\bar{H}_2 S + \bar{H}_1 P + \bar{U} + N^{-1} d \tag{11}$$

where $\bar{H}_2 = N^{-1} H_2$; $\bar{H}_1 = N^{-1} R H_1$, and $\bar{U} = N^{-1} T$. Tracking error and its dynamics can be defined as $e = q_2 - q_{2d}$ and $\dot{e} = \dot{q}_2 - \dot{q}_{2d}$, respectively; q_{2d} is the desired trajectory for the active degrees of freedom. A measure of the tracking error is also defined as

$$Q = \lambda_1 e + \dot{e} \tag{12}$$

where $\lambda_1 \in \mathbb{R}^{m \times m}$ is a positive definite matrix. The estimation errors of the uncertainty vectors are shown by $\tilde{P}$ and $\tilde{S}$ which are defined as $\tilde{S} = \bar{S} - S$ and $\tilde{P} = \bar{P} - P$; $\bar{S} \in \mathbb{R}^m$ and $\bar{P} \in \mathbb{R}^l$ are the estimations of the uncertainty vectors. Using the definitions of $\tilde{S}$ and $\tilde{P}$, (11) is rewritten as follows

$$\ddot{q}_2 = -\bar{H}_2 \left(\bar{S} - \tilde{S} \right) + \bar{H}_1 \left(\bar{P} - \tilde{P} \right) + \bar{U} + N^{-1} d \tag{13}$$

Considering (12), $\dot{Q}$ is calculated as follows

$$\dot{Q} = \lambda_1 \left(\dot{q}_2 - \dot{q}_{2d} \right) - \bar{H}_2 \left(\bar{S} - \tilde{S} \right) + \bar{H}_1 \left(\bar{P} - \tilde{P} \right) + \bar{U} + N^{-1} d - \ddot{q}_{2d} \tag{14}$$

To proceed with the control design, the following Lyapunov function is proposed

$$V = \frac{1}{2} \left(Q^{\mathrm{T}} Q + \tilde{P}^{\mathrm{T}} \tilde{P} + \tilde{S}^{\mathrm{T}} \tilde{S} + \tilde{\gamma}^2 \right) \tag{15}$$

where $\tilde{\gamma} = \hat{\gamma} - \gamma$ is the Euclidean norm of the estimation error of the uncertain dynamics of the actuators. Taking time-derivative of (15) gives

$$\begin{aligned} \dot{V} = {} & Q^{\mathrm{T}} \left(\lambda_1 \dot{q}_2 - \lambda_1 \dot{q}_{2d} - \bar{H}_2 \left(\bar{S} - \tilde{S} \right) + \bar{H}_1 \left(\bar{P} - \tilde{P} \right) + \bar{T} + u + N^{-1} d - \ddot{q}_{2d} \right) \\ & + \dot{\tilde{P}}^{\mathrm{T}} \tilde{P} + \dot{\tilde{S}}^{\mathrm{T}} \tilde{S} + \tilde{\gamma} \dot{\tilde{\gamma}} \end{aligned} \tag{16}$$

where $\bar{U}$ is decomposed into $\bar{U} = \bar{T} + u$. Given the structure of (16), the following control and adaptive laws are proposed

$$\bar{T} = -\psi Q - \lambda_1 \left(\dot{q}_2 - \dot{q}_{2d} \right) + \bar{H}_2 \bar{S} - \bar{H}_1 \bar{P} + \ddot{q}_{2d} \tag{17}$$

$$\dot{\tilde{P}}^{\mathrm{T}} - Q^{\mathrm{T}} \bar{H}_1 = 0 \tag{18}$$

$$\dot{\tilde{S}}^{\mathrm{T}} + Q^{\mathrm{T}} \bar{H}_2 = 0 \tag{19}$$

where $\psi \in \mathbb{R}^{m \times m}$ is a positive definite matrix; (17) is the proposed control law, and (18)-(19) are the adaptive laws to update the two auxiliary signals introduced in (7)-(8). Substituting (17)-(19) into (16) yields

$$\dot{V} = -Q^{\mathrm{T}} \psi Q + Q^{\mathrm{T}} u + Q^{\mathrm{T}} N^{-1} d + \tilde{\gamma} \dot{\tilde{\gamma}} \tag{20}$$

