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The Impacts of Joint Energy and Output Prices Uncertainties in a Mean-Variance Framework

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Abstract

In this paper, we analyze the impacts of joint energy and output prices uncertainties on the inputs demands in a mean-variance framework. We find that the concepts of elasticities and variance vulnerability play important roles in the comparative statics analysis. If the firms' preferences exhibit variance vulnerability, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demands for the non-risky inputs. Further, we investigate two special cases with only uncertain energy price and only uncertain output price. In the case with only uncertain energy price, we find that the uncertain energy price has no impact on the demands for the non-risky inputs. Besides, if the firms' preferences exhibit variance vulnerability, increasing the variance of energy price will surely cause the risk averse firm to decrease the demand for energy.

Keywords

Price Uncertainty, Mean-Variance, Energy Price, Risk

1. Introduction

The empirical literature that dealt with energy uncertainty is scarce, especially the energy literature under multiple sources of uncertainty. Some studies focused on the agricultural sector. Examples include Alghalith [1] [2], Kumbhakar [3], Nazlioglu and Soytas [4], Nazlioglu, *et al.* [5] and Du, *et al.* [6]. Some other papers studied the impact of oil shocks on the energy related stocks, such as Broadstock, *et al.* [7], Arouri, *et al.* [8] and Li, *et al.* [9]. Aduda, *et al.* [10] considered energy futures and spot prices and investigated the trends that underlie the price dynamics in order to gain further insights into possible nuances of price discovery and energy market dynamics by using a family of ARMA-GARCH. Examining the properties of efficient portfolios in the mean-variance framework in the presence of a cash account, Jiang, *et al.* [11] showed that investors will retain a portion of their funds in cash and found that the portion of funds invested in the intersection portfolio is more efficient than the corresponding traditional efficient portfolio. Using dynamic programming theory and considering continuous-time mean-variance portfolio selection with partial information, Pang, *et al.* [12] showed that the optimal portfolio strategy can be constructed by solving a deterministic forward Riccati-type ordinary differential equation and two linear deterministic backward ordinary differential equations. Using breakpoint regression technique, Syed and Zwick [13] found that the relationship between the oil and stock prices remains intact, the slope changes over time, thus identifying a non-linear relationship.

Alghalith [14] modeled energy price uncertainty in the U.S. manufacturing sector. In so doing, the study attempted to estimate the impact of energy price uncertainty on the manufacturing output. Alghalith [15] extended the model used by Alghalith [14] in two ways. First, it assumed that the manufacturing output price is uncertain. Second, it tested for the correlation between the energy price shocks and manufacturing price shocks. They also estimated the impact of the correlation on the manufacturing output. The firm's random profit is given by

$$\tilde{\Pi} = \tilde{p}F(\boldsymbol{x}) - \sum_{i=1}^{n-1} p_i x_i - \tilde{p}_n x_n, \qquad (1.1)$$

where $\boldsymbol{x} = (x_1, \cdots, x_{n-1}, x_n)$ is a vector of inputs, $p_i\,(i = 1, \cdots, n-1)$ is a non-random input price, $F(\boldsymbol{x})$ is a neoclassical production function with $\partial F/\partial x_j = F_j > 0$ for $j = 1, \cdots, n$, $\tilde{p}_n$ is the price of energy, and $\tilde{p}$ is the price of output. In this paper, we assume both the price of energy, $\tilde{p}_n$, and the price of output, $\tilde{p}$, to be uncertain and random.

The objective of the firm is to maximize the expected value of a von Neumann-Morgenstern utility function of profit $U(\tilde{\Pi})$, defined on the profit, $\tilde{\Pi}$. The firm is risk averse so that $U'(\tilde{\Pi}) > 0$ and $U''(\tilde{\Pi}) < 0$ for any $\tilde{\Pi} > 0$. This type of utility functions include the quadratic utility functions [16]. The firm maximizes the expected utility of the profit stated in (1.1)

$$\max_{x_1, \cdots, x_n} EU\left(\tilde{p}F(\boldsymbol{x}) - \sum_{i}^{n-1} p_i x_i - \tilde{p}_n x_n\right), \qquad (1.2)$$

