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Influence of the Elastic Modulus of the Soil and Concrete Foundation on the Displacements of a Mat Foundation

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ABSTRACT

In this paper, we suggest to study the behavior of a mat foundation on subsoil from the plate theory taking into account the soil-structure interaction. The objective is to highlight the soil-structure interaction particularly the influence of the rigidities of the soil and the concrete on the subgrade reaction (k) and the displacements of the mat foundation subjected to vertical loads. From plate theory and the soil-structure interaction, the general equation is reached. This equation depends more on the subgrade properties than the concrete foundation properties. Consequently, the behavior of the mat foundation is more influenced by soil properties than the concrete.

Keywords: Mat Foundation; Plate Theory; Soil-Structure Interaction; Mechanical Properties

1. Introduction

The structural and geotechnical calculations of civil engineering works involve the limit state method and require the determination of characteristic values for resistance and deformation criteria of structures and soils. However, the geotechnical design is mainly based on the determination of the displacement caused by the actions applied to foundation and the determination of stresses under limit state service. The structural design is strongly based on the determination of stresses and displacements. A computational approach that takes into account structural and geotechnical aspects related to the design of foundation structures must be developed. It is then question of interaction between two bodies of very different characteristics of deformability. The rupture is often followed by the formation of a thin region led in the direction of contact. This area is called soil-structure interface and it is the location of the major displacements. This work focused on the foundation slab and, more particularly, on the characterization of the soil-structure interface. A precise knowledge of moduli characterizes its deformability and stress paths which should facilitate the optimization of the structural and geotechnical design of foundation.

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2. Modelisation

A foundation is responsible for transmitting the loads from the superstructure to the soil; it provides an interface between the upper part of the structure and the soil. A mat foundation is a continuous reinforced concrete slab and the study may be governed by the theory of plates whose behavior can be studied from the Lagrange equation which take into account the soil-structure interaction. The solution of the Lagrange equation is possible with the use of the methods of Fourier series or finite differences with well-defined boundary conditions. Characterization of the interface has also allowed us to see that the soil-structure interaction is important for the design of foundation. Selvadurai [1] presented a detailed analysis of the soil-foundation interaction problem, explaining the different approaches proposed to model this interaction. These models recognize that soil reaction is a linear function of the displacement of the soil-foundation interface layer. Several models have been developed:

- Winkler model [2],
- Elastic continuum model [1];
- Biparametric model [3];
- Filonenko Borodich model [4,5];
- Hetenyi model model [6];
- Pasternak model [7];

- Reissner model [8];
- Vlazov and Leontiev model [9];
- Vlazov modified model [10].

The system is similar to a concrete slab (E_b, v_b) resting on an elastic soil (E_s, v_s). The plate is assumed to rest on a spring assembly infinitely close to each other with k as the modulus of reaction. These springs are connected by an elastic membrane of shear modulus ($2T$). Modeling of the system is shown in **Figure 1**.

The problem is governed by the following general equation:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial x^4}\right) -2T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+kw=q\left(x,y\right) \quad (1)$$

where D is the flexural rigidity of the plate and is given by:

$$D=\frac{E_b e^3}{12\left(1-v_b^2\right)} \quad (2)$$

with:

E_b: elastic modulus of the material constituting the plate;

e: the thickness of the plate;

v_b: Poisson's ratio of the plate;

k is the modulus of subgrade reaction.

Biot [11] developed an empirical formula for k expressed as follow:

$$k=\frac{0.65E_s}{1-v_s^2}\sqrt[12]{\frac{E_s B^4}{E_b I}} \quad (3)$$

Vesic [12] improved (3) by:

$$k=\frac{0.95E_s}{1-v_s^2}\left(\frac{E_s B^4}{\left(1-v_s^2\right)E_b I}\right)^{0.108} \quad (4)$$

where:

E_s is the modulus of subgrade;

v_s is the Poisson's ratio of the subgrade;

B is the width of the foundation;

E_b is the Young's modulus of the concrete foundation;

I is the moment of inertia of the cross section of the concrete.

Equation (2) can be written as:

$$k=0.65\frac{E_s}{1-v_s^2}\left(\frac{E_s B^4}{E_b I}\right)^{1/12} \quad (5)$$

and Equation (4) by:

$$k=0.95\left(\frac{1}{1-v_s^2}\right)^{0.108}\frac{E_s}{1-v_s^2}\left(\frac{E_s B^4}{E_b I}\right)^{0.108} \quad (6)$$

Generally for foundations, Poisson's ratio is between 0.15 and 0.4 [13], and the term $\left(\frac{1}{1-v_s^2}\right)^{0.108}$ is between 1.0025 and 1.019 [13] (*which leads to ignore this term in the expression*) for k in Equation (6) which can be rewritten as follows:

$$k=0.95\frac{E_s}{1-v_s^2}\left(\frac{E_s B^4}{E_b I}\right)^{0.108} \quad (7)$$

Thus by combining (3) and (6), k is expressed by the following equation:

$$k=a\frac{E_s}{1-v^2}\left(\frac{E_s B^4}{EI}\right)^{\gamma} \quad (8)$$

where a and γ are constants according to different authors (**Table 1**).