(20) can be rewritten as follows

$$\dot{V} \le -\lambda_{\min} \left(\psi \right) \left\| Q \right\|^2 + \left\| Q \right\| \left\| N^{-1} \right\| \left(\hat{\gamma} - \tilde{\gamma} \right) + Q^{\mathrm{T}} u + \tilde{\gamma} \dot{\tilde{\gamma}} \tag{21}$$

where $\|.\|$ stands for the Euclidean norm. Since N is a bounded positive definite matrix, the adaptive law to update $\tilde{\gamma}$ is proposed as

$$\dot{\hat{\gamma}} = \|Q\|\|N^{-1}\| \tag{22}$$

Employing (22), (21) becomes

$$\dot{V} \le -\lambda_{\min}(\psi)\|Q\|^2 + \|Q\|\|N^{-1}\|\hat{\gamma} + Q^{\mathrm{T}}u \tag{23}$$

To establish the asymptotic stability of the closed-loop system, the second part of the control law is introduced as follows in (24). It is to be noted that the second part of the control law compensates for the actuators uncertain dynamics.

$$u = -\frac{\hat{\gamma}^2\|N^{-1}\|Q}{\hat{\gamma}\|Q\|\|N^{-1}\| + \sigma(t)} \tag{24}$$

where $\sigma(t) \in L_1$ is an arbitrary signal where $\int_0^{\infty}\sigma(t)\mathrm{d}t < \infty$. By substituting (24) into (23), the following is obtained

$$\dot{V} \le -\lambda_{\min}(\psi)\|Q\|^2 + \frac{\hat{\gamma}\|Q\|\|N^{-1}\|\sigma(t)}{\hat{\gamma}\|Q\|\|N^{-1}\| + \sigma(t)} \tag{25}$$

Since $\|Q\|, \|N^{-1}\|, \sigma(t)$ are positive, the following inequality is concluded

$$\frac{\hat{\gamma}\|Q\|\|N^{-1}\|\sigma(t)}{\hat{\gamma}\|Q\|\|N^{-1}\| + \sigma(t)} \le \sigma(t) \tag{26}$$

Hence, (25) is simplified as follows

$$\dot{V} \le -\lambda_{\min}(\psi)\|Q\|^2 + \sigma(t) \tag{27}$$

Taking integral from both sides of (27) gives

$$\lambda_{\min}(\psi)\int_0^t\|Q\|^2\,\mathrm{d}\tau \le V(0) - V(t) + \int_0^t\sigma(\tau)\,\mathrm{d}\tau \tag{28}$$

Considering the fact that $\sigma(t) \in L_1$, the following is achieved

$$\lambda_{\min}(\lambda_2)\int_0^{\infty}\|Q\|^2\,\mathrm{d}t \le V(0) - V(\infty) + a \tag{29}$$

where a is a positive constant. It is observed that the left side of (29) is non-negative, thus, it follows that $(V(0) - V(\infty))$ is bounded. As a result, it is deduced that the right side of (29) remains bounded as well. Consequently, $Q \in L_2$. It is also easy to check that $\dot{Q} \in L_\infty$. Therefore, it is concluded from Barbalat's lemma that $\lim_{t\to\infty} Q(t) = 0$ which implies $\lim_{t\to\infty} e(t) = 0$. Thus, it was proven that adopting the extracted control laws (17), (24), and adaptive laws (18), (19)) and (22) guarantees the asymptotic stability of the closed-loop system. That is, the system is able to track desired trajectories asymptotically in the presence of model and actuators uncertainties.