where E denotes the expectation operator and all the terms are defined in (1.1). Meyer (1987), Wong and Ma [17], and Eichner and Wagener [18] showed that, under some conditions, the expected utility decision problem can be transformed into the mean (μ)-standard deviation (σ) framework. This approach has been widely used in literature including Battermann *et al.* [19], Broll *et al.* [20], Alghalith, *et al.* [21] [22]. In this paper, we extend their work by analyzing the impact of joint energy price and output price uncertainties on the demands for energy and the other non-risky inputs. We allow the dependence between energy price and output price and consider the effect of the covariance between these two random variables on the demands for inputs. In this paper, we find that the concepts of elasticities and variance vulnerability play important roles in the comparative statics analysis. Further, we also consider some special cases of our model. That is the situation with only uncertain energy price and that involving only uncertain output price. In these two special cases, clearer and intuitive results are obtained.

2. The Model

As described in the introduction we model risk preferences in a mean-variance framework (μ,σ) [23] which infers that (i) The expected utility EU stated in (1.2) can be represented by a two-parameter function $V(\mu,\sigma)$ defined over mean μ and standard deviation σ of the underlying random variable; (ii) the preference function V possesses the following properties: $\partial V(\mu,\sigma)/\partial\mu = V_\mu > 0$, $\partial^2 V(\mu,\sigma)/\partial\mu^2 = V_{\mu\mu} < 0$, $\partial V(\mu,\sigma)/\partial\sigma = V_\sigma < 0, \sigma > 0$ and $V_\sigma(\mu,0) = 0$. We assume that $\partial^2 V(\mu,\sigma)/\partial\mu\partial\sigma$ is positive, $\partial^2 V(\mu,\sigma)/\partial\sigma^2$ exists and V is a strictly concave function. The indifference curves are convex in (σ,μ)-space.[1]

Using the (μ,σ) preferences, the decision problem of the firm maximizing the expected utility of the profit as stated in (1.2) is equivalent to the following problem:

$$\max_{x_1,\cdots,x_n} V(\mu_\Pi,\sigma_\Pi), \tag{2.1}$$

where $\mu_\Pi = E(\tilde{\Pi})$, $\sigma_\Pi = \sqrt{E\left(\tilde{\Pi} - E(\tilde{\Pi})\right)^2} > 0$, and all the terms are defined in (1.1) with

$$\mu_\Pi = \mu_p F(\boldsymbol{x}) - \sum_i^{n-1} p_i x_i - \mu_{p_n} x_n,$$

$$\sigma_\Pi = \sqrt{\sigma_p^2 F^2(\boldsymbol{x}) + \sigma_{p_n}^2 x_n^2 - 2F(\boldsymbol{x}) x_n \sigma_{p,p_n}}.$$

We note that the slope S of the investor's indifference curve in (σ,μ)-space at (σ,μ) is the marginal rate of substitution between risk, σ, and expected return of profit, μ. Lajeri and Nielsen [25] and Ormiston and Schlee [26] identify S as the two-parameter analogue of the Arrow-Pratt concept of absolute risk aversion. Eichner and Wagener [27] [28] investigated properties of S further. The slope of an indifference curve in $\mu-\sigma$ space is positive. Risk aversion implies that the indifference curves are upward sloping. Therefore, S can be interpreted as a measure of risk aversion within the mean-standard deviation approach. We also note that because comparisons of risk aversion are determined only from the family of risks in (2.1), risk aversion can be measured in terms of standard deviation and mean, and thus, it can be measured by the slope S. Wagener [29], and Eichner and Wagener [18] [30] carried out some compara-

[1]See, for example, Battermann, Broll and Wahl [19], Broll, Wahl and Wong [20], Wong and Ma [17], and Eichner and Wagener [24].

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tive static analysis under uncertainty within the mean-standard deviation approach and the notation S is widely used in these analysis.