It should specify that the vertical modulus of subgrade reaction can be determined from the results of geotechnical testing. T is the horizontal elastic modulus of subgrade reaction. Vlasov [9] proposes the following relation:

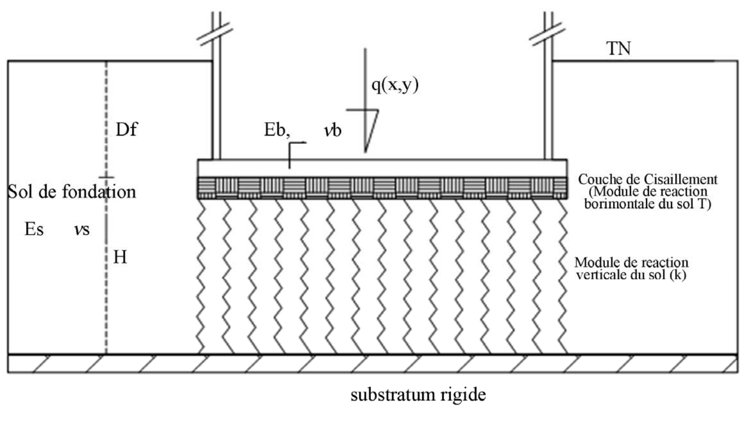

Figure 1. Discretisation of the system.

Table 1. Equations giving *k* [13].

Authors	*a*	γ	*k*
Biot (1937) [11]	0.65	1/12	$k = 0.65\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{1/12}$
Vesic (1963) [12]	0.69	0.0868	$k = 0.69\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.0868}$
Liu (2000) [14]	0.74	0.0903	$k = 0.74\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.0903}$
Daloglu *et al*. (2000) [15]	0.78	0.0938	$k = 0.78\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.0938}$
Fischer *et al*. (2000) [16]	0.82	0.0973	$k = 0.82\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.0973}$
Yang (2006) [17]	0.95	0.108	$k = 0.95\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.108}$
Henry (2007) [18]	0.91	0.1043	$k = 0.91\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.1043}$
Arul *et al.* (2008) [19]	0.87	0.1008	$k = 0.87\frac{E_s}{1-\nu^2}\left(\frac{E_s B^4}{EI}\right)^{0.1008}$

$$T = \frac{E_s}{4\left(1-\nu_s^2\right)\left(1+\nu_s\left(1-\nu_s\right)\right)}\int_0^H \Phi^2 \mathrm{d}z \qquad (9)$$

To a relatively deep layer of soil where the normal stress may vary with depth, it is possible to use, for the function $\Phi(z)$, the non-linear continuous variable defined by Equation 10(a). $\Phi(z)$ is a function which describes the variation of the displacement $w(x,y)$ along the z axis, such that:

$$\Phi(0) = 1 \; et \; \Phi(H) = 0$$

Selvadurai [1] suggests two expressions of $\Phi(z)$:

$$\Phi(z) = \left(1 - \frac{z}{H}\right) \qquad (10a)$$

$$\Phi(Z) = \frac{\sinh\left[(H-z)\frac{\gamma}{L}\right]}{\sinh\left(\frac{\gamma H}{L}\right)} \qquad (10b)$$

H: thickness of the soil layer (depth of the rigid substratum).

And for a linear variation of $\Phi(z)$, the shear parameter model is given after integration by:

$$T = \frac{E_s \cdot H}{12\left(1-\nu_s^2\right)\left(1+\nu_s\left(1-\nu_s\right)\right)} \qquad (11)$$

3. Analytical Solutions

Before the calculation of displacements due to load, it should be consider that the motion of the interface is a result of the weight of the slab. This displacement is constant on the entire extension of the interface and is a function of the thickness of the plate and the modulus of vertical subgrade reaction. The displacement w_0 is given by:

$$w_0 = 25000 \times e/k \qquad (12)$$

In the case of an elastic homogeneous soil, a uniform distribution of the forces applied to the foundation system is assumed. This amounts to admitting that the stress $q(x,y)$ is constant (Q value) from each point of the foundation. For a foundation of infinite dimension, a zero displacement at the edges of the plate is imposed. If each edge is far from one to another, this is true. Although, this questionable assumption allows an accurate resolution of the problem using the Fourier series. At first, we assume a uniform distribution of the applied foundation system forces. So $q(x,y)$ is constant (Q value) for analytical solution, and the double Fourier series is used.

