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Bayesian Estimations with Fuzzy Data to Estimation Inverse Rayleigh Scale Parameter

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Abstract

In this paper, Bayesian computational method is used to estimate inverse Rayleigh Scale parameter with fuzzy data. Based on imprecision data, the Bayes estimates cannot be obtained in explicit form. Therefore, we provide Tierney and Kadane's approximation to compute the Bayes estimates of the scale parameter under Square error and Precautionary loss function using Non-informative Jefferys Prior. Also, we provide compared numerically through Monte-Carlo simulation study to obtained estimates of the scale parameter in terms of mean squared error values.

Keywords

Inverse Rayleigh Distribution, Imprecision Data, Modified Newton Method, Tierney and Kadane's Approximation

1. Introduction

The Rayleigh distribution (RD) is originated from two parameter Weibull distribution and it is an appropriate model for life-testing. It can be shown by transformation of random variable that if the random variable X has Rayleigh distribution, Then the random variable $Y=\frac{1}{X}$ has an inverse Rayleigh distribution (IRD) [1]. The Inverse Rayleigh distribution (IRD) has been introduced by Trayer (1964) [2]. The distribution of life times of several types of experimental units can be approximated by the IRD [3]. The IRD plays an important role in many applications, including life test and reliability studies [4]. A random variable Y is said to have a one-parameter (IRD) if it has the following (PDF),

$$f_Y(y;\lambda)=\frac{2\lambda}{y^3}e^{\frac{-\lambda}{y^2}};\ y\geq 0,\lambda>0 \tag{1}$$

and (CDF), is given by:

$$F_Y(y;\lambda) = e^{\frac{-\lambda}{y^2}};\ y \geq 0, \lambda > 0 \tag{2}$$

where λ is the scale parameter.

2. Maximum Likelihood Estimators (MLE)

Given $\underline{y} = (y_1, y_2, \cdots, y_m)$ be an (i.i.d.) random vector of a random sample of size m from (IRD), the complete-data likelihood function is:

$$L(\lambda;\underline{y}) = 2^m \lambda^m \prod_{i=1}^{m} \frac{1}{y_i^3} e^{-\lambda \sum_{i=1}^{m} \frac{1}{y_i^2}} \tag{3}$$

Now if $\underline{y}$ is not observed precisely. Then, we can compute its probability by using Zadeh's definition of an imprecision event [5]. The observed-data log-likelihood function can then be obtained as,

$$L(\lambda;\underline{\tilde{y}}) = \prod_{i=1}^{m} \int f_Y(y;\lambda)\, \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y$$

$$L(\lambda;\underline{\tilde{y}}) = \prod_{i=1}^{m} \int \frac{2\lambda}{y^3} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y \tag{4}$$

where $\mu f_{\tilde{y}_i}(y)$ is the Borel measurable membership function.

Now, by take the natural logarithm for the likelihood function and differentiating with respect to λ and then equating to zero we get:

$$\frac{\partial \ln L(\lambda;\underline{\tilde{y}})}{\partial \lambda} = \frac{m}{\lambda} - \sum_{i=1}^{m} \frac{\int \frac{1}{y^5} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y}{\int \frac{1}{y^3} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y} = 0 \tag{5}$$

Since, the (MLE) of λ is the solution of Equation (5), so, we used the modified Newton's Method to determine the MLE of the parameter λ.

Where, at iteration $(h+1)$

$$\hat{\lambda}^{(h+1)} = \hat{\lambda}^{(h)} - (\upsilon) \frac{\left.\frac{\partial \ln L(\lambda;\underline{\tilde{y}})}{\partial \lambda}\right|_{\lambda=\hat{\lambda}^{(h)}}}{\left.\frac{\partial^2 \ln L(\lambda;\underline{\tilde{y}})}{\partial \lambda^2}\right|_{\lambda=\hat{\lambda}^{(h)}}},\ \upsilon > 1 \tag{6}$$

and

$$\frac{\partial^2 \ln L(\lambda;\underline{\tilde{y}})}{\partial \lambda^2} = -\frac{m}{\lambda^2} + \sum_{i=1}^{m} \frac{\int \frac{1}{y^7} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y}{\int \frac{1}{y^3} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y} - \sum_{i=1}^{n} \left[\frac{\int \frac{1}{y^5} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y}{\int \frac{1}{y^3} e^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\, \mathrm{d}y} \right]^2 \tag{7}$$

3. Bayes Estimator

In this section, we describe Bayesian method to estimate the parameter λ. In

Bayesian opinion the parameter itself is considered as a random variable from a given probability distribution whose variability can be described by the prior distribution.