To develop the model, we first define some notations for the related elasticities as follows:

$$\varepsilon_{F,x_j} = \frac{\partial F}{\partial x_j}\frac{x_j}{F} = \frac{F_j x_j}{F},\ j = 1,\cdots,n;$$

$$\varepsilon_{\mu,x_j} = \frac{\partial \mu_\Pi}{\partial x_j}\frac{x_j}{\mu_\Pi},\ j = 1,\cdots,n;$$

$$\varepsilon_{\sigma,x_j} = \frac{\partial \sigma_\Pi}{\partial x_j}\frac{x_j}{\sigma_\Pi},\ j = 1,\cdots,n; \tag{2.2}$$

$$\varepsilon_{S,\mu} = \frac{\partial S}{\partial \mu_\Pi}\frac{\mu_\Pi}{S} \text{ and } \varepsilon_{S,\sigma} = -\frac{\partial S}{\partial \sigma_\Pi}\frac{\sigma_\Pi}{S}.$$

To proceed our analysis, we then derive the first order conditions by carrying some simple computations to lead the following equations:

$$\Phi\left(x_n^*,\lambda\right) \equiv \mu_p F_n^* - \mu_{p_n} - S^*\frac{\partial \sigma_\Pi}{\partial x_n^*} = 0;$$

$$\Psi\left(x_i^*,\lambda\right) \equiv \mu_p F_i^* - p_i - S^*\frac{\partial \sigma_\Pi}{\partial x_i^*} = 0,\ i = 1,\cdots,n-1; \tag{2.3}$$

in which

$$\frac{\partial \sigma_\Pi}{\partial x_n} = \frac{\sigma_p^2 F F_n + \sigma_{p_n}^2 x_n - \sigma_{p,p_n}\left(F + x_n F_n\right)}{\sigma_\Pi};$$

$$\frac{\partial \sigma_\Pi}{\partial x_i} = \frac{\sigma_p^2 F F_i - \sigma_{p,p_n} x_n F_i}{\sigma_\Pi},\ i = 1,\cdots,n-1;$$

and $\lambda = \left(\mu_p, \mu_{p_n}, \sigma_p, \sigma_{p_n}, \sigma_{p,p_n}\right)$.

Furthermore, from equations (2.3), we have

$$\frac{\partial \sigma_\Pi}{\partial x_n} = \frac{\mu_p F_n - \mu_{p_n}}{S} = \frac{\partial \mu_\Pi / \partial x_n}{S};$$

$$\frac{\partial \sigma_\Pi}{\partial x_i} = \frac{\mu_p F_i - p_i}{S} = \frac{\partial \mu_\Pi / \partial x_i}{S},\ i = 1,\cdots,n-1.$$

We are interested in obtaining the optimal input demands responds to a changes in the parameters of the decision problems. In the following section, we provide complete characterizations of the comparative statics of $x_i^*\left(\lambda\right)$ and $x_n^*\left(\lambda\right)$ with respect to σ_p, σ_{p_n} and σ_{p,p_n}.

3. Impacts of Variances of Energy and Output Prices

Now we turn to investigate the impacts of variances of energy and output prices on the optimal inputs demand. The following results are obtained.

Theorem 3.1 *Under the model setup to maximize the expected utility of the profit* $V\left(\mu_\Pi, \sigma_\Pi\right)$ *stated in* (2.1), *we have*

1. $\partial x_j / \partial \sigma_p < 0, j = 1, \cdots, n$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than* $2/H(x_j) - 1$ *with* $H(x_j) = \varepsilon_{\sigma,x_j} / \varepsilon_{F,x_j}$ *; and*

2. *the firm will decrease the inputs when the variance of output price increases if and only if the elasticity of risk aversion with respect to the standard deviation of the final profit is less than twice of the elasticity of the production function with respect to the input over the elasticity of the standard deviation with respect to the input minus one.*

Theorem 3.2 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_n / \partial \sigma_{p_n} < 0$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than* $2/\varepsilon_{\sigma,x_n} - 1$ *;*

2. *the firm will decrease the demand for energy when the variance of energy price increases if and only if the elasticity of risk aversion with respect to the standard deviation of the final profit is less than two over the elasticity of standard deviation with respect to the energy minus one.*

Theorem 3.3 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_i / \partial \sigma_{p_n} < 0, i = 1, \cdots, n-1$ *if and only if* $S_\sigma > 0$*, and*

2. *the firm will decrease the demands for the non-risky input when the variance of energy price increases if and only if* $S_\sigma > 0$.