$q(x,y)$ can be written as:

$$q(x,y) = Q = \sum_1^\infty \sum_1^\infty a_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \qquad (13)$$

with

$$a_{mn} = \frac{4}{LB} Q \int_0^L \int_0^B \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \mathrm{d}x\mathrm{d}y \tag{14}$$

$$a_{mn} = \frac{4}{LB} Q \left[\frac{L}{m\pi} \left(-\cos\frac{m\pi x}{L} \right)_0^L \right] \left[\frac{B}{n\pi} \left(-\cos\frac{n\pi y}{B} \right)_0^B \right] \tag{15}$$

It result that for m and n impair:

$$a_{mn} = \frac{16Q}{mn\pi^2} \tag{16}$$

For the calculation of the displacements, we assume that $w(x, y)$ can also be decomposed into Fourier series:

$$w(x,y) = \sum_1^\infty \sum_1^\infty b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{17}$$

Thus:

$$\frac{\partial^2 w}{\partial x^2} = -\sum_1^\infty \sum_1^\infty \left(\frac{m\pi}{L}\right)^2 b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{18}$$

$$\frac{\partial^2 w}{\partial y^2} = -\sum_1^\infty \sum_1^\infty \left(\frac{n\pi}{B}\right)^2 b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{19}$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \sum_1^\infty \sum_1^\infty \left(\frac{m\pi}{L}\right)^2 \left(\frac{n\pi}{B}\right)^2 b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{20}$$

$$\frac{\partial^4 w}{\partial x^4} = \sum_1^\infty \sum_1^\infty \left(\frac{m\pi}{L}\right)^4 b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{21}$$

$$\frac{\partial^4 w}{\partial y^4} = \sum_1^\infty \sum_1^\infty \left(\frac{n\pi}{B}\right)^4 b_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \tag{22}$$

By replacing the differential equation governing the behavior of the system we have:

$$\begin{aligned} &\sum_1^\infty \sum_1^\infty b_{mn} \left[D\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right)^2 \right. \\ &\left. +2T\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right) + k \right] \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \\ &= \sum_1^\infty \sum_1^\infty a_{mn} \sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right) \end{aligned} \tag{23}$$

According to (23), the expression b_{mn} can be given by the following relation:

$$b_{mn} = \frac{a_{mn}}{D\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right)^2 + 2T\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right) + k} \tag{24}$$

The axial deflection is:

$$w(x,y) = \frac{16Q}{\pi^2} \sum_1^\infty \sum_1^\infty \frac{\sin\left(\frac{m\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{B}\right)}{Dmn\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right)^2 + 2Tmn\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{B}\right)^2 \right) + kmn} \tag{25}$$

The total displacement is obtained by summing the displacements given by Equations (12) and (25).

Figures 2 to **7** show the evolution of k according to the different parameters of the mechanical behavior model. **Figure 2** shows the increase of k with the increase of E_s. **Figures 3-5** show that k is sensitive to the mechanical properties of the soil foundation. These figures show that k and the displacements vary slightly with the mechanical properties of concrete foundation and are strongly dependent on elastic modulus of the soil foundation.

4. Conclusion

It appears from this study that the elastic modulus and the Poisson's ratio of the subgrade are the most influential parameters on the displacements of the plate. The results show that modulus of subgrade reaction and displacements varies slightly with the mechanical properties of concrete foundation and is more influenced by the elastic modulus of the soil. Hence the importance of mastering the property of the foundation soil is to better

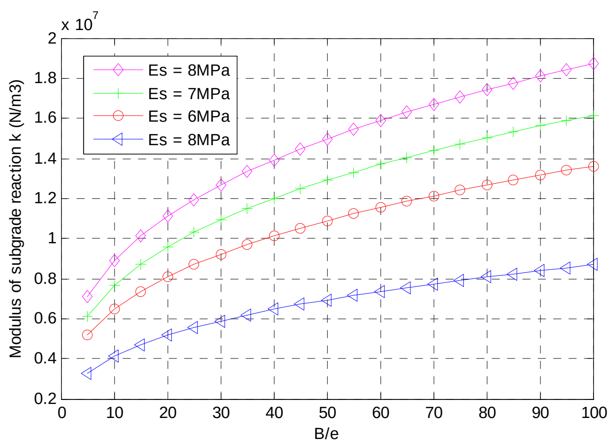

Figure 2. Modulus of subgrade reaction versus *B*/*e* ratio of the plate for various values of E_s.