Assume that the prior distribution of the unknown scale parameter λ of IRD defined as using Jeffery's prior information $\pi(\lambda)$, which is given by [2]:

$$\pi(\lambda) \propto \sqrt{I(\lambda)}$$

where $I(\lambda) = -m\,E\left[\frac{\partial^2 \ln f(y;\lambda)}{\partial\lambda^2}\right]$

$$\Rightarrow \pi(\lambda) = a\sqrt{-mE\left[\frac{\partial^2 \ln f(y;\lambda)}{\partial\lambda^2}\right]},\ a \text{ is a constant,}$$

where

$$E\left[\frac{\partial^2 \ln f(y;\lambda)}{\partial\lambda^2}\right] = \frac{-1}{\lambda^2} \Rightarrow \pi(\lambda) = \frac{a\sqrt{m}}{\lambda}, \lambda > 0$$

Now, the posterior density function of λ given imprecision data is:

$$h(\lambda \mid \underset{\sim}{\tilde{y}}) = \frac{\pi(\lambda)L(\lambda;\underset{\sim}{\tilde{y}})}{\int_0^\infty \pi(\lambda)L(\lambda;\underset{\sim}{\tilde{y}})\mathrm{d}\lambda}$$

$$\Rightarrow h(\lambda \mid \underset{\sim}{\tilde{y}}) = \frac{\frac{a\sqrt{m}}{\lambda}\prod_{i=1}^m \int \frac{2\lambda}{y^3} \mathrm{e}^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int_0^\infty \frac{a\sqrt{m}}{\lambda}\prod_{i=1}^m \int \frac{2\lambda}{y^3} \mathrm{e}^{\frac{-\lambda}{y^2}} \mu f_{\tilde{y}_i}(y)\mathrm{d}y} \tag{8}$$

In this study we consider non-informative prior density for λ based on square error and precautionary loss function as the following:

3.1. Bayes Estimator Based on Square Error Loss Function

Bayes estimation of any function of the scale parameter λ say $g(\lambda)$, based on a squared error loss function, may be written as,

$$\hat{g}_s(\lambda) = E\left[g(\lambda) \mid \underset{\sim}{\tilde{y}}\right] = \frac{\int_0^\infty g(\lambda)\pi(\lambda)L(\lambda;\underset{\sim}{\tilde{y}})\mathrm{d}\lambda}{\int_0^\infty \pi(\lambda)L(\lambda;\underset{\sim}{\tilde{y}})\mathrm{d}\lambda} \tag{9}$$

3.2. Bayes Estimator Based on Precautionary Loss Function

Precautionary loss function was proposed by Norstrom (1996) [6], as follows:

$$L(\hat{\theta},\theta) = \frac{(\theta-\hat{\theta})^2}{\hat{\theta}},$$

where $\hat{\theta}$ is an estimate of θ.

Bayes estimation of any function of the scale parameter λ say $g(\lambda)$, based on a precautionary error loss function, may be written as,

$$\hat{g}_p(\lambda)=\sqrt{E\left[g^2(\lambda)\mid \underset{\sim}{y}\right]}=\sqrt{\frac{\int_0^\infty g^2(\lambda)\pi(\lambda)L\left(\lambda;\underset{\sim}{y}\right)\mathrm{d}\lambda}{\int_0^\infty \pi(\lambda)L\left(\lambda;\underset{\sim}{y}\right)\mathrm{d}\lambda}} \tag{10}$$

Note that, Bayes estimator in (9) and (10) cannot be simplified in to a closed form. Therefore, we consider Tierney and Kadane's approximation form to obtain Bayes estimator of λ of IRD.

4. Tierney and Kadane's Approximation Form

Tierney and Kadane (1986) [7] proposed an alternative method for the evaluation of the ratio of integrals of the form (9) and (10).

$$\textit{Setting}\ \ Q(\lambda)=\ln\left(\pi(\lambda)\right)+\ln\left(L\left(\lambda;\underset{\sim}{y}\right)\right)$$

$$\hat{g}_s(\lambda)=E\left[g(\lambda)\mid \underset{\sim}{y}\right]=\frac{\int_0^\infty g(\lambda)\mathrm{e}^{Q(\lambda)}\mathrm{d}\lambda}{\int_0^\infty \mathrm{e}^{Q(\lambda)}\mathrm{d}\lambda} \tag{11}$$

$$\hat{g}_p(\lambda)=\sqrt{E\left[g^2(\lambda)\mid \underset{\sim}{y}\right]}=\sqrt{\frac{\int_0^\infty g^2(\lambda)\mathrm{e}^{Q(\lambda)}\mathrm{d}\lambda}{\int_0^\infty \mathrm{e}^{Q(\lambda)}\mathrm{d}\lambda}} \tag{12}$$