We provide a short proof for Theorems 3.1 to 3.3 as follows:

Proof: Applying the implicit function theorem, we get

$$\frac{\partial x_n}{\partial \sigma_p} = -\frac{\Phi_{\sigma_p}}{\Phi_{x_n}} = -\frac{1}{\Phi_{x_n}}\left(-S\frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_p} - \frac{\partial \sigma_\Pi}{\partial x_n} S_\sigma \frac{\partial \sigma_\Pi}{\partial \sigma_p}\right),$$

where

$$\frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_p} = \frac{\sigma_p F\left(\sigma_p^2 F^2 F_n + 2\sigma_{p_n}^2 x_n^2 F_n - 3FF_n x_n \sigma_{p,p_n} - \sigma_{p_n}^2 x_n F + \sigma_{p,p_n} F^2\right)}{\sigma_\Pi^3};$$

$$\frac{\partial \sigma_\Pi}{\partial x_n}\frac{\partial \sigma_\Pi}{\partial \sigma_p} = \sigma_p F^2 \frac{\sigma_p^2 FF_n + \sigma_{p_n}^2 x_n - \sigma_{p,p_n}\left(F + x_n F_n\right)}{\sigma_\Pi^2}.$$

Since $\Phi_{x_n} < 0$, we have

$$\begin{aligned}\operatorname{sign}\left(\frac{\partial x_n}{\partial \sigma_p}\right) &= \operatorname{sign}\left(-S\frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_p} - \frac{\partial \sigma_\Pi}{\partial x_n} S_\sigma \frac{\partial \sigma_\Pi}{\partial \sigma_p}\right) \\ &= \operatorname{sign}\left(-1 + \varepsilon_{S,\sigma}\frac{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_p}{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_p \times \sigma_\Pi}\right) \\ &= \operatorname{sign}\left(\varepsilon_{S,\sigma} - \frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_p \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_p}\right).\end{aligned}$$

Define $H(x_n) = \varepsilon_{\sigma,x_n} / \varepsilon_{F,x_n} = \partial \sigma_\Pi / \partial x_n \times F / (\sigma_\Pi F_n)$. After some simple computations, we get

$$\frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_p \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_p} = \frac{2}{H(x_n)} - 1.$$

Similarly, for the term $\partial x_i / \partial \sigma_p$, $i = 1, \cdots, n-1$, we have

$$\operatorname{sign}\left(\frac{\partial x_i}{\partial \sigma_p}\right) = \operatorname{sign}\left(\varepsilon_{S,\sigma} - \frac{\partial^2 \sigma_\Pi / \partial x_i \partial \sigma_p \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_i \times \partial \sigma_\Pi / \partial \sigma_p}\right) = \operatorname{sign}\left[\varepsilon_{S,\sigma} - \left(\frac{2}{H(x_i)} - 1\right)\right].$$

with $H(x_i) = \varepsilon_{\sigma,x_i} / \varepsilon_{F,x_i} = \partial \sigma_\Pi / \partial x_i \times F / (\sigma_\Pi F_i)$. Then, the statements in Theorem 3.1 are proved.

For the term $\partial x_n / \partial \sigma_{p_n}$, we have

$$\operatorname{sign}\left(\frac{\partial x_n}{\partial \sigma_{p_n}}\right) = \operatorname{sign}\left(-S \frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_{p_n}} - \frac{\partial \sigma_\Pi}{\partial x_n} S_\sigma \frac{\partial \sigma_\Pi}{\partial \sigma_{p_n}}\right) = \operatorname{sign}\left(\varepsilon_{S,\sigma} - \frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_{p_n} \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_{p_n}}\right)$$

with

$$\frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_{p_n}} = \frac{\sigma_{p_n} x_n \left(2\sigma_p^2 F^2 + \sigma_{p_n}^2 x_n^2 - 3F x_n \sigma_{p,p_n} - \sigma_p^2 F x_n F_n + \sigma_{p,p_n} F_n x_n^2\right)}{\sigma_\Pi^3};$$

$$\frac{\partial \sigma_\Pi}{\partial x_n} \frac{\partial \sigma_\Pi}{\partial \sigma_{p_n}} = \sigma_{p_n} x_n^2 \frac{\sigma_p^2 F F_n + \sigma_{p_n}^2 x_n - \sigma_{p,p_n}\left(F + x_n F_n\right)}{\sigma_\Pi^2}.$$

