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On Otaki's Keynesian Macroeconomic Model

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Abstract

"Otaki model" is an overlapping generations (OLG) model with fiat money and imperfect competition. It derives Keynesian implications such as involuntary unemployment, welfare-improving fiscal and monetary policy, and money non-neutrality. Though its structure can be applied to other models with productive implications, the Otaki model has not been fully understood by economists. One reason could be that in seminal works about the Otaki model, he implicitly changed the hidden assumptions, and hence, the implications of the model. By employing such assumptions, he aimed at dealing with multiplicity of equilibria in the OLG model. This paper points out the hidden assumptions and reinterprets the Otaki model from the viewpoint of multiple equilibria.

Keywords

Otaki Model, Imperfect Competition, OLG Model, Non-Neutrality of Money

1. Introduction

Economist Masayuki Otaki passed away in July 2018. In recent years, he has built a notable Keynesian macroeconomic model that we call as "Otaki model." Otaki model is an overlapping generations (OLG) model with fiat money and imperfect competition. It derives Keynesian implications such as involuntary unemployment, welfare-improving fiscal and monetary policy, and money non-neutrality. Though its structure can be applied to other models with productive implications, the "Otaki model" has not been fully understood by macroeconomists. This paper sheds lights on an unrecognized but important aspect of Otaki model for economists to understand and apply to other models.

From Otaki's early career, he had attempted to build a macroeconomic model of dynamic general equilibrium that reflected a Keynesian view of economy. Otaki [1] was eventually successful in this attempt; his work was first published in *Economics Letters*. Expansions to Otaki's research [1] include further studies

by Otaki [2]-[7] in *Theoretical Economics Letters*, where the Otaki model is extensively used. Later, Otaki published *Keynesian Economics and Price Theory: Re-Orientation of a Theory of Monetary Economy* [7], which predominantly focused on the Otaki model. Chapter 2 of this book provides an excellent explanation of his model. However, the Otaki model has not been fully understood by economists. One reason could be that in seminal works, he implicitly changed the hidden assumptions, and hence, the implications of the model. In this article, we name the Otaki model from 2007 to 2010 the "early Otaki model", and that from 2011 to 2018 the "late Otaki model". The problem lies in the fact that the early Otaki model implicitly employs one hidden assumption, while the late Otaki model substitutes another assumption for it. This problem is partly introduced in Tamura [8].

By the Way, The Otaki model has two main implications:

- Fiscal policy that is financed by seigniorage improves economic welfare.
- The core concept of Keynesian economics is not based on price rigidity or imperfect competition, but on non-neutrality of money.

The first implication is derived from the early Otaki model, while the second one is derived from the late Otaki model. From the next section, we emphasize the assumption change that can affect the model's implications.

2. Otaki Model in 2007-2010 and Its Assumptions

Mankiw [9] and Reinhorn [10] show that multiplier effects arising from government expenditure do occur in a general equilibrium model with imperfect competition. In other words, multiplier effects in a 45-degree line analysis still occur in the general equilibrium model, as well as the classical IS-LM (investment-saving and liquidity preference-money supply) model. Its mechanism is illustrated in Figure 1.

The government purchases the firm's product, and hence, increases the firm's profit, because it exhibits imperfect competition. The profit is distributed to consumers, and increases the consumers' income. Then, part of the increased income goes to consumption. Thus, the firm's profit increases again, which is distributed to consumers and consumers' income. This cycle continues infinitely.

However, the authors above found that fiscal policy (that is assumed to be a "waste of money" in the sense that does not directly contribute to the consumers' utility) worsens economic welfare because the profit increase goes with the production increase. Further, the production increase goes with a labor supply increase, which in turn, decreases the consumers' utility to some extent.

Figure 1. Multiplier effect in general equilibrium.

In addition to Mankiw [9] and Reinhorn [10], the original version of the early Otaki model [1] adds an OLG setting (and hence, money) and indivisible labor to the model. Therefore, the main structure of the Otaki model is a general equilibrium model with

- Overlapping generations with fiat money.
- Indivisible labor.
- Imperfect competition.

The main difference between the Otaki model and Mankiw [9] is the OLG setting and indivisible labor. Otaki [11] that is an early explanation about the Otaki model adopts the same settings. Then, the fiscal policy and multiplier effect were found to increase economic welfare in the Otaki model, even if it is a "waste of money" in the sense that does not directly contribute to the consumers' utility. In the Otaki model, we can say that fiscal policy is effective in improving economic welfare. Hence, we ask: Why does this occur? Which of the three assumptions above changes the welfare implications?

