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New Efficient Estimators of Population Mean Using Non-Traditional Measures of Dispersion

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Abstract

One of the aims in survey sampling is to search for the estimators with highest efficiency. In the present paper, three improved estimators of population mean have been proposed using some non-traditional measures of dispersion of auxiliary variable such as Gini's mean difference, Downton's method and probability weighted moments early given by Abid [1] with a special population parameter of auxiliary variable. The large sample properties that are biased and mean squared errors of the proposed estimators have been derived up to the first order of approximation. A theoretical comparison of the proposed estimators has been made with the other existing estimators of population mean using auxiliary information. The conditions under which the proposed estimators perform better than the other existing estimators of population mean have been given. A numerical study is also carried out to see the performances of the proposed and existing estimators of population mean and verify the conditions under which proposed estimators are better than other estimators. It has been shown that the proposed estimators perform better than the existing estimators as they are having lesser mean squared error.

Keywords

Study Variable, Auxiliary Variable, Bias, Mean Squared Error, Efficiency

1. Introduction

Sampling is done when the population is very large and we have to get the result very soon. The population parameters are estimated by the corresponding statistics in a natural sense. As it has been mentioned that the most suitable estimator for the estimation of population parameter is the corresponding statistics so to estimate population mean the most suitable estimator is the sample mean. Although he sample mean is an unbiased estimator of population mean and it has reasonably large variance and our aim is to search for the estimator with minimum variance or may be biased but with minimum mean squared error. This purpose is solved through the use of auxiliary information. Auxiliary information is obtained from auxiliary variable which is highly positively or negatively correlated with main variable under study. When the auxiliary variable is positively correlated with the main variable under study, ratio type estimators are used for improved estimation of population parameters. When it is negatively with the main variable under consideration, product type estimators are used for improved estimation of population parameters. In the present manuscript, we have confined our study to positively correlated populations only and proposed three ratio type estimators for improved estimation of population mean with higher efficiencies.

Let the population under consideration consists of N distinct and identifiable units and let $(x_i, y_i), i = 1, 2, \cdots, n$ be a two variable sample of size n taken from bivariate variables (X, Y) through simple random sampling without sampling scheme. Let $\bar{X}$ and $\bar{Y}$ be the population means of the auxiliary and the study variables respectively, and let $\bar{x}$ and $\bar{y}$ be the respective sample means and both are unbiased estimators of $\bar{X}$ and $\bar{Y}$ respectively. Let the correlation coefficient between the variables X and Y be denoted by ρ.

2. Existing Estimators under Review

As mentioned above most appropriate estimator of population mean is the sample mean $\bar{y}$ given by,

$$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

The above estimator is unbiased for population mean of the study variable and its variance up to the first order of approximation is given by,

$$V(t_0) = \frac{1-f}{n} S_y^2 \tag{1}$$

Cochran [2] proposed the following usual ratio estimator of population mean by using positively correlated auxiliary variable as,

$$t_R = \bar{y}\frac{\bar{X}}{\bar{x}}$$

This estimator is biased and the bias and mean squared error of this estimator, up to the first order of approximation respectively are given by,

$$B(t_R) = \frac{1-f}{n}\frac{1}{\bar{X}}\left[R_1 S_x^2 - \rho S_y S_x\right]$$

$$MSE(t_R) = \frac{1-f}{n}\left[S_y^2 + R_1^2 S_x^2 - 2R_1\rho S_y S_x\right], \tag{2}$$

where $R_1 = \dfrac{\bar{Y}}{\bar{X}}$

Many estimators of population mean have been given by various authors in the literature for improved estimation. The latest references can be made of Yadav [3], Yadav and Kadilar [4] [5], Yadav *et al.* [6] [7] [8] [9], Yadav and Mishra [10], Misra and Gupta [11] [12] and Misra *et al* [13]. The **Table 1** below represents different estimators of population mean using auxiliary variable along with their constants, biases and their mean squared errors.

Table 1. Various estimators of population mean, bias, mean squared error and constant.