Thus, we have

$$\frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_{p_n} \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_{p_n}} = \frac{2}{\varepsilon_{\sigma,x_n}} - 1.$$

For the last term $\partial x_i / \partial \sigma_{p_n}$, we have

$$\operatorname{sign}\left(\frac{\partial x_i}{\partial \sigma_{p_n}}\right) = \operatorname{sign}\left(-\frac{\partial \sigma_\Pi}{\partial x_i} \frac{\partial \sigma_\Pi}{\partial \sigma_{p_n}} S_\sigma\right).$$

Thus, the assertions of Theorems 3.1 to 3.3 hold.

Theorems 3.1 to 3.3 tell us the impact of the variance of energy and output price on the input demands are complex and relates to several elasticities. What we should pay attention to is the fact that under the situation with joint energy and output price uncertainties, the change of the variance of energy price can have some impacts on the demands of the inputs with fixed prices. Eichner and Wagener [27] show that the convexity of the slope of (μ, σ)-indifference curves with respect to σ, *i.e.*, $S_{\sigma,\sigma}(\mu, \sigma) > 0$ together with $S_\sigma > 0$, generally characterizes the comparative static effect that individuals behave in a more risk-averse way when they are confronted with an increase in an independent background risk. Inspired by the concept of "risk vulnerability" in the expected- utility framework, Eichner and Wagener [26] refer $S_{\sigma,\sigma}(\mu, \sigma) > 0$ as variance vulnerability. Following Tobin [31] and Sinn [32], we assume that (μ, σ)-indifference curves enter the μ-axis with slope zero, to be precise; that is, $S(\mu, 0) = V_\sigma(\mu, 0) = 0$. Under these assumptions, Eichner and Wagener [27] showed that

$S_{\sigma,\sigma}(\mu,\sigma) > 0$ is equivalent to $S - \sigma_\Pi S_\sigma < 0$. Since S is always non-negative, it can lead $S_\sigma > 0$. Consequently, if the firms' preferences exhibit variance vulnerability, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demands for the non-risky input.

4. Impacts of Covariance of Energy and Output Prices

Next, we consider the impact of the covariance of energy and output prices on the demand for the inputs. We have the following observations for the impact of the covariance of energy and output prices as follows:

Theorem 4.1 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_n / \partial \sigma_{p,p_n} < 0$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than* $1/G(x_n) - 1$ *with* $G(x_n) = \varepsilon_{\sigma,x_n} / (\varepsilon_{F,x_n} + 1)$, *and*

2. *the firm will decrease the demand of the energy when the covariance of output and energy price increases if and only if the elasticity of risk aversion with respect to the standard deviation of final profit is less than the elasticity of production function with respect to the energy plus the inverse of the elasticity of standard deviation with respect to the energy minus one.*

Theorem 4.2 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_i / \partial \sigma_{p,p_n} < 0, i = 1, \cdots, n$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than* $1/H(x_i) - 1$ *with* $H(x_i) = \varepsilon_{\sigma,x_i} / \varepsilon_{F,x_i}$, *and*

2. *the firm will decrease the demand for the non-risky inputs when the covariance of output and energy price increases if and only if the elasticity of risk aversion with respect to the standard deviation of the final profit is less than the elasticity of production function with respect to the input over the elasticity of standard deviation with respect to the input minus one.*

We provide a short proof for Theorems 4.1 and 4.2 as follows:

