Theoretical Economics Letters, 2019, 9, 2019-2032
http://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Measuring Asymmetric Nature of Beta Using a Smooth Linear Transformation

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How to cite this paper: Mitra, S.K. (2019) Measuring Asymmetric Nature of Beta Using a Smooth Linear Transformation. *Theoretical Economics Letters*, **9**, 2019-2032.
https://doi.org/10.4236/tel.2019.96128

Received: May 16, 2019
Accepted: August 26, 2019
Published: August 29, 2019

Abstract

Prior studies have found weak evidence on the asymmetric nature of the beta coefficient based on upward and downward movements of the market by classifying market movements into two mutually exclusive and exhaustive series using a fixed threshold. Instead of using a directional measure, we used a smooth linear transformation function to measure both magnitude and direction of market movements which is scaled on the basis of the highest and lowest monthly market return during the preceding three years. Proposed classification can capture the asymmetric behavior of beta in a better way.

Keywords

Asymmetric, Beta, Linear Transformation

1. Introduction

Finance theory has long been established a positive relationship between the expected return and risk from an investment. The Capital Asset Pricing Model (CAPM) [1] [2] and [3] has provided a framework to model a linear relationship between risk of a stock and return from the overall market as per the following equation.

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$$

The model attempted to measure the risk of the security (i) relative to the movements of the overall market by regressing the excess return of a particular security (r_{it}) with the excess returns of a stock market proxy say S&P 500 (r_{mt}). The risk of a stock is measured by the value of beta (β_i) coefficient using ordinary least squares (OLS). The OLS method measures the impact of stock returns on account of market price changes near the centre of the distribution and provides a measure of the average risk of the stocks return compared to the overall market return [4]. A stock is considered risky if the beta of a stock is more than 1 and similarly it is considered relatively safe compared to market when beta of the stock is less than 1. This measure of beta has regularly been used for portfolio management, measuring performance of mutual funds, measurement of returns in event studies, etc.

Following the CAPM model, the expected return from any security could be explained solely by beta that captures its level of systematic risk. However, several studies attempted to determine whether the Beta value of a stock using the standard model differs in different market conditions, such as, in bull and bear markets. In one of the earliest attempts, [5] using a sample of 700 stocks in New York Stock Exchange (NYSE) examined whether beta varied significantly between bull and bear market conditions between 1966 and 1971. They estimated separate betas for bull and bear markets and concluded that no significant difference exists between bull and bear conditions. Kim and Zumwalt [6] also could not find support for beta instability but reached to a conclusion that while an investor would be ready for a negative premium in an up-market condition, a positive premium was associated with the down-market beta. [7] also found a similar result as [6], where they observed that decomposition of the systematic risk in up-market risk and the down-market risk was more appropriate to capture time-varying beta. [8] examined differences of beta estimates of individual stocks and unlike the results of [5] stocks exhibited significant differences over beta measures between the bull and bear market conditions during 1972-1977.

While some studies found evidence that beta changes by phases of the market, the evidence was varied and weak. Most of the studies used dual-beta based on a binary classification of up and down phases of the market, and such classifications can easily be influenced by the noisy movements of the market. As stock market movements are noisy, a minor fluctuation around the threshold value can be misinterpreted as signals for upward and downward movements of the market. As evidence to date on the topic is hardly reassuring, it justifies further research on the topic. Therefore, we revisit the presence of an asymmetric relationship between stock price movements and market movements. First, similar to other studies, we analyzed whether beta differed significantly during upward and downward movements of the market using a predetermined threshold to classify market movements into two phases. This indicator used could capture only the direction of the market return. Second, we used a smooth linear transformation function to measure both the magnitude and direction of market movements, which is scaled on the basis of the highest and lowest monthly market return during the preceding three years. This smooth function captures the up and down movements of the market between −1 and +1 depending on the direction and magnitude of change exhibited by the index. The current month's movement was measured on a linear scale that takes the highest monthly return of the index in the preceding three years as +1 and lowest monthly return as −1. The function would yield value close to zero for minor fluctuations around thresholds and give proportionate weight depending on the degree of market movements. Thirdly, we augmented the beta measure of the capital asset pricing model by adding both the above-mentioned indicators based on a threshold and smooth transition function.

