



# Three-Dimensional Temperature in Pipe Heated Concrete

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## Abstract

Heating by pipes embedded in concrete is considered in three-dimensional space. A concrete floor resting on the ground or held at a constant temperature has heating pipes located midway between the top and bottom of the concrete. The top of the concrete is cooled by Newton's law of cooling and the sides are insulated.

## Keywords

Heat Flow, Three-Dimensional Solution, Concrete Floor, Heating Pipes, Curved Pipes, Finite Elements, End Effects and Temperature Distribution

**Subject Areas:** Applied Physics, Electric Engineering

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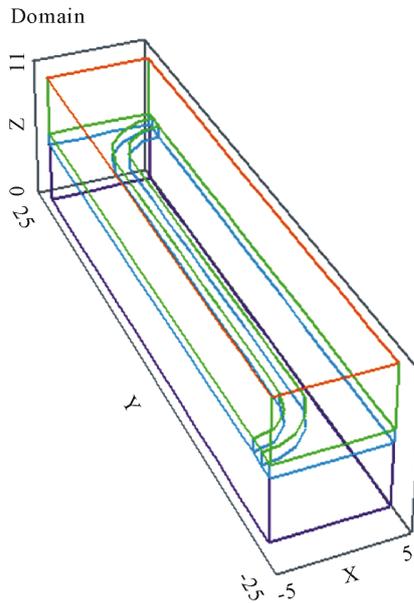
## 1. Introduction

In the past there have been many papers written on floor heating by heating pipes embedded in concrete. It is not easy to find any that solve the Laplace equation of heat flow in either two- or three-dimensional space. It is doubtful that any work has been done in three-dimensional space. That is done here in order to find out how uniform the floor temperature is.

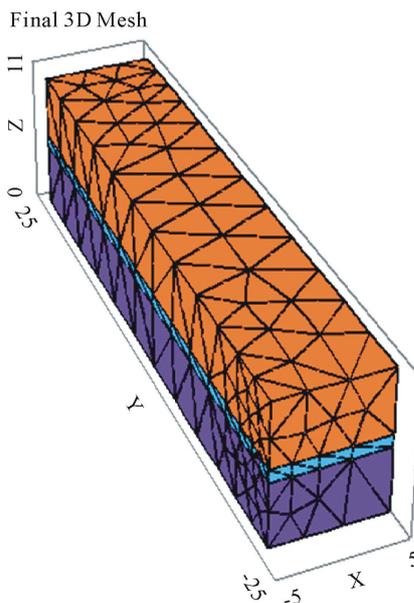
## 2. The Geometry Considered

A three-dimensional geometry considered. It is exhibited in **Figure 1** comprising three layers. The bottom layer in purple is below the heating pipe, the thin green layer is the layer containing the heating pipe and the top red rimmed layer is over the heating pipe. The surface bounded by red lines is the floor surface. In **Figure 2** the complete object is shown after being split into a large number of finite elements.

Here no heat is allowed to flow out of the YZ or XZ planes by making the normal derivative of the temperature be zero. This also makes them planes of symmetry filling all of space with heating pipes of the same



**Figure 1.** Outline of the 3D geometry.



**Figure 2.** The complete geometry in finite elements.

geometry. This is what you see in a rectangular room with mirrors on all vertical walls. The partial differential equations of heat flow are given by Hughes and Gaylord [1]. They are solved here in three-dimensional space.

### 3. The Laplace Equation for Temperature

We let temperature be denoted as  $T$ , thermal conductivity of concrete as  $k$  and the coordinates as  $X$ ,  $Y$  and  $Z$  as shown in the figures.

$$\nabla(k\nabla T) = 0 \tag{1}$$

Here  $k = 0.80016$  for concrete and we have used a Newton coefficient such that

$$\partial T / \partial Z = 1.35(68 - T) \quad (2)$$

In the Newton cooling law, given above, we have assumed the desired floor temperature is 68 degrees Fahrenheit. In Pennsylvania the temperature of the dirt below in the ground under the concrete varies with the season. In the summer it is about two degrees Fahrenheit lower than in winter. The average ground temperature for a few feet below the surface in the higher elevations of Pennsylvania is 40 degrees Fahrenheit. In other places the average yearly temperature of the ground may be as low as 32 degrees Fahrenheit. Thus, given the author's location the bottom of the concrete is held at 40 degrees Fahrenheit.