Proof: For the term $\partial x_n / \partial \sigma_{p,p_n}$, similar to the above arguments, we can have

$$\operatorname{sign}\left(\frac{\partial x_n}{\partial \sigma_{p,p_n}}\right) = \operatorname{sign}\left(\varepsilon_{S,\sigma} - \frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_{p,p_n} \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_{p,p_n}}\right)$$

with

$$\frac{\partial^2 \sigma_\Pi}{\partial x_n \partial \sigma_{p,p_n}} = \frac{x_n F^2 \sigma_{p,p_n} - \sigma_p^2 F^3 + x_n^2 F F_n \sigma_{p,p_n} - x_n^3 F_n \sigma_{p_n}^2}{\sigma_\Pi^3},$$

$$\frac{\partial \sigma_\Pi}{\partial x_n} \frac{\partial \sigma_\Pi}{\partial \sigma_{p,p_n}} = -x_n F \frac{\sigma_p^2 F F_n + \sigma_{p_n}^2 x_n - \sigma_{p,p_n}(F + x_n F_n)}{\sigma_\Pi^2}.$$

Define $G(x_n) = \varepsilon_{\sigma,x_n} / (\varepsilon_{F,x_n} + 1) = \partial \sigma_\Pi / \partial x_n \times x_n F / [\sigma_\Pi (x_n F_n + F)]$. Conducting some simple computations yields

$$\frac{\partial^2 \sigma_\Pi / \partial x_n \partial \sigma_{p,p_n} \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_n \times \partial \sigma_\Pi / \partial \sigma_{p,p_n}} = \frac{1}{G(x_n)} - 1.$$

For the term $\partial x_i / \partial \sigma_{p,p_n}$, we obtain the following equation:

$$\operatorname{sign}\left(\frac{\partial x_i}{\partial \sigma_{p,p_n}}\right) = \operatorname{sign}\left(\varepsilon_{S,\sigma} - \frac{\partial^2 \sigma_\Pi / \partial x_i \partial \sigma_{p,p_n} \times \sigma_\Pi}{\partial \sigma_\Pi / \partial x_i \times \partial \sigma_\Pi / \partial \sigma_{p,p_n}}\right)$$
$$= \operatorname{sign}\left[\varepsilon_{S,\sigma} - \left(\frac{1}{H(x_i)} - 1\right)\right].$$

Again, the impact of the covariance of energy and output price greatly depends on several elasticities.

5. Some Special Cases

In this section, we consider two special cases of our model. First, we deal with the situation with only uncertain energy price. In this case, we can have $\sigma_p = \sigma_{p,p_n} = 0$ and $\sigma_\Pi = \sigma_{p_n} x_n$. We have the following observations for the impacts of the variance of energy price as shown in the following theorems:

Theorem 5.1 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_n / \partial \sigma_{p_n} < 0$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than one*;
2. *the firm will decrease the demand for energy when the variance of energy price increases if and only if the elasticity of risk aversion with respect to the standard deviation of the final profit is less than one*; *and*
3. *if* $S_\sigma > 0$, $\partial x_n / \partial \sigma_{p_n} < 0$; *that is*, *if* $S_\sigma > 0$, *increasing the variance of energy price will surely cause the risk averse firm to decrease the demand for energy*.

Theorem 5.2 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *we have*

1. $\partial x_i / \partial \sigma_{p_n} \equiv 0, i = 1, \cdots, n-1$.
2. *In addition*, *increasing the variance of energy price has no effect on the demands for inputs with fixed prices*.

The proofs of Theorems 5.1 to 5.2 are simple and similar to arguments in Section 3. We omit the details.

Compared with the results in Theorems 3.1 to 3.3, Theorems 5.1 and 5.2 here give us clearer findings. If the firms' preferences exhibit variance vulnerability, increasing the variance of energy price will surely cause the risk averse firm to decrease the demand for energy. From Theorem 5.2, we can see that the energy price uncertainty has no effect on the demands for the inputs with fixed prices. The results here are different from those under joint energy and output price uncertainties. To be specific, under joint energy and output price uncertainties, even the firms's preferences exhibit variance vulnerability, increasing the variance of energy price may not necessarily cause the risk averse firm to decrease the demand for energy. Instead in this case, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demands for the non-risky input.