Now, set

$$H(\lambda)=\frac{Q(\lambda)}{m}$$

$$H_s^*(\lambda)=\frac{\ln\left(g(\lambda)\right)}{m}+H(\lambda) \tag{13}$$

And

$$H_p^*(\lambda)=\frac{\ln\left(g^2(\lambda)\right)}{m}+H(\lambda) \tag{14}$$

$$\Rightarrow \hat{g}_s(\lambda)=\frac{\int_0^\infty \mathrm{e}^{nH_s^*(\lambda)}\mathrm{d}\lambda}{\int_0^\infty \mathrm{e}^{nH(\lambda)}\mathrm{d}\lambda} \tag{15}$$

$$\hat{g}_p(\lambda)=\left[\frac{\int_0^\infty \mathrm{e}^{nH_p^*(\lambda)}\mathrm{d}\lambda}{\int_0^\infty \mathrm{e}^{nH(\lambda)}\mathrm{d}\lambda}\right]^{\frac{1}{2}} \tag{16}$$

Now, the Equation (15) and Equation (16) can be written as

$$\hat{g}_s^T(\lambda)=\sqrt{\frac{\tau^*}{\tau}}\exp\left\{n\left(H_s^*\left(\hat{\lambda}^*\right)-H\left(\hat{\lambda}\right)\right)\right\} \tag{17}$$

$$\hat{g}_p^T(\lambda)=\sqrt{\sqrt{\frac{\tau^*}{\tau}}\exp\left\{n\left(H_p^*\left(\hat{\lambda}^*\right)-H\left(\hat{\lambda}\right)\right)\right\}} \tag{18}$$

where, τ^*: is the minus the inverses of the second derivative of $H_s^*(\lambda)$ or $H_p^*(\lambda)$ at $\hat{\lambda}^*$ depending on what loss function have been used. τ: is the minus the inverses of the second derivative of $H(\lambda)$ at $\hat{\lambda}$. And $\hat{\lambda}^*$ maximize

$H_s^*(\lambda)$ and $H_p^*(\lambda)$ as well as $\hat{\lambda}$ maximize $H(\lambda)$

Now, the function $H(\lambda)$ is given by,

$$H(\lambda)=\frac{1}{m}\left[k+(m-1)\ln(\lambda)+\sum_{i=1}^{m}\ln\int\frac{1}{y^3}e^{\frac{-\lambda}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y\right] \tag{19}$$

where,

$$k=\ln(a)+m\ln(2)+\frac{1}{2}\ln(m) \tag{20}$$

and $\hat{\lambda}$ that maximize $H(\lambda)$, can be obtained by solving the following equation,

$$\frac{\partial H(\lambda)}{\partial\lambda}=\frac{1}{m}\left[\frac{m-1}{\lambda}-\sum_{i=1}^{m}\frac{\int\frac{1}{y^5}e^{\frac{-\lambda}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int\frac{1}{y^3}e^{\frac{-\lambda}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right]=0 \tag{21}$$

It is clear there is no explicit solution to Equation (21). Therefore, modified Newton method is applied to solve the required equation.

$$\hat{\lambda}^{(h+1)}=\hat{\lambda}^{(h)}-(\upsilon)\frac{\left.\frac{\partial H(\lambda)}{\partial\lambda}\right|_{\lambda=\hat{\lambda}^{(h)}}}{\left.\frac{\partial^2 H(\lambda)}{\partial\lambda^2}\right|_{\lambda=\hat{\lambda}^{(h)}}},\ \upsilon>1 \tag{22}$$

where

$$\frac{\partial^2 H(\lambda)}{\partial\lambda^2}=\frac{1}{m}\left[\frac{-(m-1)}{\lambda^2}+\sum_{i=1}^{m}\frac{\int\frac{1}{y^7}e^{\frac{-\hat{\lambda}}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int\frac{1}{y^3}e^{\frac{-\hat{\lambda}}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}-\sum_{i=1}^{m}\left(\frac{\int\frac{1}{y^5}e^{\frac{-\hat{\lambda}}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int\frac{1}{y^3}e^{\frac{-\hat{\lambda}}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right)^2\right] \tag{23}$$

then,

$$\tau=-\left[\left.\frac{\partial^2 H(\lambda)}{\partial\lambda^2}\right|_{\lambda=\hat{\lambda}^{(h)}}\right]^{-1}$$

Now, following the same argument with $g(\lambda)=\lambda$

4.1. Tierney and Kadane's Approximation of *λ* Based on Square Error Loss Function (TKS)

Set $(\lambda)=\lambda$, Equation (13) will be,

$$H_s^*(\lambda)=\frac{\ln(\lambda)}{m}+H(\lambda)$$

$$H_s^*(\lambda)=\frac{1}{m}\left[k+(m)\ln(\lambda)+\sum_{i=1}^{m}\ln\int\frac{1}{y^3}e^{\frac{-\lambda}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y\right] \tag{24}$$

where k is a constant as in (20).