Otaki [1] does not explicitly answer to the question. However, we can say that the key lies in the equilibrium multiplicity of OLG models. As many studies in the literatures have noted, the OLG model possesses equilibrium multiplicity. The OLG setting in Otaki model also yields multiple equilibria, and one of the equilibria gives the Keynesian implications above. However, it should be noted that Otaki did not mention equilibrium multiplicity in his seminal works. He solved the model and found only one equilibrium. This is because he unconciously puts an additional hidden assumption on the model, and hence, results in selecting one equilibrium. In summary, 1) the early Otaki model that is a variation of OLG models must have multiple equilibria; 2) Otaki unconciously employed a hidden assumption; 3) it results in finding a Keynesian equilibrium.

To highlight the discussions so far, we introduce a part of the structure of the early Otaki model[1]. The firm's profit maximization problem gives

$$p_t = \frac{W^R}{1 - \eta^{-1}}. \tag{1}$$

Here p_t represents prices in period t, W^R is R reservation wage, and η the parameter. This can be regarded as markup principle. Reservation wage is determined by the labor supply of consumers, and hence, current and future price. So it can be rewritten as

$$P_t = \left(\frac{A^{-1}\beta}{1 - \eta^{-1}} \right)^{\frac{1}{1-\alpha}} P_{t+1}, \tag{2}$$

where A, β, and α are the parameters. P_t and P_{t+1} are the current and future price level. We find that the inflation rate ($\frac{P_{t+1}}{P_t}$) is determined only in the real economy. However, while the inflation rate is determined above, the price levels have not been determined yet. This is one representation of multiple equi-

[1]We can see the full version of the early model in Otaki [1].

libria problems. Multiple equilibria are the typical results in an OLG environment. To determine one equilibrium, additional assumptions are required, including the transversality condition and the interest rate condition. Also in the Otaki model, one more assumption is required. Otaki [1] unconciously added additional hidden assumptions. As a result, he found one Keynesian equilibrium among multiple OLG equilibria. Otaki [1] unconcously added the following assumption. "If government can arbitrarily control the real money balance $m_{t+1} = \dfrac{M_t}{P_t}$". Here, m_{t+1} is real money balance, M_t is money supply. To see that this is a hidden assumption, we review one equation in Otaki [1]. Equation (15) of Otaki [1] describes the multiplier effect as follows:

$$\frac{Y_t}{P_t} = \alpha \frac{Y_t}{P_t} + \rho m_{t+1}, \qquad (3)$$

here, Y_t is output. We can see that real money balance affects Y_t through the multiplier effect. The more real money balance (that increases fiscal policy) the government provides, the more output the economy produces. However, it is still undetermined whether the government can control m_{t+1} or not. The policy variable is not m_{t+1}, but M_t. Controlling M_t would be equivalent to controlling m_{t+1}, if P_t would be fixed. However, as we see in Equation (2), P_t is not undetermined because of multiple equilibria, therefore, P_t is not necessarily fixed. Then, controlling M_t is not equivalent to controlling m_{t+1}. For example, if increasing M_t would be followed by increasing P_t, m_{t+1} could not be controlled by the government. This is why "If government can arbitrarily control the real money balance m_{t+1}" is an additional assumption.

Later, Professor Otaki seems to have recognized that it is an assumption, and additional assumption (one example is the assumption above) is required to solve the model. However, Is this an adequate and realistic assumption? If the government can choose the cash balance ($\dfrac{M_t}{P_t}$), is it equivalent to assuming non-neutrality of money itself? It is clearly a strong assumption. In fact, seminal works including Otaki [1] [11] focuses not on this assumption, but on the implications of the model—the multiplier effects arising from government expenditures improve economic welfare. This claim is completely different from previous studies, such as Mankiw [9] and Reinhorn [10].

3. Otaki Model in 2011-2018 and Its Assumptions

Professor Otaki recognized a hidden assumption, and sought other assumptions to pin down one of multiple equilibria. Then, he focused on the expectation formations since multiple equilibria are consistent with multiple expectation formations. Because each equilibrium arises from its rational expectation, each equilibrium has its specific expectation formation. In this sense, expectation formations can be considered assumptions of the model.

Otaki implicitly changed the assumption of the Otaki model. While the hidden assumption of the early Otaki model is that the government can control the real cash balance, which of the late Otaki model is that people have expectations that money supply does not affect the price level. Otaki called this assumption "credibility of money". He substituted this assumption for the former one from 2011.