The uniqueness of this study lies in its proposition of a smooth linear transformation function which is found to be yielding better result over that of the traditional binary classification of beta for capturing the asymmetry. The smooth transition measure would give less weight to unsystematic and noisy movements and thus, expected to capture long-run departures in a better way. This indicator not only captured the direction of the market return but also contained a measure to capture the magnitude of the market return. It was observed from the study that the proposed method of market classification could capture the asymmetric characteristics of beta in a better way. In the study with 777 stocks, the asymmetric influence was significant in the case of 112 stocks, whereas the number was much lower when binary up and down classification was used in accordance with conventional procedures. Thus, our proposition of a smooth linear transformation function is proved to be yielding a superior result for capturing the asymmetry. Whether these asymmetries can be used for investment decisions, need further exploration.

The rest of the paper is arranged as follows. Section 2 reviews the literature. In Section 3, we explain data and methodology. Section 4 discusses the results. And Section 5 concludes.

2. Literature Review

A large number of empirical studies validated the usefulness of the CAPM model [2] [9]. The model is still treated as a theoretically sound model and is used as a benchmark for empirical investigations. The CAPM model asserts that only the systematic risk is rewarded as unsystematic risk can be reduced through diversification. The expected return of any risky security is the sum of the risk-free rate and risk premium estimated using beta. This interrelationship between the risk and the return can also be used to test the relationship between the variables. The original study of [10] used a three-step approach to establish the validity of CAPM. First, they estimated the beta for individual securities. In the second step, they estimated each portfolio beta for a subsequent period, and in the last step, they regressed portfolio returns on portfolio betas. Using monthly data from 1935 through 1968, they found the existence of a positive relationship between returns and beta and concluded that the model adequately describes the relationship between risk and return in capital markets.

A number of studies [11] [12] however, found weak evidence in favour of beta and observed that the above relationship can be spurious as the difference in returns across many portfolios for a sample of monthly returns studies by him was not significant, and further the relationship was not consistent across various sub-periods. Beta as the most effective measure of systematic risk for individual securities was challenged by [7] and they suggested the use of several macroeconomic variables, namely industrial production risk premium, twists in the yield curve, inflation, consumption, and oil prices. [13] studied beta for mutual fund and concluded that it reacts differently in bull and bear market conditions. Other studies [14] [15] [16] examined the portfolio return build upon the dual-beta approach proposed by [5]. [14] observed that the dual-beta model improves return predictions formed by size, past beta, and historic portfolio-return performance. [15] concluded that "small firms stocks underperform large firm stocks when beta risk is allowed to vary in bull and bear markets" (p. 270).

Several other studies [17] [18] found insignificant evidence between beta and average returns. [18] even concluded that the CAPM model does not describe the last 50 years of average stock returns. They investigated several market variables' ability to explain the cross-sectional return of stocks. The market value of equity and ratio of book value to the market value of equity evolved to have the most significant effect on return. This result was countered by others, which argue that in the model specified by [18], beta measures were mis-specified. [16] found significant size effect and in contrast to [18]; they found that beta could significantly explain the cross-sectional returns.

Another set of studies tried to establish a relationship between return and beta using a conditional relationship. [19] argued that if realized market return is above the risk-free rate, portfolio betas should be positively related, however, when the realized market return is less than the risk-free interest rate, beta and return should be inversely related.

In order to address the situation when realized excess returns are not always positive, [19] argued that while CAPM model postulated a positive relationship between beta and expected returns, the empirical investigation of [18] used realized returns instead of expected returns. Working on the US market data, they found a significant relationship between conditional beta and returns. In a more recent attempt, [20] tested the modified CAPM proposed by [19] in eleven emerging markets and found corroborative evidence supporting a positive estimated risk premium in up-market conditions and a negative estimated risk premium in down-market conditions.