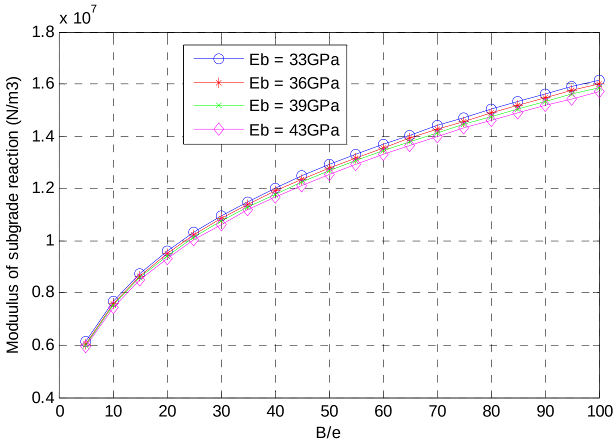

Figure 3. Modulus of subgrade reaction versus *B*/*e* ratio of the plate for various values of E_b.

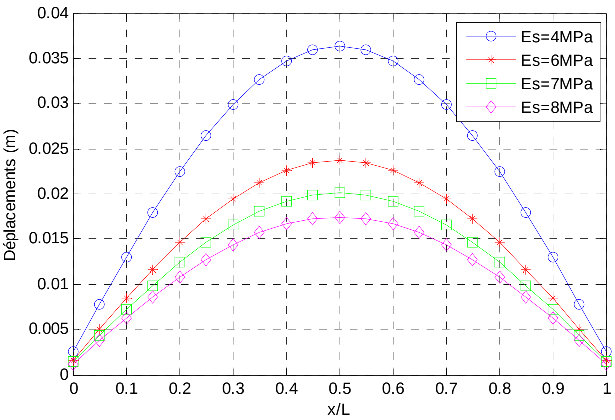

Figure 4. Displacements along the median of the plate for various values of E_s.

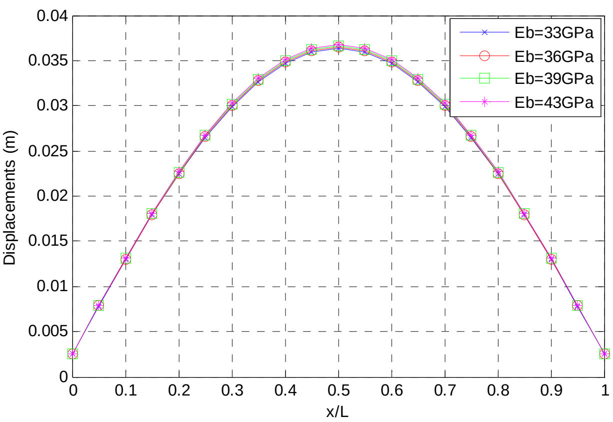

Figure 5. Displacements along the median of the plate for various values of E_b.

understand the behavior of foundation structures for optimal sizing of these and especially in order to limit the displacements, which are the vectors of disorder in the structures.

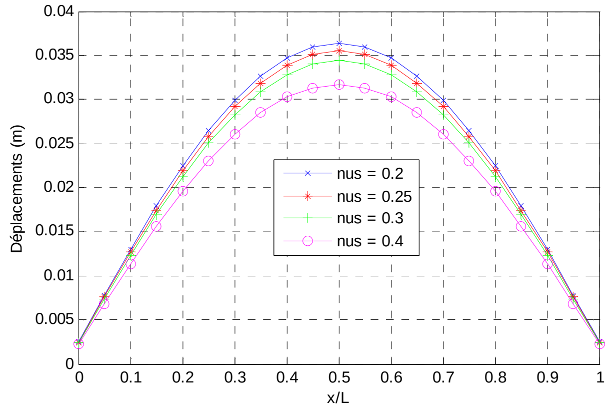

Figure 6. Displacements along the median of the plate for various values of ν_s.

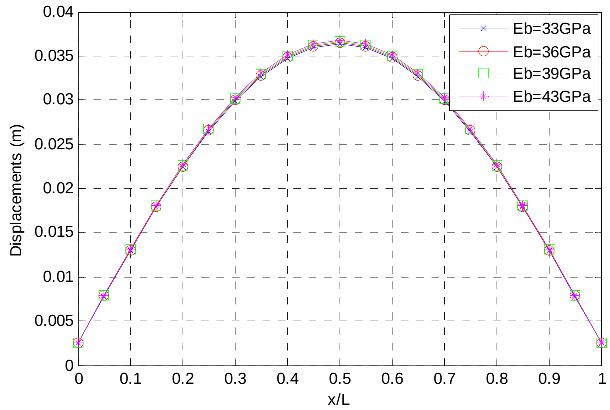

Figure 7. Displacements along the median of the plate for various values of ν_b.

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