#### 4. The Computer Program

```

title '3D Floor Heating by hot water pipes' coordinates
cartesian3 SELECT
errlim=1e-04 thermal colors=on VARIABLES
T {Temperature in Degrees Fahrenheit} definitions
Zc = 5 P = 0.5 Ta = 0.20 Wf = 20 Lf = 36 Uc = 5
k = 0.80016 Ym = 25
equations
T: div(k*grad(T)) = 0 extrusion
z = 0, Zc, Zc+2*P, Zc+2*P+Uc BOUNDARIES
surface 1 value(T) = 40
surface 4 natural(T)=1.35*(68-T)
region 1
start(-Wf/4,-Ym) line to (Wf/4,-Ym) to (Wf/4,Ym) to (-Wf/4,Ym) to close
Limited Region 2
layer 2 VOID
start(P,-Lf/2-2*P) layer 2 value(T) = 88 line to (P,Lf/2) arc(center=Wf/4,Lf/2) angle=-90
layer 2 natural(T)=0 line to (Wf/4,12*P+Lf/2) layer 2 value(T)=88 arc(center=Wf/4,Lf/2) angle=90
line to (-2*P,-Lf/2) arc(center=-Wf/4,-Lf/2) angle=-90 layer 2 natural(T)=0 line to (-Wf/4,-(Lf/2+11*P))
layer 2 Value(T)=88 arc(center=-Wf/4,-Lf/2) to close monitors
contour (T) on z=Zc+2*P+Uc plots
contour (T) on z=Zc+2*P+Uc painted contour (T) on z=Zc+P painted contour(T) on x=0 painted contour(T)
on x=-Wf/4 painted
end

```

It is easy to change the parameters for the size of the geometry, room temperature, bottom temperature, Newton cooling coefficient and thermal conductivity. In the results given here the fluid in the heating pipes is heated to 88 degrees Fahrenheit.

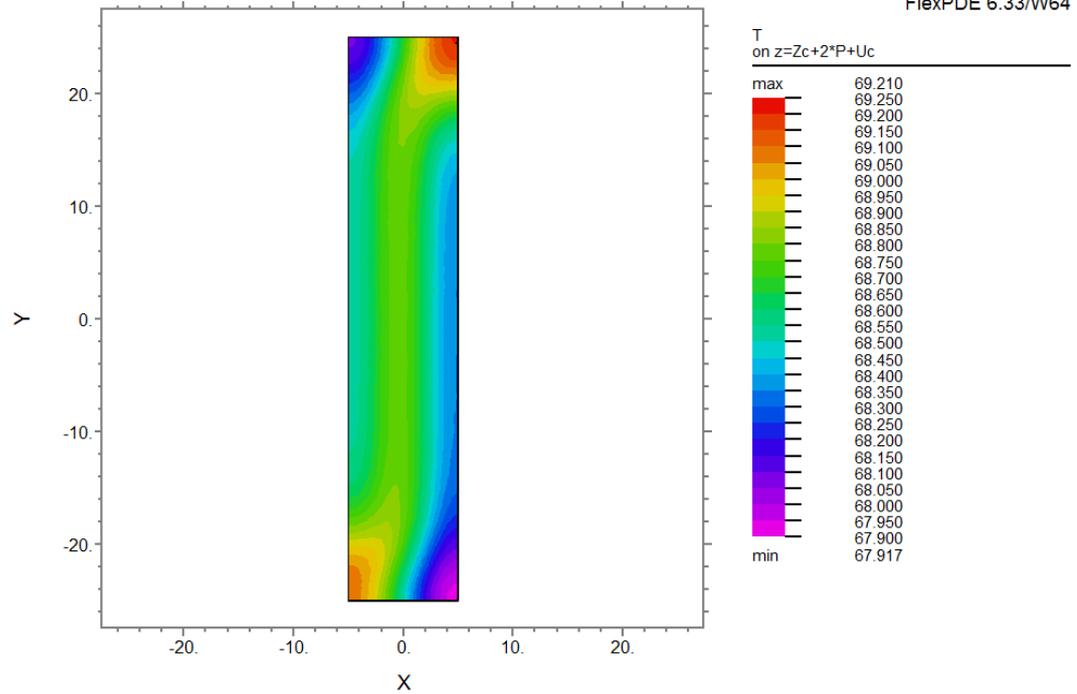
#### 5. Results

The distribution of temperature in the room floor is of great importance. It is shown in **Figure 3** where the temperature varies from 67.917 to 69.25 and is almost constant for  $-14 \leq Y \leq 14$  inches. The Y dimension could be altered to match a room of any size and there would be only a small temperature variation in the last 10 or 12 inches of the floor. **Figure 4** exhibits temperature variation in the XY plane taken through the center of the heating pipe. In **Figure 5** gives a view in the YZ plane through the center of the pipe. It shows the cooling effect of the 40 degree Fahrenheit side of the concrete. **Figure 6** shows the temperature on the YZ plane for  $X = -Wf/4 = -5$ . This shows the influence of the circular part of the heat pipe on the edges of the floor.