Now, $\hat{\lambda}^*$ that maximize $H_s^*(\lambda)$ in (24) can be obtained by solving following equation iteratively as in

$$\hat{\lambda}^{*(h+1)} = \hat{\lambda}^{*(h)} - (\upsilon)\frac{\left.\frac{\partial H_s^*(\lambda)}{\partial \lambda}\right|_{\lambda=\hat{\lambda}^{*(h)}}}{\left.\frac{\partial^2 H_s^*(\lambda)}{\partial \lambda^2}\right|_{\lambda=\hat{\lambda}^{*(h)}}},\ \upsilon > 1 \tag{25}$$

where

$$\frac{\partial H_s^*(\lambda)}{\partial \lambda} = \frac{1}{m}\left[\frac{m}{\hat{\lambda}^*} - \sum_{i=1}^{m}\frac{\int \frac{1}{y^5}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right]$$

$$\frac{\partial^2 H_s^*(\lambda)}{\partial \lambda^2} = \frac{1}{m}\left[\frac{-(m)}{\hat{\lambda}^{*2}} + \sum_{i=1}^{m}\frac{\int \frac{1}{y^7}e^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y} - \sum_{i=1}^{m}\left(\frac{\int \frac{1}{y^5}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right)^2\right]$$

And

$$\tau^* = -\left[\left.\frac{\partial^2 H_s^*(\lambda)}{\partial \lambda^2}\right|_{\lambda=\hat{\lambda}^{*(h)}}\right]^{-1} \tag{26}$$

Now, Bayes estimate of λ of IRD based on square error loss function, denoted by $\hat{\lambda}_s^{TK}$, can be obtained from Equation (17), where all the H and H_s^* elements are evaluated in $\hat{\lambda}$ and $\hat{\lambda}^*$ respectively.

4.2. Bayes Estimate of λ Based on Precautionary Loss Function (TKP)

Set, $g(\lambda) = \lambda$, Equation (14) will be,

$$H_p^*(\lambda) = \frac{\ln(\lambda^2)}{m} + H(\lambda)$$

$$H_p^*(\lambda) = \frac{1}{m}\left[k + (m+1)\ln(\lambda) + \sum_{i=1}^{m}\ln\int \frac{1}{y^3}\mathrm{e}^{\frac{-\lambda}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y\right] \tag{27}$$

where k is a constant as in (20).

Now, $\hat{\lambda}^*$ that maximize $H_p^*(\lambda)$ in (27) can be obtained by solving following equation

$$\frac{\partial H_p^*(\lambda)}{\partial \lambda} = \frac{1}{m}\left[\frac{m+1}{\hat{\lambda}^*} - \sum_{i=1}^{m}\frac{\int \frac{1}{y^5}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right]$$

iteratively as in

$$\hat{\lambda}^{*(h+1)} = \hat{\lambda}^{*(h)} - (\upsilon)\frac{\left.\frac{\partial H_p^*(\lambda)}{\partial \lambda}\right|_{\lambda=\hat{\lambda}^{*(h)}}}{\left.\frac{\partial^2 H_p^*(\lambda)}{\partial \lambda^2}\right|_{\lambda=\hat{\lambda}^{*(h)}}},\ \upsilon > 1 \tag{28}$$

where

$$\frac{\partial^2 H_p^*(\lambda)}{\partial \lambda^2} = \frac{1}{m}\left[\frac{-(m+1)}{\hat{\lambda}^{*2}} + \sum_{i=1}^{m}\frac{\int \frac{1}{y^7}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y} - \sum_{i=1}^{m}\left(\frac{\int \frac{1}{y^5}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}{\int \frac{1}{y^3}\mathrm{e}^{\frac{-\hat{\lambda}^*}{y^2}}\mu f_{\tilde{y}_i}(y)\mathrm{d}y}\right)^2\right]$$

And

$$\tau^* = -\left[\left.\frac{\partial^2 H_p^*(\lambda)}{\partial \lambda^2}\right|_{\lambda=\hat{\lambda}^{*(h)}}\right]^{-1} \tag{29}$$

Now, Bayes estimate of λ of IRD based on prec. loss function, denoted by $\hat{\lambda}_p^{TK}$, can be obtained from Equation (18), where all the H and H_p^* elements are evaluated in $\hat{\lambda}$ and $\hat{\lambda}^*$ respectively.