Segmenting the Market into Different Phases

In the absence of any sacrosanct definition for defining up and down market conditions, [13] segregated sampled periods into two mutually exclusive and exhaustive series by placing months in which r_{mt} was non-negative as up months and conversely, when r_{mt} was negative was categorised as down months. This classification ignored market trends and return of each month was considered independent of past months return. In another classification, they segregated months as substantially up or down months, where market returns were divided into three categories. When returns of the market are greater than 0.5 times of its standard deviation, that month was categorised as substantially up months and similarly, a month was categorised as substantially down months, when the market return was less than 0.5 times of standard deviation. Months when movements were in ±0.5 standard deviation were considered normal months.

Several studies noted that the time series of market prices are noisy, and a simple threshold-based cut-off cannot measure the cyclical nature of the data. [21] used a trend-based approach to capture the status of market conditions. [22] suggested classifications based on non-overlapping trend-based bull and bear phases. They had taken daily price changes of the Index to determine months in which peaks and troughs were found. [13] separated bull and bear months based on the median return of the market portfolio returns. [23] categorized the market into "Bullish", "Bearish," and "Usual" sections based on the quantiles of the returns series.

[24] employing a Markov regime switching model, examined the instability of beta and concluded that CAPM was stable only in the low-risk state, while it was unstable in the high-risk state. [25] investigated the validity of a conditional three-beta model in the high, flat, and low volatility regimes and found that the majority of portfolio betas were stable across the regimes. [26] also found the stable beta for Indian stocks in all market conditions. [27] used a logistic smooth transition market model on Australian industry portfolios to investigate the difference of beta measure between the bull and bear markets. They found that 'bull' and 'bear' beta were significantly different across most industries and the up-market risk, in all cases, was not lower than the down-market risk.

To investigate the asymmetric relationship, studies in the literature analyzed whether beta differed significantly during upward and downward movements of the market. In the study, a smooth linear transformation function was used to measure both the magnitude and direction of market movements. Use of the transformation function could capture the up and down movements of the market between −1 and +1 depending on the direction and magnitude of change exhibited by the index. The function would yield value close to zero for minor fluctuations around thresholds and give proportionate weight depending on the degree of market movements.

3. Data and Methodology

We collected daily closing stock prices for top 1000 firms traded in U.S. exchanges from February 2005 to March 2019 from the Bloomberg database and S&P 500 index from S&P Dow Jones Indices. Out of 1000 securities, closing data for the full period, *i.e.*, February 2005 to March 2019 was available in case of 777 stocks, and thus the sample size of the stocks was reduced to 777 stocks. The daily price series were converted to the monthly price series by taking data applicable to the first trading day of the month. The secondary market rate for the three month Treasury bill, without seasonal adjustment, was used as a proxy for the risk-free rate. This data was sourced from http://research.stlouisfed.org/fred2 for the corresponding period.

As daily returns of 777 individual stocks and S&P 500 returns are used in the study and providing descriptive statistics of all series would consume space, the descriptive statistics of daily returns of S&P 500 index and first 10 stocks selected on the basis of alphabetic order of their trading symbols are presented in Table 1.

Descriptive statistics of the monthly returns of the securities used are presented in Table 1.

We estimated beta of the stocks using the following approaches:

First, the beta value was estimated using simple OLS following Equation (1) which is the most common measure of beta.

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \tag{1}$$

Secondly, we captured the asymmetric nature of beta related to the upward and downward movements of the market using Equation (2). When beta during upward movements (β_{UP}) of the market is higher than the beta during downward movements (β_{DN}), the stock is considered attractive by the investors as it tends to offer high payoffs at the time of the rising market but would fall at a lower rate when the market falls. On the contrary, a stock with higher β_{DN} would be unattractive as it would give a lower return when the market is falling.

$$r_{it} = \alpha_i + \beta_{UPi} r_{mt} + \beta_{DNi} r_{mt} + \varepsilon_{it} \tag{2}$$

The up and down movements of the market can also be captured by adding a variable to the Equation (1) as follows.

$$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} D_t r_{mt} + \varepsilon_{it} \tag{3}$$

where D_t will assume a value of +1 when the excess return of the market is nonnegative and −1 when the market return is negative. A positive and significant value of the coefficient β_2 would signify that beta is higher during upward movements of the market compared to the downward trends of the market and vice versa. In this measure, $\beta_{UP} = \beta_1 + \beta_2$ and $\beta_{DN} = \beta_1 - \beta_2$. This is a minor

Table 1. Descriptive Statistics of Daily returns of the S&P 500 Index and 10 other stocks.