#### 6. Conclusion

The solution to the Laplace partial differential equation in three dimensions confirms the relatively high thermal conductivity of concrete or flagstone allows the use of buried heating pipes to result in an almost constant temperature on the floor surface. Many variations of this problem could be solved involving different heating pipe configurations and different thermal boundary conditions dictated by geographic considerations. The interested

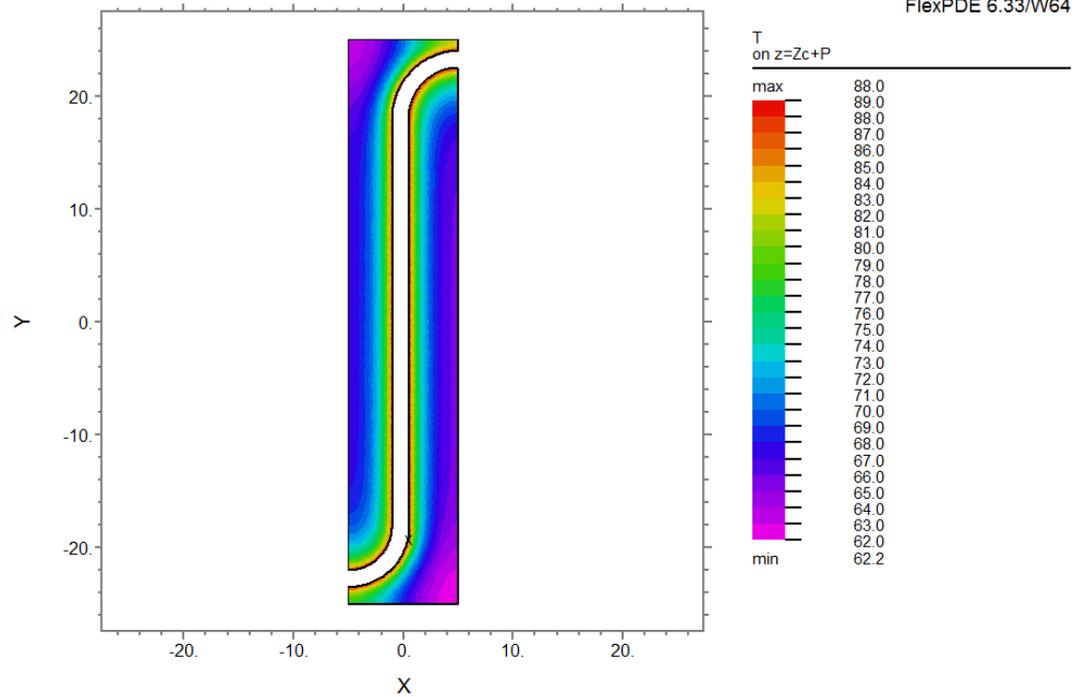
3D Floor Heating by hot water pipe



3d\_floorheatc: Grid#10 P2 Nodes=3397409 Cells=2407847 RMS Err= 9.2e-5  
Integral= 34309.62

Figure 3. Floor surface temperature.