Estimator	Bias	Mean Squared Error	Constant
$t_1=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$ Kadilar and Cingi [14]	$B(t_1)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_1^2$	$MSE(t_1)=\frac{1-f}{n}\left[R_1^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_1=\frac{\bar{Y}}{\bar{X}}$
$t_2=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x)$ Kadilar and Cingi [14]	$B(t_2)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_2^2$	$MSE(t_2)=\frac{1-f}{n}\left[R_2^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_2=\frac{\bar{Y}}{\bar{X}+C_x}$
$t_3=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2)$ Kadilar and Cingi [14]	$B(t_3)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_3^2$	$MSE(t_3)=\frac{1-f}{n}\left[R_3^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_3=\frac{\bar{Y}}{\bar{X}+\beta_2}$
$t_4=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$ Kadilar and Cingi [14]	$B(t_4)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_4^2$	$MSE(t_4)=\frac{1-f}{n}\left[R_4^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_4=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$
$t_5=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi [14]	$B(t_5)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$MSE(t_5)=\frac{1-f}{n}\left[R_5^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_5=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$t_6=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho)$ Kadilar and Cingi [15]	$B(t_6)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$MSE(t_6)=\frac{1-f}{n}\left[R_6^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_6=\frac{\bar{Y}}{\bar{X}+\rho}$
$t_7=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho)$ Kadilar and Cingi [15]	$B(t_7)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$MSE(t_7)=\frac{1-f}{n}\left[R_7^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_7=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$
$t_8=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x)$ Kadilar and Cingi [15]	$B(t_8)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_8^2$	$MSE(t_8)=\frac{1-f}{n}\left[R_8^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_8=\frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$
$t_9=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho)$ Kadilar and Cingi [15]	$B(t_9)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_9^2$	$MSE(t_9)=\frac{1-f}{n}\left[R_9^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_9=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$
$t_{10}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2)$ Kadilar and Cingi [15]	$B(t_{10})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$	$MSE(t_{10})=\frac{1-f}{n}\left[R_{10}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{10}=\frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$
$t_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1)$ Yan and Tian [16]	$B(t_{11})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$	$MSE(t_{11})=\frac{1-f}{n}\left[R_{11}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{11}=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$t_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\beta_2)}(\bar{X}\beta_1+\beta_2)$ Yan and Tian [16]	$B(t_{12})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2$	$MSE(t_{12})=\frac{1-f}{n}\left[R_{12}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{12}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2}$

Continued

$t_{13}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+M_d)}(\bar{X}+M_d)$ Subramani and Kumarpandiyan [17]	$B(t_{13})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{13}^2$	$MSE(t_{13})=\frac{1-f}{n}\left[R_{13}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{13}=\frac{\bar{Y}}{\bar{X}+M_d}$
$t_{14}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(C_x\bar{x}+M_d)}(C_x\bar{X}+M_d)$ Subramani and Kumarpandiyan [17]	$B(t_{14})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2$	$MSE(t_{14})=\frac{1-f}{n}\left[R_{14}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{14}=\frac{\bar{Y}C_x}{\bar{X}C_x+M_d}$
$t_{15}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+M_d)}(\bar{X}\beta_1+M_d)$ Subramani and Kumarpandiyan [17]	$B(t_{15})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2$	$MSE(t_{15})=\frac{1-f}{n}\left[R_{15}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{15}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+M_d}$
$t_{16}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+M_d)}(\bar{X}\beta_2+M_d)$ Subramani and Kumarpandiyan [17]	$B(t_{16})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{16}^2$	$MSE(t_{16})=\frac{1-f}{n}\left[R_{16}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{16}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+M_d}$
$t_{17}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+QD)}(\bar{X}\beta_1+QD)$ Jeelani *et al.* [18]	$B(t_{17})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{17}^2$	$MSE(t_{17})=\frac{1-f}{n}\left[R_{17}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{17}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+QD}$
$t_{18}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+G)}(\bar{X}+G)$ Abid *et al.* [1]	$B(t_{18})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{18}^2$	$MSE(t_{18})=\frac{1-f}{n}\left[R_{18}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{18}=\frac{\bar{Y}}{\bar{X}+G}$
$t_{19}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho+G)$ Abid *et al.* [1]	$B(t_{19})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{19}^2$	$MSE(t_{19})=\frac{1-f}{n}\left[R_{19}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{19}=\frac{\bar{Y}\rho}{X\rho+G}$
$t_{20}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+G)}(\bar{X}C_x+G)$ Abid *et al.* [1]	$B(t_{20})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{20}^2$	$MSE(t_{20})=\frac{1-f}{n}\left[R_{20}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{20}=\frac{\bar{Y}C_x}{\bar{X}C_x+G}$
$t_{21}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+D)}(\bar{X}+D)$ Abid *et al.* [1]	$B(t_{21})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{21}^2$	$MSE(t_{21})=\frac{1-f}{n}\left[R_{21}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{21}=\frac{\bar{Y}}{\bar{X}+D}$
$t_{22}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D)}(\bar{X}\rho+D)$ Abid *et al.* [1]	$B(t_{22})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{22}^2$	$MSE(t_{22})=\frac{1-f}{n}\left[R_{22}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{22}=\frac{\bar{Y}\rho}{X\rho+D}$
$t_{23}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D)}(\bar{X}C_x+D)$ Abid *et al.* [1]	$B(t_{23})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{23}^2$	$MSE(t_{23})=\frac{1-f}{n}\left[R_{23}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{23}=\frac{\bar{Y}C_x}{\bar{X}C_x+D}$
$t_{24}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+S_{pw})}(\bar{X}+S_{pw})$ Abid *et al.* [1]	$B(t_{24})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{24}^2$	$MSE(t_{24})=\frac{1-f}{n}\left[R_{24}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{24}=\frac{\bar{Y}}{\bar{X}+S_{pw}}$
$t_{25}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+S_{pw})}(\bar{X}\rho+S_{pw})$ Abid *et al.* [1]	$B(t_{25})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{25}^2$	$MSE(t_{25})=\frac{1-f}{n}\left[R_{25}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{25}=\frac{\bar{Y}\rho}{X\rho+S_{pw}}$
$t_{26}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+S_{pw})}(\bar{X}C_x+S_{pw})$ Abid *et al.* [1]	$B(t_{26})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{26}^2$	$MSE(t_{26})=\frac{1-f}{n}\left[R_{26}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{26}=\frac{\bar{Y}C_x}{\bar{X}C_x+S_{pw}}$

