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A General Cournot-Bertrand Model with Homogeneous Goods*

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Abstract

We analyze a Cournot-Bertrand model where one firm competes in output and the other competes in price. With general demand functions and perfectly homogeneous products, we show that the unique Nash equilibrium is the perfectly competitive equilibrium. Equilibrium price equals marginal cost, the Cournot-type firm produces the perfectly competitive level of market output, and the Bertrand-type firm exits the market. Even with just one firm in the market, the presence of a potential Bertrand-type competitor provides sufficient discipline to guarantee a competitive outcome.

Keywords: Cournot-Bertrand Model, Product Differentiation

1. Introduction

There has been increasing interest in the static Cournot-Bertrand model in which one firm competes in output (a la Cournot) and the other competes in price (a la Bertrand). When two firms have the choice to compete in output or in price, Singh and Vives [1] show that under certain demand and cost conditions the dominant strategy is for each firm to compete in output rather than price (*i.e.*, Cournot dominates Bertrand behavior and Cournot-Bertrand behavior). More recently, Tremblay *et al.* [2] show how different institutional and technological conditions can change firm payoffs so that Bertrand behavior or Cournot-Bertrand behavior becomes optimal.[1]

Given that there are many theoretical possibilities, Kreps and Scheinkman [3] argue that whether firms compete in output or in price is ultimately an empirical question. In the real world, Cournot, Bertrand, and Cournot-Bertrand behavior are observed. Vegetable producers set quantities at local farmers' markets, while restaurants set prices. In the market for small cars, Saturn and Scion dealers set prices and Honda and Subaru dealers set quantities.[2] Given this observation, additional research on the Cournot-Bertrand model is warranted and may further our understanding of oligopoly markets.

Tremblay and Tremblay [6] investigate the Cournot-Bertrand model when the degree of product differentiation is allowed to vary. An interesting result emerges from their work: when products are perfectly homogeneous, the perfectly competitive outcome results in which the Cournot-type firm produces the competitive level of market output and the Bertrand-type firm exits the market. The mere threat of a Bertrand-type firm ensures the perfectly competitive outcome, demonstrating that a potential competitor can have a dramatic effect on market power. The main weakness with their work is that demand functions are assumed to be linear. In this paper, we show that their result holds with a general demand specification.

2. The Model

Two firms, 1 and 2, compete in a static Cournot-Bertrand game where firm 1 is the Cournot-type firm that competes in output and firm 2 is the Bertrand-type firm that competes in price. Firm i's output level is q_i and its price is p_i $\forall$ i = 1, 2. The goal of each firm is to maximize its profit (π_i). Information is complete.

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[1]That is, Tremblay *et al*. [2] show that when firms are given the choice of competing in output or in price, cost asymmetries can lead to a Nash equilibrium where one firm competes in output and the other competes in price.

[2]See Palmeri [4] and Tremblay *et al.* [5] for further discussion of why some small car dealers compete in price and others compete in output.

[3]Restaurants provide one example, where an Italian restaurant is darkly lit and serves spaghetti, while a Chinese restaurant is brightly decorated and serves stir-fried dishes.

Products are substitutes, and products may vary over a variety of characteristics.[3] Product differentiation of this sort can be incorporated into a linear demand system, as found in Dixit [7], Singh and Vives [1], and Beath and Katsoulacos [8]. The inverse demand function for firm i is $p_i = a - q_i - dq_j$, where j is firm i's rival, a is a positive constant, and d is an index of product differentiation, $d \; \varepsilon \; [0, 1]$. Products 1 and 2 are perfectly homogeneous when $d = 1$, and each firm is a monopolist when $d = 0$. Thus, product differentiation diminishes as $d \to 1$. In the Cournot-Bertrand model, this system must be solved so that demand is a function of the strategic variables, q_1 and p_2: $p_1 = \alpha - bq_1 + dp_2$ and $q_2 = a - p_2 - dq_1$, where $\alpha \equiv a - ad$ and $b \equiv 1 - d^2$. Firms face the same linear cost function, where $c \; \varepsilon \; (0, a)$ is defined as average and marginal cost. The profit equation for firm i is $\pi_i = (p_i - c)\, q_i$.

Tremblay and Tremblay [6] investigate this model. In Proposition 1, they show that the model has a stable Nash equilibrium (NE) when there is sufficient product differentiation (d is sufficiently close to 0). When d is sufficiently close to 1, the NE becomes unstable. Once $d = 1$, however, their Proposition 2 demonstrates that the equilibrium becomes stable once again. In this case of perfectly homogeneous goods, the equilibrium price equals marginal cost, only firm 1 survives (firm 2 produces no output), and firm 1 produces the perfectly competitive level of output (Q_{pc}). Like a contestable market (Baumol *et al.* [9]), this demonstrates how important a potential entrant can be to the level of price competition.

