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The Role of Money: Credible Asset or Numeraire?

Masayuki Otaki

Institute of Social Science, University of Tokyo, Tokyo, Japan
Email: ohtaki@iss.u-tokyo.ac.jp

ABSTRACT

It is well known that money is neutral if 1) people hold the extraneous belief that it is an only numeraire and does not possess intrinsic value; and 2) new money is injected into an economy as its own interest in the OLG model under perfect information (Lucas [1] Theorem (2)). We find that whenever 1) is not satisfied and money is rationally held to have substance value, money becomes non-neutral even if we use the same model as Lucas [1].

1. Introduction

This article shows that the multiplicity of rational beliefs concerning the value of money decides whether money is neutral under perfect information structure in the two-period OLG model. The result is kept intact even if new money injected into the economy is subject to the model of Lucas [1]. This result contrasts with Otani [2] and Otaki [3-5].

Even if the nominal rate of interest on money increases (*i.e.*, money supply increases), people can consistently believe that the purchasing power of money (the inverse of the next period price level) is retained.[1] Then the real interest rate becomes higher, and thereby matching the supply, and the demand for money increases.

From assumptions concerning the utility function, this situation also implies the reduction of current consumption and leisure. Thus, the monetary expansion increases current total output, and hence, money becomes non-neutral.

We must note that the attained equilibrium is stationary in the sense that values of real endogenous variables such as current/future consumption and leisure are entirely time-independent. This assertion holds, since once the real interest rate is heightened by an increase of the nominal interest rate (increment of nominal money supply per capita), one may expect that the change in the inflation rate will equal that of the nominal interest rate; the heightend real interest rate is kept intact, and thus, the equilibrium becomes self-enforcing and stationary.

The rest of paper is organized as follows. In Section 2, we construct the same model as Lucas [1], except for the formation of rational expectation concerning the value of money, and proves the non-neutrality of money. A

welfare economics implication is also analyzed. Section 3 contains brief concluding remarks.

2. The Model

2.1. The Structure of the Model

We use essentially the same model as Lucas [1], excluding uncertainty. In every period a unit individual is born and lives two periods. Each individual has an indentical utility function $\tilde{U}$:

$$\tilde{U} \equiv U(c_1, n) + V(c_2) \qquad (1)$$

where c_1 and c_2 are the current and future consumption level, and n denotes the hours worked per individual.

Furthermore, U and V satisfy the following properties:

$$U_c > 0, U_n < 0, \qquad (2)$$

$$U_{cc} < 0, U_{nn} < 0, U_{cc} + U_{cn} < 0, U_{cn} + U_{nn} < 0, \qquad (3)$$

$$V''(c_2)c_2 + V'(c_2) > 0, \frac{c_2 V''(c_2)}{V'(c_2)} \leq -a < 0, \qquad (4)$$

$$\lim_{c_2 \to 0} V'(c_2) = +\infty, \lim_{c_2 \to \infty} V'(c_2) = 0. \qquad (5)$$

2.2. The Maximization Problem of Representative Individual

Each individual maximizes his/her lifetime utility $\tilde{U}$ subject to the following budget constraint:

$$n_t \leq c_{1t} + \frac{m_{t-1}}{p_t}, \; p_{t+1}c_{2t+1} \leq m_{t-1}x,$$

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[1]Otaki [5] defines such a rational expectation as money being *credible*.

$$\Leftrightarrow n_t \le c_{1t} + \rho_t c_{2t+1}, \tag{6}$$

$$\rho_t \equiv \frac{p_{t+1}}{p_t x}, \tag{7}$$

where x denotes the increment of money supply per capita. ρ_t is the inverse of the real interest rate.

The Kuhn-Tucker condition implies that the optimal decision $\left(c_{1t}^*, c_{2t}^*, n_t^*\right)$ satisfies

$$\frac{\mathrm{d}n_t^*}{\mathrm{d}c_{1t}^*} = -\frac{U_c\left(c_{1t}^*, n_t^*\right)}{U_n\left(c_{1t}^*, n_t^*\right)} = 1, \tag{8}$$

$$\frac{V'\left(c_{2t}^*\right)}{U_c\left(c_{1t}^*, n_t^*\right)} = \rho_t, \tag{9}$$

$$n_t^* = c_{1t}^* + \rho_t c_{2t}^*. \tag{10}$$

2.3. Market Equilibrium

There are two markets in the above model: the money market and good market. By Walras' law, we can neglect the equilibrium condition for the good market. The money market equilibrium condition is

$$m_{t-1}x = p_{t+1}^* c_{2t+1}^*. \tag{11}$$

Instead of the quality-theoretic equilibrium price function imposed by Lucas [1], let us assume that money is *credible* in the sense of Otaki [5]—That is, the rational expectation concerning the current purchasing power of money $\frac{1}{p_{t+1}^*}$ is not purturbed by an increse of x:

$$\frac{\mathrm{d}p_{t+1}^*}{\mathrm{d}x} = 0. \tag{12}$$

The general equilibrium of markets is attained by five equations: (8), (9), (10), (11), and (12). Endogenous variables are $\left(c_{1t}^*, c_{2t}^*, p_t^*, p_{t+1}^*, n_t^*\right)$.