Securities	Mean	Median	Std. Dev	Kurtosis	Skewness	Minimum	Maximum	No. of Obs.
SP 500	0.35%	1.32%	3.98%	9.64	−2.04	−22.86%	11.34%	170
ABT	0.46%	1.29%	4.93%	0.72	−0.70	−16.68%	9.26%	170
ACN	0.88%	1.36%	6.46%	0.35	−0.50	−17.55%	17.10%	170
ACE	0.72%	1.69%	6.14%	2.84	−0.82	−22.17%	17.88%	170
ACT	1.66%	1.69%	6.42%	0.03	0.20	−15.12%	18.38%	170
ATVI	1.01%	0.62%	8.51%	0.28	−0.01	−20.59%	24.23%	170
AYI	1.30%	2.12%	10.23%	1.35	−0.26	−31.41%	32.46%	170
ADBE	0.62%	2.39%	9.94%	1.68	−0.57	−33.92%	31.40%	170
AAP	1.25%	1.79%	8.16%	2.15	−0.79	−28.47%	17.96%	170
AES	−0.11%	0.44%	9.90%	3.26	−0.61	−35.43%	32.31%	170
AET	0.76%	2.41%	10.19%	4.64	−1.06	−42.84%	36.61%	170

variation from using a dummy variable where the dummy takes two values: 0 and 1.

In the next step, to avoid sharp differentiation of market movements, we used a normalizing measure to capture the degree of market changes, where the magnitude and direction of market movements were transformed between two user-specified values. Supposing that "A" and "B" are the minimum and maximum values of the scale in which the actual values of market return (r_{mt}) would be transformed; the following formula can be used.

$$N_t = -A + \frac{\left(r_{mt} - r_m\left(\text{Lowest}\right)\right)}{\left(r_m\left(\text{Highest}\right) - r_m\left(\text{Lowest}\right)\right)}\left(B - A\right) \tag{4}$$

In the proposed conversion, the highest return of a month during the past three years was taken as $r_m\left(\text{Highest}\right)$ and similarly, the lowest monthly return of three years was taken as $r_m\left(\text{Lowest}\right)$. We set the value of the lower limit A to −1 and the upper limit B to +1 so that converted values lie between the limits of ±1. The normalisation function was, therefore, simplified as follows.

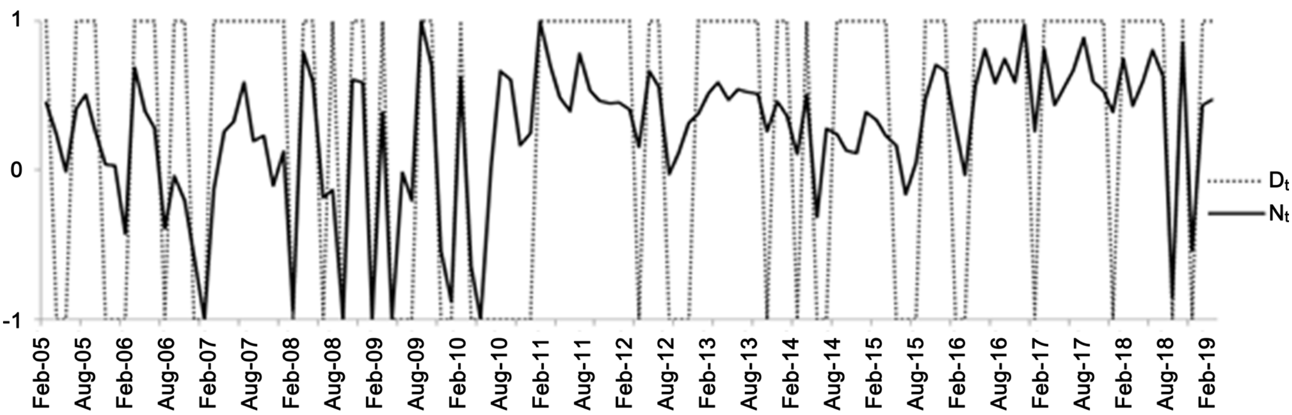
$$N_t = -1 + \frac{\left(r_{mt} - r_m\left(\text{Lowest}\right)\right)}{\left(r_m\left(\text{Highest}\right) - r_m\left(\text{Lowest}\right)\right)} \times 2 \tag{5}$$