3.2. Control Design for Passive Degrees of Freedom

In this section, a robust nonlinear controller is designed for the passive degrees of freedom alongside extracting adaptive laws to update uncertainties vectors. To do such, (8) is rewritten as

$$\ddot{q}_2 = M_{22}^{-1}\left(-M_{21}\ddot{q}_1 - H_2S + T + d\right) \tag{30}$$

Substituting (30) into (7) yields

$$F\ddot{q}_1 - GH_2S + h_1P + GT + Gd = 0 \tag{31}$$

where $G = M_{12}M_{22}^{-1}$; $F = M_{11} - M_{12}M_{22}^{-1}M_{21}$. It follows from the structure of M that M_{22} is nonsingular. In addition, it can be easily shown that F is a positive definite matrix as well. Therefore (31) can be expressed as follows

$$\ddot{q}_1 = KS - DP - V - F^{-1}Gd \tag{32}$$

where $K = F^{-1}GH_2$, $D = F^{-1}H_1$, $J = F^{-1}GT$, and J is a control signal yet to be defined. The actual control input can be retrieved from J as follows

$$T = G^{-1}FJ \tag{33}$$

It is concluded from (33) that G should be nonsingular. Thus, according to the definition of G, M_{12} needs to be invertible as well. Hence, in order to extract a non-singular control law for the passive degrees of freedom, the following assumption is made:

Assumption 1 It is assumed that the number of passive degrees of freedom is less or equal than the number of active degrees of freedom.

The tracking error for the passive degrees of freedom and its dynamics are defined as $\bar{e} = q_1 - q_{1d}$ and $\dot{\bar{e}} = \dot{q}_1 - \dot{q}_{1d}$ where q_{1d} is the desired trajectory for the passive degrees of freedom. Moreover, a measure of the tracking error is defined as $\bar{Q} = \lambda_2\bar{e} + \dot{\bar{e}}$ where $\lambda_2 \in \mathbb{R}^{l\times l}$ is a positive definite matrix. Using the definitions of $\tilde{S}, \tilde{P}$, which are defined in the previous section, (32) is expressed as follows

$$\ddot{q}_1 = K\left(\bar{S} - \tilde{S}\right) - D\left(\bar{P} - \tilde{P}\right) - J - F^{-1}Gd \tag{34}$$

Based on the structure of (34), the following Lyapunov function is introduced

$$\bar{V} = \frac{1}{2}\left(\bar{Q}^{\mathrm{T}}\bar{Q} + \tilde{P}^{\mathrm{T}}\tilde{P} + \tilde{S}^{\mathrm{T}}\tilde{S} + \tilde{\gamma}^2\right) \tag{35}$$

The time-derivative of (35) is calculated as

$$\begin{aligned}\dot{\bar{V}} &= \bar{Q}^{\mathrm{T}}\left(\lambda_2\left(\dot{q}_1 - \dot{q}_{1d}\right) + K\bar{S} - D\bar{P} - J_1 - J_2\right) + \left(\dot{\tilde{P}}^{\mathrm{T}} + \bar{Q}^{\mathrm{T}}D\right)\tilde{P} \\ &\quad + \left(\dot{\tilde{S}}^{\mathrm{T}} - \bar{Q}^{\mathrm{T}}K\right)\tilde{S} + \tilde{\gamma}\dot{\tilde{\gamma}} - \bar{Q}^{\mathrm{T}}\left(F^{-1}Gd - \ddot{q}_{1d}\right)\end{aligned} \tag{36}$$

where $J = J_1 + J_2$. Given (36), the following control and adaptive laws are proposed

$$J_1 = \bar{\psi}\bar{Q} + \lambda_2\left(\dot{q}_1 - \dot{q}_{1d}\right) + K\bar{S} - D\bar{P} - \ddot{q}_{1d} \tag{37}$$

$$\dot{\tilde{P}}^{\mathrm{T}} + \bar{Q}^{\mathrm{T}}D = 0 \tag{38}$$

$$\dot{\tilde{S}}^{\mathrm{T}} - \bar{Q}^{\mathrm{T}}K = 0 \tag{39}$$

where $\bar{\psi} \in \mathbb{R}^{l\times l}$ is a positive definite matrix. Substituting (37)-(39) into (36) and taking the Euclidean norm form both sides lead to the following

$$\dot{\bar{V}} \le -\lambda_{\min}\left(\bar{\psi}\right)\left\|\bar{Q}\right\|^2 + \left\|\bar{Q}\right\|\left\|F^{-1}\right\|\left\|G\right\|\left(\hat{\gamma} - \tilde{\gamma}\right) - \bar{Q}^{\mathrm{T}}J_2 + \tilde{\gamma}\dot{\tilde{\gamma}} \tag{40}$$