3. Proposed Estimators

Motivated by Abid *et al.* [1] and Subramani [19] and searching for the improved estimators, we have used a specific parameter as the ratio of correlation coefficient and coefficient of skewness of auxiliary variable along with some non-traditional parameters of auxiliary variable given by Abid *et al.* [1] as,

$$t_{p_1} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\tau \bar{x} + G\right)}\left(\tau \bar{X} + G\right),$$

$$t_{p_2} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\tau \bar{x} + D\right)}\left(\tau \bar{X} + D\right),$$

$$t_{p_3} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\tau \bar{x} + S_{pw}\right)}\left(\tau \bar{X} + S_{pw}\right),$$

where, $\tau = \rho / \beta_1$

To study the large sample approximations, we have used the following approximations as,

$$\bar{y} = \bar{Y}\left(1 + e_0\right) \quad \text{and} \quad \bar{x} = \bar{X}\left(1 + e_1\right)$$

such that

$$E\left(e_i\right) = 0, i = 0,1$$

and

$$E\left(e_0^2\right) = \frac{1-f}{n} C_y^2, \quad E\left(e_1^2\right) = \frac{1-f}{n} C_x^2,$$

and

$$E\left(e_0 e_1\right) = \frac{1-f}{n} C_{yx} = \frac{1-f}{n} \rho C_y C_x,$$

where $f = \frac{n}{N}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, and $C_x^2 = \frac{S_x^2}{\bar{X}^2}$.

Using above approximation and up to the first order of approximations, the biases and the mean squared errors of proposed estimators are given by,

$$B\left(t_{p_j}\right) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_{p_j}^2, \left(j = 1,2,3\right)$$

$$MSE\left(t_{p_j}\right) = \frac{1-f}{n}\left[R_{p_j}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right], \left(j = 1,2,3\right) \tag{3}$$

where,

$$R_{p_1} = \frac{\bar{Y}\tau}{\bar{X}\tau + G}, \quad R_{p_2} = \frac{\bar{Y}\tau}{\bar{X}\tau + D}, \quad R_{p_3} = \frac{\bar{Y}\tau}{\bar{X}\tau + S_{pw}}$$

4. Efficiency Comparison

In this section, the proposed estimators have been compared theoretically with the other existing estimators of population mean in terms of theirs variances and mean squared errors under simple random sampling without replacement scheme.

From Equation (3) and the from the Equation (1), the proposed estimators performs better than the mean per unit estimator if,

$$MSE\left(t_{p_j}\right)-V\left(\bar{y}\right)\leq 0$$

or,

$$\left[R_{p_j}^2 S_x^2-\rho^2 S_y^2\right]\leq 0$$

or,

$$R_{p_j}^2\leq\frac{\rho^2 S_y^2}{S_x^2}$$

or,

$$R_{p_i}\leq\pm\frac{\rho S_y}{S_x},\left(j=1,2,3\right) \tag{4}$$

The proposed estimators $t_{p_j}\left(j=1,2,3\right)$ in Equation (3) are better than the ratio estimator by Cochran [2] t_r in Equation (2) under the condition if,

$$MSE\left(t_{p_j}\right)-MSE\left(t_r\right)\leq 0$$

or,

$$\left[\left(R_{p_j}^2-R_1^2\right)S_x^2-\rho^2 S_y^2+2R_1\rho S_y S_x\right]\leq 0$$

or,

$$\left(R_{p_j}^2-R_1^2\right)S_x^2\leq\rho^2 S_y^2-2R_1\rho S_y S_x,\left(j=1,2,3\right) \tag{5}$$