The highest market price change during a past specified period would be valued at +1, and the lowest change would be taken as −1. Other periods will assume the value between −1 and +1 depending on the magnitude of change. This measure of market movement N_t would be different from the measure D_t used in Equation (2). **Figure 1** shows the difference between D_t and N_t values of monthly returns of the S&P 500 index from February 2005 to March 2019.

The following regression was performed to capture the asymmetric influence market movements on the beta coefficient.

$$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} N_t r_{mt} + \varepsilon_{it}, \tag{6}$$

where, N_t measures the state of market movement (both direction and magnitude) estimated using Equation (5). Similar to the earlier approach, β_2 coefficient



Source: plotted by the Author based on the analysis.

Figure 1. State of market movements using conventional and proposed measures.

measures the asymmetric impact of market movements and $\beta_{UP} = \beta_1 + \beta_2$ and $\beta_{DN} = \beta_1 - \beta_2$. A significant value of β_2 would indicate the asymmetric nature of beta even when a smooth linear function was used to capture the state of market movement.

Finally, to find the joint influence of both the indicators (D_t and N_t), the following regression was used.

$$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} D_t r_{mt} + \beta_{3i} N_t r_{mt} + \varepsilon_{it}, \quad (7)$$

4. Results and Discussion

Results of regression Equations ((1), (3), (6), and (7)) are analyzed for analysis of the asymmetric nature of beta. However, producing a complete table for 777 securities would be space consuming, and hence, the estimated coefficients of 10 stocks (based on the alphabetic order of their trading symbols) are presented in Table 2.

Prior studies have found a positive relationship between risk and return, and in line with those studies, we examined the relationship between return and beta by estimating the slope coefficient (β_i) used in Equation (1). It was found that beta value was significant at 5 percent level for 729 stocks out of 777 stocks and not significant only in 48 cases during the sample period and thus, a relationship between risk and return was found in most of the price series.

In our endeavor to find whether the beta coefficient associated with up-market differs from beta for the down- market, Equation (3) was used. The value of β_2 in the Equation (3) is supposed to measure the difference between the betas when $r_{mt} > r_f$ and $r_{mt} < r_f$. This method was similar to the method used by earlier studies, such as Fabozzi and Francis (1977).

As the stock price movements are noisy and monthly return from a market fluctuates above and below the risk-free rate of return on a month to month basis, instead of using a simple threshold to classify the market, we used a smooth function that also measures the quantum of upward and downward movements using Equation (5).

A comparison of the asymmetric component of beta when upward and downward movements of the market were measured by using a threshold approach as also when market movements were captured using a smooth linear transformation function, for all the price series investigated is presented in Table 3.

The value of β_2 coefficient was significant in more number of cases where upward and downward movements were captured using the smooth linear transformation function. Percentage of stocks shown significant asymmetric beta component was higher when the status of market movements was measured the proposed smooth linear transformation function.

According to Fabozzi and Francis (1977), the beta value significantly (at 5 percent significance level), differed in upward and downward movements of the market only in 3.9 percent of the cases and for the remaining cases, the asymmetric impact of beta for upward and downward movements of the market was

Table 2. Coefficients of daily returns as per equations.