5. Simulation Study

In trying to illustrate and compare the methods as described above, a Monte-Carlo simulation study was perform to generate an (i.i.d) random samples, say $\underline{y}$, according to IRD through the adoption of inverse transformation method with size n = 10, 30 and 90 to take care of small, medium and large data sets. The scale parameter λ = 0.3, 0.5, 1, 1.5, 2. Then, each observation of $\underline{y}$ was made Imprecision based on an appropriate selected membership function among four membership functions in the Imprecision Information System as the following **Figure 1**.

The simulation program has been written by using MATLAB (R2010b) program. The results of Monte-Carlo simulation have been summarized in **Table 1**.

The initial values required for proceeding modified Newton-Raphson method chosen to be the symmetrical rank regression estimators. The comparisons between the parameter estimates were based on values from MSE where [8]:

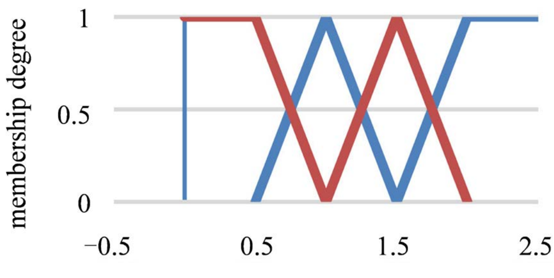

Figure 1. Imprecision information system.

Table 1. MSE values for estimates of the scale parameter (λ) of IRD with different cases.

n	$\hat{\lambda}_{MNR}$	Bayes Estimates		Best Estimate
		$\hat{\lambda}_{TKS}$	$\hat{\lambda}_{TKP}$	
		$\lambda = 0.3$		
10	0.3016845	0.4254903	0.0999919	TKP
30	0.2213556	0.3593909	0.0999881	TKP
90	0.1380426	0.2075041	0.0899999	TKP
		$\lambda = 0.5$		
10	0.4445674	0.6759990	0.1456790	TKP
30	0.3999456	0.5888219	0.1444456	TKP
90	0.2000347	0.3211106	0.1000000	TKP
		$\lambda = 1$		
10	0.9238011	0.6187163	0.6395949	TKS
30	0.6336614	0.5504253	0.5527095	TKS
90	0.0333695	0.0299999	0.0300257	TKS
		$\lambda = 1.5$		
10	2.0795793	1.3326324	1.387364	TKS
30	1.4255994	1.2227846	1.2287826	TKS
90	0.0750782	0.0656017	0.065692	TKS
		$\lambda = 2$		
10	4.0827706	2.7752958	2.8630777	TKS
30	0.8928044	0.7653234	0.7693016	TKS
90	0.0883097	0.0864630	0.0864936	TKS

n: sample size; $\hat{\lambda}_{MNR}$: maximum likelihood estimate of λ by newton-raphson; $\hat{\lambda}_{TKS}$: Bayes estimate of λ of IRD based on square error loss function; $\hat{\lambda}_{TKP}$: Bayes estimate of λ of IRD based on prec. loss function.

$$\text{MSE}\left(\hat{\lambda}\right) = \frac{\sum_{j=1}^{L}\left(\hat{\lambda}_j - \lambda\right)^2}{L} \tag{30}$$

$\hat{\lambda}_j$: is the estimate of λ respectively at the j^{th} run.

L: is the number of sample replicated chosen to be (500).

6. Conclusions and Recommendations

The most important conclusions of Monte-Carlo simulation results are:

Tierney and Kadane's approximation based on square error loss function (TKS) estimate introduced the best perform compared with the different estimates for all sample sizes and for all cases except $\lambda = 0.3$ and $\lambda = 0.5$, where Bayes Estimate based on Precautionary loss function (TKP) is the best.

Based on this, we recommend,

1) Using the TKS estimate to compute estimates of the scale parameter of IRD for all sample sizes and with cases $\lambda = 1$, $\lambda = 1.5$ and $\lambda = 2$.

2) Using the TKP estimate to compute estimates of the scale parameter of IRD for all sample sizes and with the cases $\lambda = 0.3$ and $\lambda = 0.5$.

3) For further study, we suggest such type of work can be done by using other informative priors for the parameter of the IRD and also the parameter can be estimated by other methods.

4) Research can be applied to real data and demonstrate the importance of this distribution in practice.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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