To make (40) negative semi-definite, the following control and adaptive laws are proposed

$$J_2 = \frac{\hat{\gamma}^2 \bar{Q} \left\| F^{-1} \right\| \left\| G \right\|}{\hat{\gamma} \left\| \bar{Q} \right\| \left\| F^{-1} \right\| \left\| G \right\| + \sigma(t)} \tag{41}$$

$$\dot{\hat{\gamma}} = \left\| \bar{Q} \right\| \left\| F^{-1} \right\| \left\| G \right\| \tag{42}$$

where $\sigma(t)$ is a positive arbitrary signal belonging to L_1. Substituting (40)-(42) into (40) gives

$$\dot{V} \le -\lambda_{\min}(\psi) \left\| Q \right\|^2 + \sigma(t) \tag{43}$$

Similar to the stability analysis presented in the previous section, it is concluded from (43) that $\lim_{t \to \infty} \bar{Q}(t) = 0$, which implies that $\lim_{t \to \infty} \bar{e} = 0$. Hence, it is guaranteed that the tracking error converges to zero asymptotically in the presence of uncertainties.

4. Simulation Results

In this section, a simulation is conducted to validate the analytical results. The simulated case study is a one-link flexible-arm robot with a payload at the tip of the arm. It is assumed that the system is subject to uncertainties and unmodeled dynamics in the actuators. The strategy is, first, to control the active degrees of freedom to their desired values. Then, the control law and adaptive laws switch to the non-collocated control to dampen the vibrations of the flexible arm. The physical properties of the investigated arm are as follows: Arm s Length = 1 m; Density = 7850 kg/m^3; Beams section area = 2e−4 m^2; Elasticity = 2.07e11; Load mass = 0.1 kg. The actual values uncertainty signals are $S = 8.2$ and $P = 5.5$.

It is shown in **Figure 1** that the flexible arms hub angle is successfully controlled to the desired angle. It is also observed from **Figure 1** that the hubs angle remains in its desired angle after the controller switches.

In **Figure 2**, the vibrations of the flexible arm are displayed, and its zoomed-in version is depicted in **Figure 3**. It follows from **Figure 3** that the arm residual vibrations are dampened as the controller switches.

Figure 4 and **Figure 5** correspond to the estimation of the introduced auxiliary signals to compensate for uncertainties. As noticed from the figures, the estimation of the uncertain parameters have converged to their limits after about two seconds through the simulation.

Finally, the required actuator torque to control the arm is shown in **Figure 6**. It follows from the simulations results that the arm tip can be controlled to its desired position more accurately. The reason stems from the fact that the non-collocated control helps decrease the response time. It is because that it would dramatically take a longer time for the residual vibrations to dampen naturally.

It was shown both analytically, in Section 3, and through simulation that the proposed controller can achieve asymptotic tracking in the presence of unknown

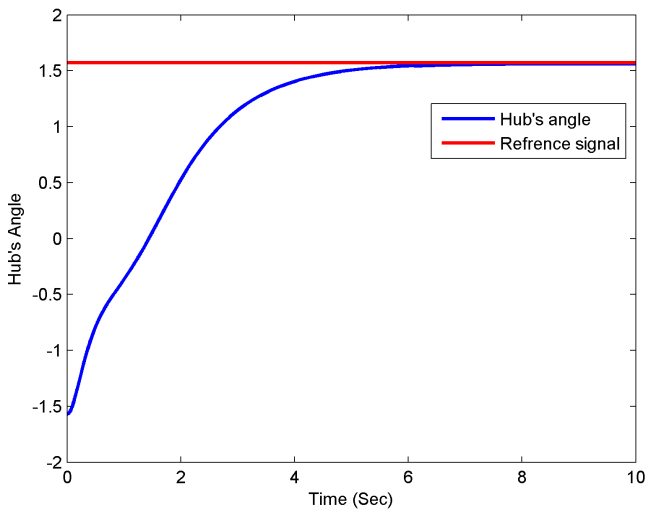

Figure 1. Arm's hub angle (rad).