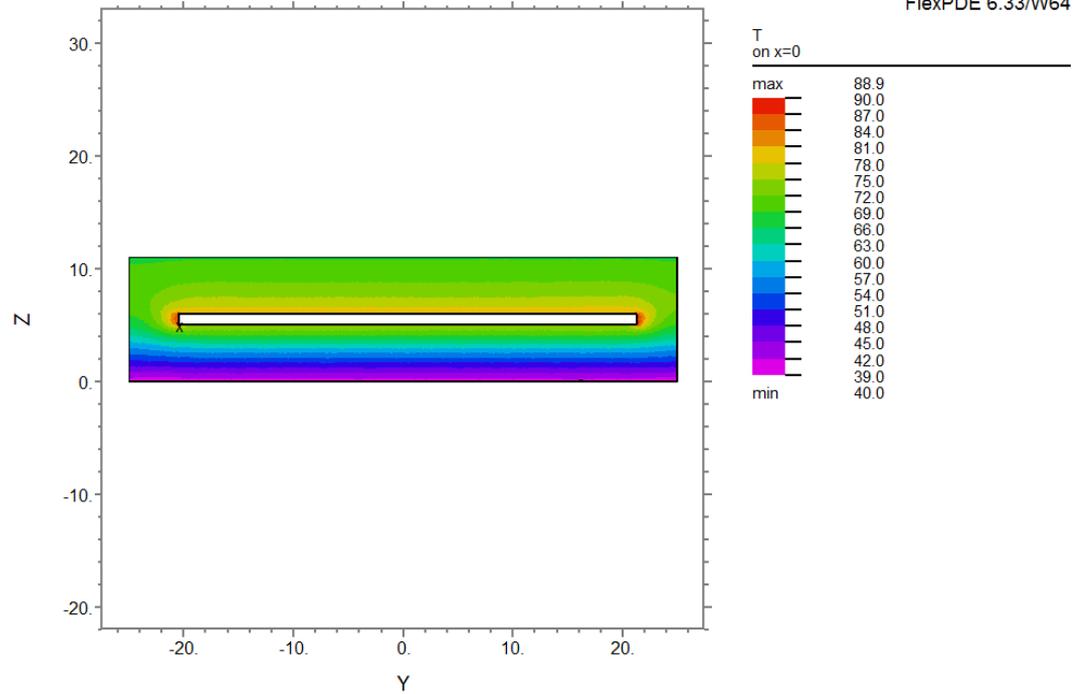
3D Floor Heating by hot water pipe



3d\_floorheatc: Grid#10 P2 Nodes=3397409 Cells=2407847 RMS Err= 9.2e-5  
Integral= 30363.30

Figure 4. Temperature in the middle of the pipe layer.

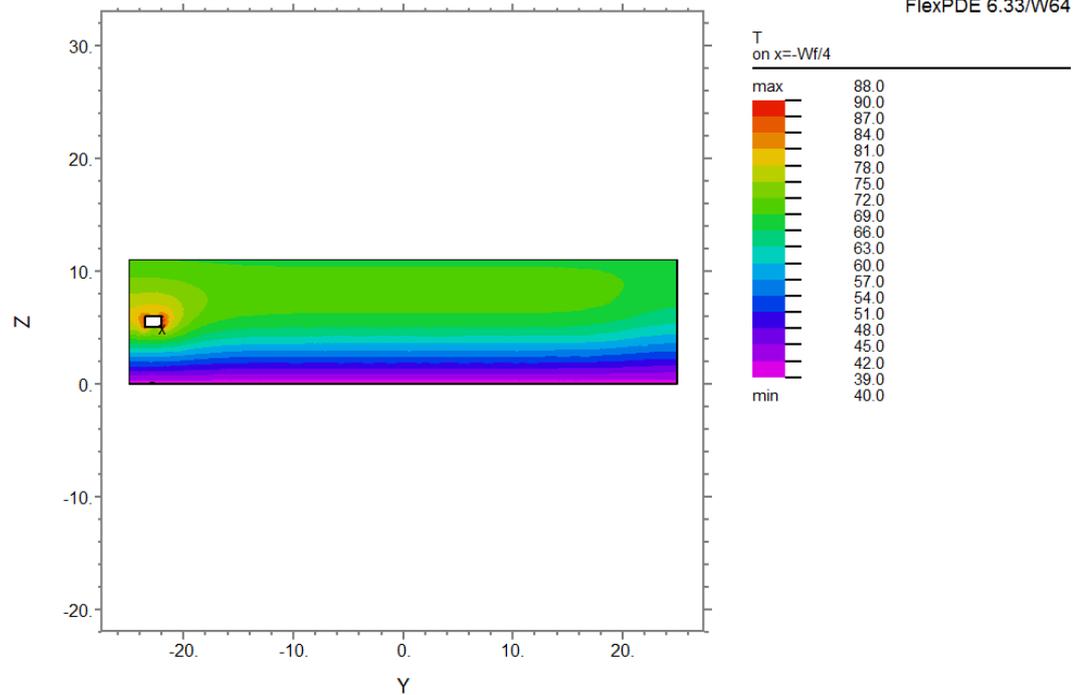
3D Floor Heating by hot water pipe



3d\_floorheat: Grid#10 P2 Nodes=3397409 Cells=2407847 RMS Err= 9.2e-5  
Integral= 33116.70

Figure 5. Temperature in the YZ plane for X = 0.

3D Floor Heating by hot water pipe



3d\_floorheat: Grid#10 P2 Nodes=3397409 Cells=2407847 RMS Err= 9.2e-5  
Integral= 34397.48

Figure 6. Temperature in the YZ plane for X = -Wf/4.

reader can use the program given here by down loading the free student version of FlexPDE finite element model builder from [www.pdesolutions.com](http://www.pdesolutions.com).

## References

- [1] Hughes, W.F. and Gaylord, E.W. (1964) Basic Equations of Engineering Science. Schaum Publishing Company, New York.