This new assumption implies that consumers do not care about money supply and monetary policy by the central bank, or do not believe that they affect the consumer goods' prices they face in their daily lives. This may seem to be an irrational assumption from the perspective of neoclassical economics. However, Otaki proves that this expectation is rational, that is, in a particular equilibrium, this expectation comes true.

From 2011, the Otaki model and its variations, such as Otaki [2]-[7], have included this assumption. Under "credibility of money," expectations are self-fulfilled, that is, money supply actually does not affect price level. This indicates the non-neutrality of money. In short, the expectation of money credibility results in non-neutrality of money, which in turn, is equivalent to the government being able to control real cash balance. These explanations are illustrated in Figure 2.

To observe this, suppose the money supply does not affect the price level. Then, $\dfrac{M_t}{p_t}$ can be controlled at any level by the government's choosing, M_t. Thus, the government decides on the real cash balance. In short, the assumption of "credibility of money" works similar to the assumption of the early Otaki model.

In 2011, the Otaki model's assumption was implicitly changed from government control of real cash balance to money credibility expectation. This does not change the results of the early Otaki model, but alters its implications. The early Otaki model's hidden assumption is ambiguous, and hence, he puts emphasis on the results themselves, that is, the multiplier effects arising from government expenditures improve economic welfare. In contrast, the late Otaki model has a meaningful assumption of credibility of money, and hence, he emphasizes both the assumptions and results. From 2011, Otaki insisted that the foundation of Keynesian economics is "credibility of money" or "neutrality of money".

So far, we have known that the Otaki model has multiple equilibria; one assumption, the expectation of money credibility (money non-neutrality), leads to Keynesian equilibrium in this model. Then, it is natural to consider what the other assumption, the expectation of money neutrality, leads to in this model. Otaki [7], however, answers this question. In the Otaki model, the following is evident.

- Expectations of "credibility of money" result in non-neutrality of money and theory of effective demand (multiplier effect and 45-degree analysis).
- Expectations of "quantity theory of money" result in neutrality of money.

In short, Otaki found another neoclassical equilibrium. In addition, Otaki [3]
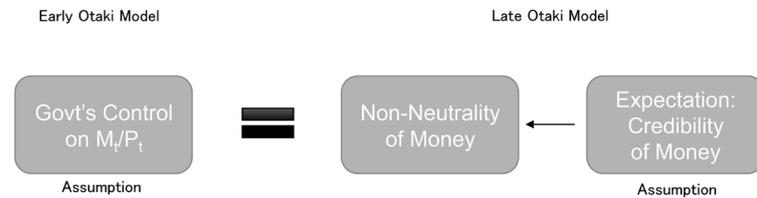
Figure 2. Two assumptions and their equivalence.

proves the same results in a perfect competition environment. Thus, we can state that the competition conditions do not matter when determining which economy occurs: Keynesian or classical. In other words, imperfect competition and price rigidity are not the core concepts of Keynesian economics. According to Otaki, expectations determine which of the above two economies occur.

Here, we can summarize the implications of the late Otaki model and its variations.

- Fiscal policy that is financed by seigniorage improves economic welfare.
- The core concept of Keynesian economics is not price rigidity or imperfect competition, but non-neutrality of money.

Finally, we consider: is credibility of money truly relevant to other economic models or is it only applicable to the Otaki model? Otaki [5], however, did attempt to apply it to other macroeconomic models. Credibility of money was introduced into the model developed by Lucas [12], who assumed the expectation of "the quantity theory of money." Then, it is found that, in equilibrium, this expectation is rational, leading to a neoclassical world. However, as Otani [13] noted earlier, Lucas [12] dealt with an OLG model, and hence, multiple equilibria occurred. In fact, Lucas [12] only found one equilibrium. Otaki [5] applied the "credibility of money" assumption to Lucas [12], and observed the occurrence of a Keynesian world. Therefore, "credibility of money" works well in other OLG macroeconomic models.

4. Concluding Remarks

The early Otaki model [1] [11] [14] has an important hidden assumption, while the late Otaki model [2]-[7] solves this problem by introducing a "credibility of money" assumption as a substitute. Therefore, the late Otaki model claims that the core concept of Keynesian economics is not imperfect competition or price rigidity, but non-neutrality of money.

An introduction to the Otaki model is difficult; the original article by Otaki [1] includes the early Otaki model. However, Chapter 2 of Otaki's [7] includes the best introduction to the late Otaki model. I expect this work to help many economists understand the Otaki model and apply it to future economic models.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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