The partial equilibrium of labor n_t^* and the young-generation's consumption c_{1t}^* is illustrated by **Figure 1**. The downward sloping curve AA is the locus of Equation (8), which is easily derived from Assumption (3).

The upward sloping curve BB is the locus of Equation (10), which is combined with Equation (9). The procedure is as follows: Substituting Equation (9) into (10), we obtain

$$n_t^* = c_{1t}^* + c_{2t}^* \frac{V'}{U_c}.$$

Differentiating both sides of the above equation,

$$\frac{\mathrm{d}n_t^*}{n_t^* - c_{1t}^*} = \frac{\mathrm{d}c_{1t}^*}{n_t^* - c_{1t}^*} + \frac{\mathrm{d}\left[V'c_{2t}^*\right]}{V'c_{2t}^*} - \frac{U_{cc}\mathrm{d}c_{1t}^* + U_{cn}\mathrm{d}n_t^*}{U_c}$$

$$= \frac{\mathrm{d}n_t^*}{n_t^* - c_{1t}^*} + \frac{\mathrm{d}\left[V'c_{2t}^*\right]}{V'c_{2t}^*} - \frac{c_{1t}^*\left[U_{cc} + U_{cn}\dfrac{\mathrm{d}n_t^*}{\mathrm{d}c_{1t}^*}\right]\mathrm{d}c_{1t}^*}{U_c} \tag{13}$$

$$\Leftrightarrow \frac{\mathrm{d}n_t^*}{n_t^* - c_{1t}^*} = \left[\frac{c_{1t}^*}{n_t^* - c_{1t}^*} - \frac{c_{1t}^*\left[U_{cc} + U_{cn}\right]}{U_c}\right]\frac{\mathrm{d}c_{1t}^*}{c_{1t}^*} + \frac{\mathrm{d}\left[V'c_{2t}^*\right]}{V'c_{2t}^*} \tag{14}$$

holds. Hence Curve BB is upward sloping for any fixed c_{2t}^*. When the money market equibrates, the equilibrium consumption of younger generation and output is determined at the intersection of Curves AA and BB (Point E_0).

Whenever money is *credible*, it is facile to depict the property of money market equilibrium. From Equations (11) and (12), we obtain

$$\frac{\mathrm{d}c_{2t}^*}{c_{2t}^*} = \frac{\mathrm{d}x}{x}. \tag{15}$$

Since $V'c_{2t}^*$ is an increasing function of c_{2t}^* from Assumption (4), Equations (14) and (15) imply that Curve BB shifts toward the south-east, like Curve $B'B'$, by an increase of x. Thus, the economy moves from Point E_0 to E_1.

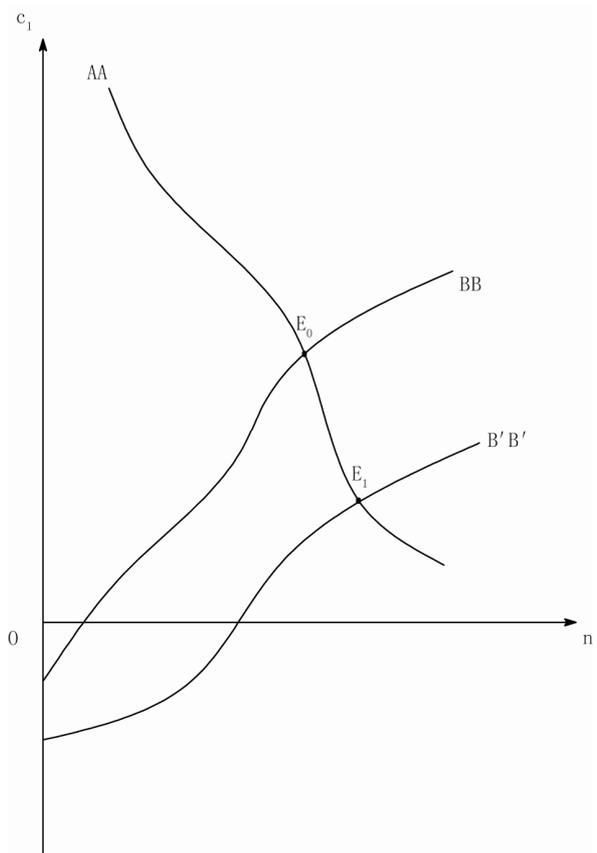Accordingly, as long as money is *credible*, a monetary

Figure 1. The determination of $\left(c_1^*, n^*\right)$.

expansion increases the output n_t^* and future consumption c_{2t}^*, and decreses the younger-generation's consumption c_{1t}^*. To sum up:

Theorem 1. *If money is a credible asset, it becomes non-neutral to the real economy. An acceleration of monetary growth hightens the real interest of money, and hence, stimulates future consumption and output/labor supply, economizes current consumption.*

Next we shall show that the equilibrium above depicted is a stationary rational expectation equilibrium. Suppose that the economy is located at Point E_1 by an increase of x, and individuals believe that the higher equilibrium real interest rate $\dfrac{1}{\rho^{1*}}$ prevails thereafter.