The goal of this paper is to prove that the conclusion in Proposition 2 is not conditional on the assumption that demand functions are linear. Here, we consider a general demand system: $p_1 = p_1(q_1, p_2)$ and $q_2 = q_2(q_1, p_2)$. Each demand function is differentiable and has a negative slope ($\partial p_1/\partial q_1 < 0$ and $\partial q_2/\partial p_2 < 0$), and products are substitutes ($\partial p_1/\partial p_2 > 0$ and $\partial q_2/\partial q_1 < 0$).[4] For notational convenience, the demand price is defined as $p(q_1')$ when products are perfect substitutes, $q = q_1'$ and $q_2 = 0$.[5] Under these conditions, the following proposition holds.

Proposition: In this duopoly market with perfectly homogeneous goods, there is a unique NE in which the equilibrium price equals marginal cost, firm 1 produces the perfectly competitive level of market output, and firm 2 produces zero output.

Proof: We investigate each of the possible strategy profiles.

1) First, we consider strategy profiles in which $q_1 \gtrless Q_{pc}$ and $p_2 \gtrless c$.

a) For firm 1:

$q_1 > Q_{pc}$ cannot be a NE strategy. At this level of output and given a negatively sloped demand function, $p(q_1 > Q_{pc}) < c$ and firm 1 earns negative profits. In this case, firm 1 can earn zero profit by exiting the industry.

$q_1 < Q_{pc}$ cannot be a NE strategy. If this is all that is produced ($q_2 = 0$), then $p(q_1 < Q_{pc}) > c$ (given a negatively sloped demand function). In this case, firm 2's best reply is to set $p_2 = p(q_1 < Q_{pc}) - \varepsilon > c$ for $\varepsilon > 0$. This enables firm 2 to produce a positive level of output and for both firms to earn a positive profit. Given $p_2 > c$, however, firm 1 can earn greater profit by increasing its production so that it is supplying all that is demanded at p_2. This leaves no residual demand for firm 2 (*i.e.*, $q_2 = 0$). Thus, firm 2 has an incentive to lower p_2 even further. This process of lowering p_2 and raising q_1 will continue until $q_1 = Q_{pc}$ and $p_2 = c$.

b) For firm 2:

$p_2 < c$ cannot be a NE strategy. At this price, firm 2 earns a negative profit and can earn zero profit by exiting the industry.

$p_2 > c$ cannot be a NE strategy. As demonstrated above, firm 1's best reply to $p_2 > c$ is to produce all that is demanded at p_2, such that $q_1 < Q_{pc}$. This in turn makes it profitable for firm 2 to charge a lower price than p_2. The process of lowering p_2 and raising q_1 will continue until $q_1 = Q_{pc}$ and $p_2 = c$.

2) Next, we consider the strategy profile $q_1 = Q_{pc}$ and $p_2 = c$.[6] This is a NE because a small deviation cannot increase the profit of either firm. As shown above, firm 1 cannot increase its profit by increasing or decreasing its output from Q_{pc}, and firm 2 cannot increase its profit by increasing or decreasing its price from c. These are the only alternative strategy profiles. Thus, $q_1 = Q_{pc}$ and $p_2 = c$ is the only NE. Q.E.D.

This result demonstrates the dramatic effect that a potential competitor can have on a market. In the Cournot-Bertrand model, the threat of a price competitor that produces a homogeneous good ensures that a monopolist will behave as a perfectly competitive firm. In this case, the potential entrant completely eliminates market power.

3. Conclusions

The Cournot-Bertrand model has several interesting qualities and is receiving renewed interest in the literature. Previous theoretical studies show that technological and institutional forces can make it profitable for firms within the same industry to choose different strategic variables. In addition, there is evidence that some firms compete in output and others compete in price in the U.S. market for small cars.

[4]The only restriction is that for the second-order conditions of profit maximization to hold, demand functions cannot be too convex. That is, the second derivative of each demand function with respect to its own price must be sufficiently small.

[5]Note that because products are perfect substitutes, p_1 will equal p_2 in equilibrium.

[6]Because products are perfectly homogeneous, $p_2 = p_1$.

In a model with general demand functions, we show that the unique NE in the Cournot-Bertand model with homogeneous goods is the perfectly competitive equilibrium. The equilibrium price equals marginal cost, the Cournot-type firm produces the perfectly competitive level of market output, and the Bertrand-type firm exits the market. Even with just one firm serving the market, the presence of a potential Bertrand-type competitor provides sufficient discipline to guarantee a competitive outcome.

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