From Equation (3) and the mean squared error of the estimators given by Kadilar and Cingi [14] in **Table 1**, the proposed estimators perform better than the Kadilar and Cingi [14] estimators under the condition if,

$$MSE\left(t_{p_j}\right)-MSE\left(t_i\right)\leq 0$$

or,

$$\left[R_{p_j}^2 S_x^2-R_i^2 S_x^2\right]\leq 0$$

or,

$$R_{p_j}\leq\pm R_i,\left(j=1,2,3\right),\left(i=1,2,3,4,5\right) \tag{6}$$

From the mean squared errors of proposed estimators and Kadilar and Cingi [15] estimators respectively in Equation (3) and in **Table 1**, the proposed estimators are better than the Kadilar and Cingi [15] estimators if,

$$MSE\left(t_{p_j}\right)-MSE\left(t_i\right)\leq 0$$

or,

$$\left[R_{p_j}^2 S_x^2-R_i^2 S_x^2\right]\leq 0$$

or,

$$R_{p_j} \le \pm R_i, \left(j=1,2,3\right), \left(i=6,7,8,9,10\right) \tag{7}$$

From Equation (3) and the mean squared error of the estimators given by Yan and Tian [16] in **Table 1**, the proposed estimators are better than Yan and Tian [16] estimators if,

$$MSE\left(t_{p_j}\right) - MSE\left(t_i\right) \le 0$$

or,

$$\left[R_{p_j}^2 S_x^2 - R_i^2 S_x^2\right] \le 0$$

or,

$$R_{p_j} \le \pm R_i, \left(j=1,2,3\right), \left(i=11,12\right) \tag{8}$$

From Equation (3) and the mean squared errors of the estimators given by Subramani and Kumarpandiyan [17] in **Table 1**, the proposed estimators perform better than Subramani and Kumarpandiyan [17] estimators if,

$$MSE\left(t_{p_j}\right) - MSE\left(t_i\right) \le 0$$

or,

$$\left[R_{p_j}^2 S_x^2 - R_i^2 S_x^2\right] \le 0$$

or,

$$R_{p_j} \le \pm R_i, \left(j=1,2,3\right), \left(i=13,14,15,16\right) \tag{9}$$

The proposed estimators are better than the estimators by Jeelani *et al.* [18] in **Table 1** under the condition if,

$$MSE\left(t_{p_j}\right) - MSE\left(t_{17}\right) \le 0$$

or,

$$\left[R_{p_j}^2 S_x^2 - R_{17}^2 S_x^2\right] \le 0$$

or,

$$R_{p_j} \le \pm R_{17}, \left(j=1,2,3\right) \tag{10}$$

From MSE of the proposed estimators in Equation (3) and the estimators given by Abid *et al.* [1], it is found the proposed estimators are better than Abid *et al.* [1] estimators if,

$$MSE\left(t_{p_j}\right) - MSE\left(t_i\right) \le 0$$

or,

$$\left[R_{p_j}^2 S_x^2 - R_i^2 S_x^2\right] \le 0$$

or,

$$R_{p_j} \le \pm R_i, \left(j=1,2,3\right), \left(i=18,19,\cdots,26\right) \tag{11}$$

5. Empirical Example

To judge the performances of the proposed and the existing estimators of population mean and to verify the conditions under which proposed estimators performs better than the existing estimators, we have considered the population given by Kadilar and Cingi [14]. The numerical values of the constants, biases and the mean squared error of the proposed and the existing estimators have been calculated for this data. The population parameters for the above population are as follows:

$$N = 106,\ n = 40,\ \bar{Y} = 2212.59,\ \bar{X} = 27421.70$$

$$\rho = 0.860,\ \rho = 0.860,\ C_y = 5.22,\ S_x = 57460.61$$

$$C_x = 2.10,\ \beta_1 = 2.122,\ \beta_2 = 34.572,\ M_d = 7297.50$$

$$QD = 12156.25,\ G = 40201.69,\ D = 35634.99,\ S_{pw} = 35298.81$$

Table 2 represents the numerical values of constants, biases and the mean squared errors of proposed and other existing estimators of population mean using auxiliary variable for the above data.

6. Results

Form Table 2, we see that the proposed estimators are having lesser biases and mean squared errors as compared to all existing estimators. So the proposed estimators are more efficient than the other estimators for estimating population mean. Our purpose to search for the estimator with higher efficiency is achieved.