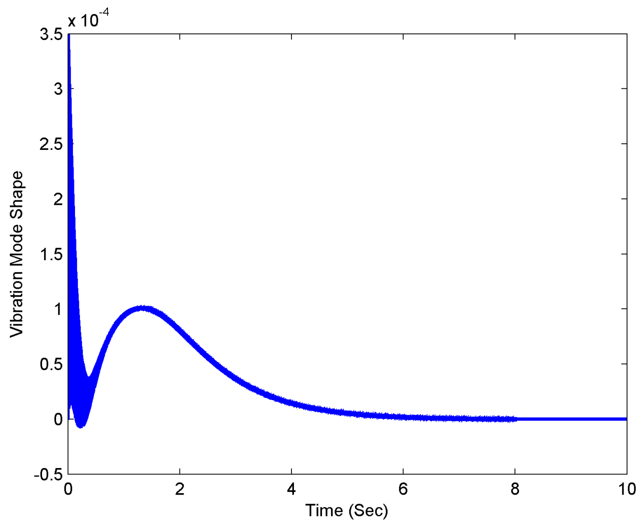

Figure 2. Flexible arm's vibrations (m).

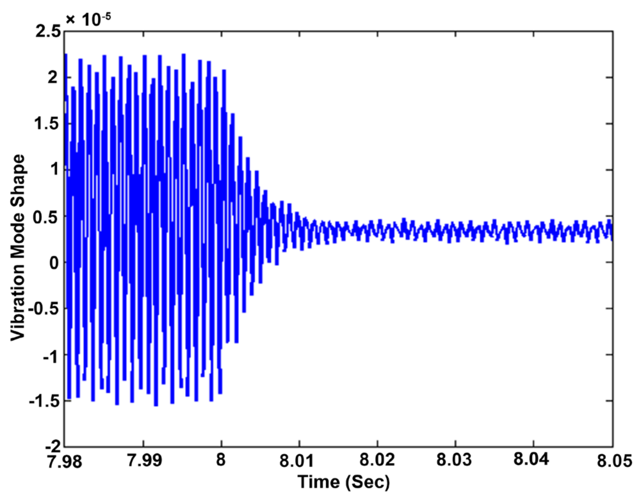

Figure 3. Zoomed-in version of the arm's vibrations.

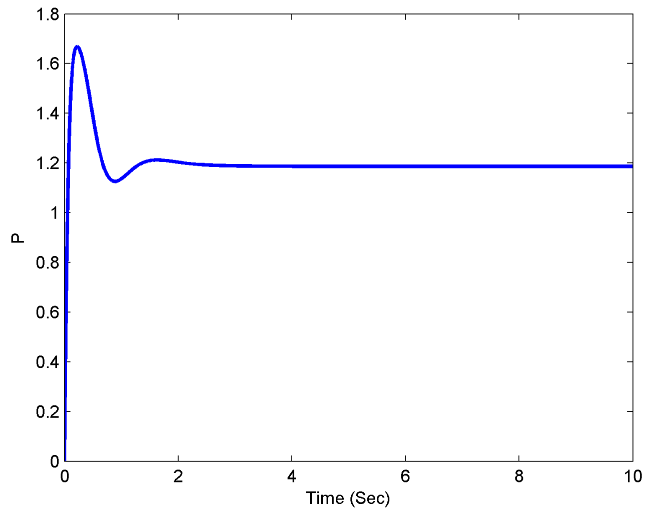

Figure 4. Auxiliary signal estimation (P).