Equation	Security	α_i	$\beta_i r_{mt}$	$\beta_{2i} D_t r_{mt}$	$\beta_{2i} N_t r_{mt}$
$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$ (Equation (1))	ABT	0.004	0.220		
	ACN	0.007	0.581		
	ACE	0.005	0.567		
	ACT	0.015	0.546		
	ATVI	0.008	0.464		
	AYI	0.010	0.841		
	ADBE	0.002	1.199		
	AAP	0.010	0.591		
	AES	−0.005	1.071		
	AET	0.004	0.927		
$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} D_t r_{mt} + \varepsilon_{it}$ (Equation (3))	ABT	0.000	−0.025	0.014	
	ACN	0.005	0.472	0.006	
	ACE	0.001	0.278	0.017	
	ACT	0.017	0.710	−0.010	
	ATVI	0.004	0.130	0.019	
	AYI	0.009	0.783	0.003	
	ADBE	0.001	1.103	0.006	
	AAP	0.010	0.596	0.000	
	AES	−0.005	1.045	0.002	
	AET	0.004	0.911	0.001	
$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} N_t r_{mt} + \varepsilon_{it}$ (Equation (6))	ABT	0.003	0.190		0.003
	ACN	0.009	0.671		−0.010
	ACE	0.003	0.477		0.010
	ACT	0.004	0.140		0.046
	ATVI	0.008	0.447		0.002
	AYI	0.018	1.156		−0.036
	ADBE	0.007	1.398		−0.023
	AAP	0.002	0.268		0.037
	AES	0.009	1.616		−0.062
	AET	0.009	1.118		−0.022
$r_{it} = \alpha_i + \beta_{1i} r_{mt} + \beta_{2i} D_t r_{mt} + \beta_{3i} N_t r_{mt} + \varepsilon_{it}$ (Equation (7))	ABT	−0.004	0.013	0.015	−0.006
	ACN	−0.001	0.571	0.009	−0.016
	ACE	−0.016	0.281	0.017	0.000
	ACT	0.001	0.349	−0.018	0.057
	ATVI	−0.011	0.201	0.021	−0.011
	AYI	0.003	1.045	0.009	−0.042
	ADBE	−0.049	1.284	0.010	−0.029
	AAP	−0.015	0.341	−0.006	0.041
	AES	−0.026	1.480	0.012	−0.069
	AET	−0.023	1.066	0.004	−0.025

Table 3. Comparison of asymmetric beta coefficient (when asymmetric component (β_2) was measured separately using Equations (2) & (5)). (a) Summary statistics β_2 value; (b) percentage of stocks where the β_2 coefficient was significant*.

(a)

	β_2 (Equation (3))	β_2 (Equation (6))
Average	0.0066	−0.0233
std. Deviation	0.0138	0.0343
Maximum	0.0923	0.1086
Minimum	−0.0687	−0.1719

(b)

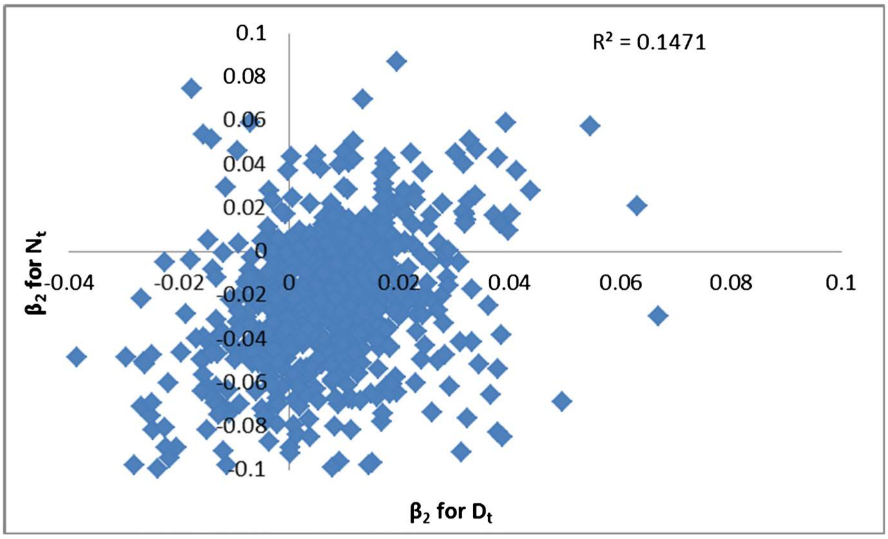
Significance Level	Using Equation (1)		Using Equation (4)	
	No. of Stocks	β_2	No. of Stocks	β_2
1% Level	16	2.06%	37	4.76%
5% Level	67	8.62%	112	14.41%
10% Level	101	13.00%	178	22.91%