Table 2. Constants, Biases and MSE of Proposed and other estimators.

Estimator	Constant	Bias	Mean Squared error	Estimator	Constant	Bias	Mean Squared error
t_0	Nil	0	2077627.25	t_{15}	0.0767	136.64	857402.20
t_r	0.0807	171.32	984589.70	t_{16}	0.0801	148.10	884526.80
t_1	0.0807	151.20	889617.50	t_{17}	0.0742	128.08	838466.80
t_2	0.0807	151.18	889566.40	t_{18}	0.0327	24.87	610126.10
t_3	0.0806	150.82	888775.70	t_{19}	0.0475	52.34	670914.00
t_4	0.0807	151.20	889616.00	t_{20}	0.0297	20.59	600579.70
t_5	0.0806	151.02	889215.30	t_{21}	0.0320	23.85	607875.10
t_6	0.0807	151.19	889596.60	t_{22}	0.0498	57.60	682552.70
t_7	0.0807	151.20	889607.50	t_{23}	0.0351	28.59	618381.50
t_8	0.0807	151.17	889557.80	t_{24}	0.0322	24.12	608480.30
t_9	0.0867	151.20	889616.90	t_{25}	0.0500	58.02	683478.00
t_{10}	0.0806	150.76	888634.40	t_{26}	0.0353	28.90	619061.50
t_{11}	0.0807	151.14	889492.50	t_{P_1}	0.00245	0.062	565007.83
t_{12}	0.0807	151.13	889452.90	t_{P_2}	0.00262	0.064	565132.91
t_{13}	0.0637	94.32	763783.60	t_{P_3}	0.00273	0.067	565334.48
t_{14}	0.0715	119.04	818477.40				

Further it is to be mentioned that among the proposed estimators, t_{p_1} is the best as it has smallest bias and the mean squared error.

7. Conclusion

This paper deals with the estimation of population mean of the study variable using auxiliary variable in the form of a special parameter along with some non-traditional measures of dispersion of auxiliary variable used by Abid *et al.* [1]. The expressions for the biases and mean squared errors of these proposed estimators have been derived up to the first order of approximation. A theoretical comparison of the proposed estimators has been made with the existing estimators of population mean under simple random sampling scheme. An empirical study is also carried out to judge the performances of the proposed and existing estimators of population mean. Through this numerical study, it has been found that the proposed estimators are more efficient than the other existing estimators. As proposed estimators are more efficient estimators for population mean, so they should be used for the improved estimation of population mean of study variable using auxiliary variable under simple random sampling scheme.

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Notations

The following given by Abid [1] have been used in this manuscript and are as,

N - Size of the population,

n - Size of the sample,

Y - Study variable,

X - Auxiliary variable,

$\bar{Y}, \bar{X}$ - Population means,

$\bar{y}, \bar{x}$ - Sample means,

S_y, S_x - Population Standard Deviations,

S_{yx} - Population Covariance between Y and X,

C_y, C_x - Coefficients of Variation,

M_d - Median of the auxiliary variable,

ρ - Correlation coefficient between X and Y,

$b = \frac{s_{yx}}{s_x^2}$ - Regression coefficient of y on x,

$\beta_1 = \frac{N \sum_{i=1}^{N} (X_i - \bar{X})^3}{(N-1)(N-2) S_x^3}$ - Coefficient of Skewness of auxiliary variable,

$\beta_2 = \frac{N(N+1) \sum_{i=1}^{N} (X_i - \bar{X})^4}{(N-1)(N-2)(N-3) S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$ - Coefficient of Kurtosis of auxiliary variable,

$QD = \frac{Q_3 - Q_1}{2}$ - Quartile Deviation,

$G = \frac{4}{N-1} \sum_{i=1}^{N} \left(\frac{2i - N - 1}{2N} \right) X_i$ - Gini'sMean Difference,

$D = \frac{2\sqrt{\pi}}{N(N-1)} \sum_{i=1}^{N} \left(i - \frac{N+1}{2} \right) X_i$ - Downton's Parameters,

$S_{pw} = \frac{\sqrt{\pi}}{N^2} \sum_{i=1}^{N} (2i - N - 1) X_i$ - Probability Weighted Moments,

$B(.)$ - Bias of the estimator,

$V(.)$ - Variance of the estimator,

$MSE(.)$ - Mean squared error of the estimator,

$PRE(t_e, t_p) = \frac{MSE(t_e)}{MSE(t_p)} * 100$ - Percentage relative efficiency of the estimator t_p over t_e.