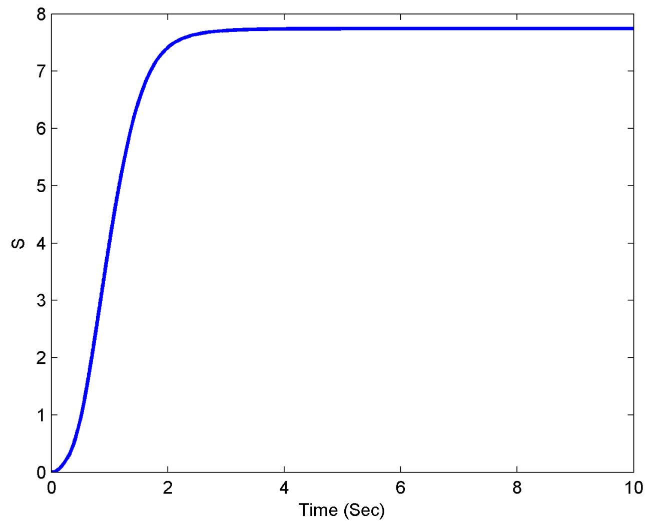

Figure 5. Auxiliary signal estimation (S).

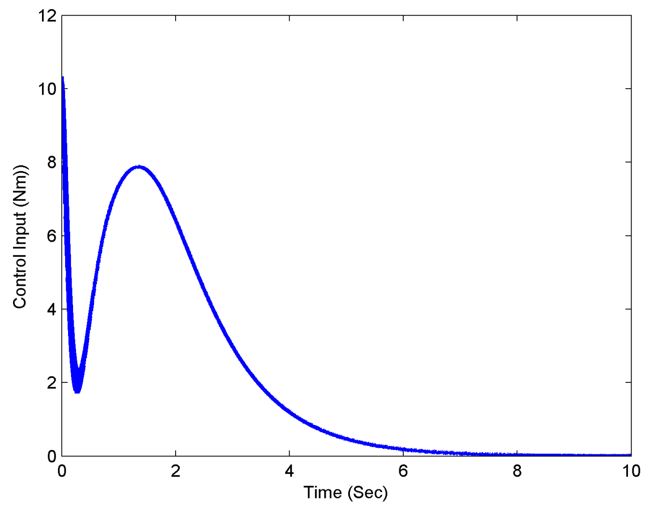

Figure 6. Control signal (Nm).

and bounded disturbance sources. As a comparison to other recent robust approaches to the control problem of such flexible manipulators, authors in [24] linearize the dynamics to simplify the complexity of their model. However, the nonlinear dynamics of the system is considered in the present study, and the controller is designed accordingly. It is to be noted the asymptotic tracking in this study is established on the postulate that no prior knowledge of the bound of the disturbance is available. Be that as it may, existing methods in the literature merely guarantee the boundedness of control and state signals in case of unknown disturbance sources. That is, they cannot prove the output can converge to desired trajectories asymptotically. As an example, authors in [25] take a robust adaptive approach to control a flexible manipulator using the Lyapunov stability theory. Their analytical analysis suggests the boundedness of all signals in the presence of disturbance. Thus, given a desired trajectory, the output can stay in the vicinity of the trajectory. However, the convergence of the output to the desired trajectory is not guaranteed.

5. Conclusion

As presented in detail, an adaptive robust nonlinear controller was designed for a class of flexible-arm robots based on the Lyapunov stability theory. A new numerical technique was presented to facilitate the extraction of the governing equations of flexible-arm robots. It was shown how computational errors caused by numerical operations could be compensated and approximated as uncertainty signals. In the control design process, first, an adaptive controller was designed for the active degrees of freedom to enable the system to follow desired trajectories in the presence of uncertainties. Further, it was shown that under some feasible conditions, another adaptive robust controller could be designed for the passive degrees of freedom. Therefore, as the system was being controlled, its responses were utilized to improve the accuracy of the mathematical model. Thus, the presented method leads to improvements in the controller performance and increases tracking accuracy. Furthermore, due to a numerical approach to extracting the dynamics, the amount of analytical computations is reduced dramatically.

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