*Significance is based on two-tailed t-tests.

not found significant. The current study found differences in 8.62 percent cases (at 5 percent significance level) when a threshold approach was used to capture the direction of market movements. However, the asymmetric impact became more prominent in 14.41 percent of the cases (at 5 percent significance level) when a smooth linear transformation function was used. It may, therefore, be inferred that although the asymmetric influence is low, the smooth transition function can capture the asymmetric characteristics of beta in a better way.

The null hypothesis of no difference of beta between up and down market; $H_0 = \beta_2 = 0$ can be rejected at 5 percent level in 67 out of 777 securities when D_t is used. However, when N_t is used to differentiate up and down market, the null hypothesis of $H_0 = \beta_2 = 0$ can be rejected in 112 out of 777 cases at 5 percent level. The difference between up and down market betas using the two methods described in Equation (2) and Equation (6) for 777 securities is plotted in Figure 2. It is observed that β_2 measures using two different propositions were highly scattered and the correlation between two measures was only 38.36 percent. Thus, the variable N_t in Equation (6) measures somewhat different information regarding the state of market compared to the variable D_t in Equation (2).

In order to find the influence of up and down markets, using both D_t (threshold-based criteria) and N_t (criteria that counts both direction and magnitude) in a single equation, Equation (7) was used for regression. Table 4 incorporates the value of coefficients β_2 and β_3 values using Equation (7). It was found that β_3 coefficient attached to the smooth linear transformation could capture the asymmetric dynamics of beta in a better way compared to the coefficient β_2 (coefficient attached to the binary classification of market return).



Source: plotted by the author based on analysis.

Figure 2. Scatter plot of β_2 coefficients associated with D_t (Equation (2)) and N_t (Equation (6)).

Table 4. Comparison of asymmetric beta coefficient (when the asymmetric component was measured simultaneously using Equation (6)). (a): Summary statistics of coefficients β_2 and β_3; (b): percentage of stocks where β_2 and β_3 coefficients were significant*.

(a)

	β_2	β_3
Average	0.0001	−0.0221
std. Deviation	0.0094	0.029
Maximum	0.0221	0.0757
Minimum	−0.0367	−0.177

(b)

Significance Level	β_2 Coefficient Significant		β_3 Coefficient Significant	
	No. of Stocks	% of Stocks	No. of Stocks	β_2
1% Level	12	1.54%	42	5.41%
5% Level	90	11.58%	147	18.92%
10% Level	152	19.56%	227	29.21%

*Significance is based on two-tailed t-tests.

From the result, it may be interpreted that, in addition to the sign of excess market return, the magnitude of market return also becomes a determining factor in the conditional beta literature.

5. Conclusions

Previous studies attempted to establish a relationship between beta and different phases of the market have found weak evidence that betas are influenced by the upward and downward movements of the market. The majority of the studies classified up and down or bull and bear classifications using a binary threshold. Movements above and below the threshold are classified into two distinct categories. As movements of the markets are noisy, the returns of the market may fluctuate around the threshold due to random shocks and are likely to produce incorrect results. The smooth function used in the study avoids the use of fixed benchmarks and allocates weights based on both the direction and magnitude of market movements. As a result, minor fluctuations around the mean would weigh close to zero, whereas major movements will carry higher weights with relevant positive and negative signs.

It was observed from the study that the proposed method of market classification could capture the asymmetric characteristics of beta in a better way. In the study with 777 stocks, the asymmetric influence was significant in the case of 112 stocks, whereas the number was much lower when binary up and down classification was used in accordance with conventional procedures. Thus, our proposition of a smooth linear transformation function is proved to be yielding a superior result for capturing the asymmetry.

The support of the empirical findings in favor of the proposed smooth transition method of market classification, thus, advocates for the use of this measure of market segmentation as an additional component to the asset pricing models in forming portfolios as well as measuring their performance.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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