Then by the definition of ρ (Equation (7)),

$$\frac{\mathrm{d}\rho^{1*}}{\rho^{1*}} = \frac{\mathrm{d}x}{x} - \left[\frac{\mathrm{d}p_{t+j+1}^*}{p_{t+j+1}^*} - \frac{\mathrm{d}p_{t+j}^*}{p_{t+j}^*} \right] = 0, \ j \geq 1 \qquad (16)$$

holds. That is, individuals consider that the change in the equilibrium inflation rate is equal the acceleration rate of monetary growth because there is no substantial change in the economic environment after period $t+1$. Since $m_{t+j-1}x = p_{t+j+1}^* c_{2t+j+1}^*$ and $m_{t+j} = m_{t+j-1}x$,

$$\frac{\mathrm{d}x}{x} = \left[\frac{\mathrm{d}p_{t+j+1}^*}{p_{t+j+1}^*} - \frac{\mathrm{d}p_{t+j}^*}{p_{t+j}^*} \right] + \left[\frac{\mathrm{d}c_{2t+j+1}^*}{c_{2t+j+1}^*} - \frac{\mathrm{d}c_{2t+j}^*}{c_{2t+j}^*} \right] \qquad (17)$$

also holds. Combining Equation (17) with (16), we finally obtain

$$\frac{\mathrm{d}c_{2t+j+1}^*}{c_{2t+j+1}^*} = \frac{\mathrm{d}c_{2t+j}^*}{c_{2t+j}^*}, \ j \geq 1. \qquad (18)$$

Thus, the equilibrium consumption of an old individual c_2^* is time-independent.

It is clear from Equations (8) and (14) that the rest of the two endogenous variables $\left(c_{1t+j}^*, n_{t+j}^* \right)$ are also time-independent. Consequently, the equilibrium illustrated by Point E_1 is stationary in the sense that every equilibrium value of endgenous variables is time-independent. One can thus affirm

Theorem 2. *The rational expectation equilibrium defined by Equations (8)-(10), (12), and (16) is stationary (i.e., time-independent). Hence the heightned real interest rate caused by an increase of the nominal interest on money x permanently affects the real variables.*

2.4. A Welfare Implication of the Model

By Theorem 1, a monetary expansion (an increase in x) stimulates the equilibrium real GDP n^* through the rise of the real rate of interest. We here consider its welfare economics implication. Let the Lagrangean of individual decision L that is evaluated at the equilibrium value.

Then, using the envelop theorem, we obtain

$$\frac{\mathrm{d}\tilde{U}}{\mathrm{d}\rho^*} = \frac{\mathrm{d}L}{\mathrm{d}\rho^*} = \frac{\partial L}{\partial \rho^*} = -\lambda^*(x) c_2^*(x) < 0,$$

where λ^* is the Lagrangean multiplier. Accordingly, a monetary expansion improves the economic welfare since it makes future goods cheaper.

3. Concluding Remarks

This paper shows that money is non-neutral as long as it is *credible* even if we obey the money-supply rule proposed by Lucas [1]. A monetary expansion (an acceleration of the money growth rate) surely hightens the real rate of interest of money whenever people believe that money is *credible*. The effect of intertempral substitution leads them to work more to prepare for more future consumption, and thus, the aggregate products increases. It also implies that the economic welfare is improved by a monetary expansion.

REFERENCES

[1] R. E. Lucas Jr., "Expecations and the Neutrality of Money," *Journal of Economic Theory*, Vol. 4, No. 2, 1972, pp. 103-124. doi:10.1016/0022-0531(72)90142-1

[2] K. Otani, "Rational Expectations and Non-Nuetrality of Money," *Weltwirschaftliches*, Vol. 121, 1985, pp. 207-216.

[3] M. Otaki, "The Dynamically Extended Keynesian Cross and the Welfare-Improving Fiscal Policy," *Economics Letters*, Vol. 96, No. 1, 2007, pp. 23-29. doi:10.1016/j.econlet.2006.12.005

[4] M. Otaki, "A Welfare Economic Foundation for the Full-Employment Policy," *Economics Letters*, Vol. 102, No. 1, 2009, pp. 1-3. doi:10.1016/j.econlet.2008.08.003

[5] M. Otaki, "A Pure Theory of Aggregate Price Determination," *Theoretical Economics Letters*, Vol. 1, No. 3, 2011, pp. 122-128. doi:10.4236/tel.2011.13026