Now we turn to the case with only uncertain output price. In this situation, we can have $\sigma_{pn} = \sigma_{p,pn} = 0$ and $\sigma_\Pi = \sigma_p F$. We have the following observations for the impacts of variance of energy price as shown in the following theorem:

Theorem 5.3 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in* (2.1), *the impacts of the variance of energy price follows*:

1. $\partial x_j / \partial \sigma_p < 0,\ j = 1, \cdots, n-1$ *if and only if* $\varepsilon_{S,\sigma}$ *is less than one*;
2. *the firm will decrease the demand for energy when the variance of energy price increases if and only if the elasticity of risk aversion with respect to the standard deviation of the final profit is less than one*; *and*
3. *If* $S_\sigma > 0$, $\partial x_n / \partial \sigma_{p_n} < 0$; *that's, if* $S_\sigma > 0$, *increasing the variance of energy price will surely cause the risk averse firm to decrease the demand for energy*.

Theorems 5.1 to 5.3 demonstrate that the concept of variance vulnerability is important in describing the behaviours of the risk averse firm under price uncertainties.

6. An Empirical Example

We used U.S. natural gas monthly data data for the period March 2001-March 2010 (obtained from Henry Hub). We used the method of Alghalith [1] to generate corresponding data series for μ_{p_n} and σ_{p_n}. Also adopting the method of Alghalith [33], we estimated the following comparative statics for each month (and we calculated the average values for the entire period)

$$\frac{\partial x_n}{\partial \mu_{p_n}} \text{ and } \frac{\partial x_n}{\partial \sigma_{p_n}}.$$

For March 2010, we get

$$\frac{\partial x_n}{\partial \mu_{p_n}} = 409229.7 \text{ and } \frac{\partial x_n}{\partial \sigma_{p_n}} = -503985.2,$$

and obtain the average values to be

$$\frac{\partial x_n}{\partial \mu_{p_n}} = 459511.6504 \text{ and } \frac{\partial x_n}{\partial \sigma_{p_n}} = -2.70805 \times 10^{19}.$$

We note that $\partial x_n / \partial \mu_{p_n} > 0$ which is consistent with our theoretical result. That is, an increase in the energy price does not necessarily reduce the energy demand. Also, $\partial x_n / \partial \sigma_{p_n} < 0$ implying that $\varepsilon_{S,\sigma}$ is less than $2/\varepsilon_{\sigma,x_n} - 1$.

7. Concluding Remarks

As documented in the literature such as Alghalith [14] and Alghalith [2] [15] [33], the energy price is uncertain. Furthermore, the price of output can be random also. In this paper, we analyze the impacts of joint energy and output price uncertainties in a mean-variance framework. The concept of elasticity plays a central role in the analysis. Furthermore, if the firms's preferences exhibit variance vulnerability, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demand for the non-risky inputs. As for the impacts of the covariance of energy price and output price, the results are unclear and greatly depend on several elasticities.

In this paper, we also consider two special cases of our model. In the first case of only uncertain energy price, we can assert that the uncertain energy price has no impact on the demands for non-risky inputs. These results are very different from the results obtained under the case of joint energy and output price uncertainties and they are intuitive. We also consider the case of only uncertain output price. Again, the concept of variance vulnerability is important in describing the behaviors of a risk aversion firm under multiple price uncertainties. Investors could incorporate other investment approaches, e.g., see Kung, *et al.* [34] into the approach introduced in this paper to get a better investment decision making.

We note that the theory developed in our paper could be used in many areas, for example, Vorotnikova and Asci [35] developed an empirical estimation for multi-output production decision using multiple inputs in the profit maximizing firm. Extension could extend their model by incorporating the theory developed in this paper. Other areas that can be improved by incorporating the theory developed in this paper including futures [36] [37], portfolio allocation among REITs, stocks, and bonds [38] [39], exchange rate [40], trade [41] [42].

We also note that mean-variance framework is related to stochastic dominance (SD) theory, see, for example, Wong [43] and Wong and Ma [44] for more information. Nonetheless, Rrisk measures are found to be interesting because they could be related to stochastic dominance theory and thus it is well-known that domination by risk measures could be related to expected utility maximization, see, for example, Ma and Wong [44]. However, most of the risk measures are only related to second-order SD, see, for example, Ma and Wong [44] and Guo, *et al.* [45] and Guo and Wong [46]. Nonetheless, recently, Niu, *et al.* [47] find that risk measures could be related to first-order SD, while Niu, *et al.* [48] find that risk measures could also be related to high-order SD. Extension could include developing mean-variance framework to be related to first and higher-order SD.

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