Journal of Modern Physics, 2013, 4, 754-756
http://dx.doi.org/10.4236/jmp.2013.46102 Published Online June 2013 (http://www.scirp.org/journal/jmp)

Study of Effective Edge Safety Factor Using Analytical Solution of Grad-Shafranov Equation for Circular Cross Section Tokamak

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Received April 5, 2013; revised May 7, 2013; accepted June 2, 2013

ABSTRACT

In this work, we present effective edge safety factor using analytical solution of the Grad-Shafranov equation based on expansion of free functions of first order and magnetic probes for circular cross section HT-7 tokamak.

Keywords: Horizontal Displacement; Tokamak

1. Introduction

The majority of research efforts in the area of controlled nuclear fusion, especially in tokamak research, are focused on confinement of hot plasmas by means of strong magnetic fields. Control of plasma position plays an important role in plasma confinement [1-16], and the achievement of optimized tokamak plasma operation. Therefore, plasma equilibrium study is one of the fundamental problems in magnetically confined plasmas. In tokamak equilibrium studies, there are many available experimental methods, and analytical solutions [1-15] for the steady state magnetohydrodynamic (MHD) equations, in particular, the Grad-Shafranov equation [17-19]. In this work, we present effective edge safety factor using analytical solutions [17-19] of the Grad-Shafranov equation [17] based on expansion of free functions of first order and magnetic probes for circular cross section HT-7 tokamak [16] (see **Table 1**).

2. Extended Grad-Shafranov Equation

Maxwell's equations together with the force balance equation from MHD equations, in the cylindrical coordinates (R, Z) reduce to the two-dimensional, nonlinear, elliptic partial differential equation, or Grad-Shafranov equation [4]: As in the linear case, the procedure to derive the Grad-Shafranov equation can be followed obtaining an extended Grad-Shafranov equation [17-25]

$$\Delta^*\psi = \mu_0 R J_\phi = -\mu_0(\gamma-1)R^2\frac{\mathrm{d}u(\psi)}{\mathrm{d}\psi} - F(\psi)\frac{\mathrm{d}F(\psi)}{\mathrm{d}\psi} \quad (1)$$

where Δ^* is

$$\Delta^* = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial z^2} \quad (2)$$

The internal energy in this extended Grad-Shafranov equation is a function of ψ. The $u(\psi)$ and $F(\psi)$ are two free functions, and where μ_0 and J are the vacuum permeability and plasma current density respectively.

3. Effective Edge Safety Factor by Solution of GSE

For circular cross section HT-7 Tokamak [16,26] (see **Table 1**), which is the ohmically heated tokamak, the Grad-Shafranov equation [17,27-28], is solved by formally expanding as follows [4,29]:

$$\psi(r,\theta) = \psi_0(r) + \psi_1(r)\cos\theta + \cdots, \quad (3)$$

Table 1. Parameter of the HT-7 tokamak.

Parameter	Value
Major Radius	1.22 m
Minor Radius	0.27 m
Toroidal Field	1 - 2.5 T
Plasma Current	100 - 250 kA
Discharge Time	~ 300 s
Electron Density	$1 - 6\times10^{19}$ m^{-3}

$$u(\psi) = u_2(\psi_0) + \frac{\mathrm{d}u_2(\psi_0)}{\mathrm{d}\psi_0}\psi_1 \cos\theta + \cdots, \tag{4}$$

$$F(\psi) = RB_\phi = R_0\left[B_0 + B_{\phi 2}(\psi_0) + \cdots\right], \tag{5}$$

where $B_0 = \text{const}$, is the vacuum toroidal field at $R = R_0$, $B_{\phi 2}(\psi)$ is a new free function replacing $F(\psi)$. In the first order solution or toroidal force balance approximation and if plasma were surrounded by a perfectly conducting shell located at $r = b$, then first order flux function [29] is:

$$\psi_1(r) = B_{\theta 1}(r)\int_r^b \frac{\mathrm{d}x}{xB_{\theta 1}^2(x)} \times \int_0^x \left[2\mu_0(\gamma - 1)y^2\frac{\mathrm{d}u_2(y)}{\mathrm{d}y} - yB_{\theta 1}^2(y)\right]\mathrm{d}y \tag{6}$$

If there are external coils to produce vertical magnetic field, the boundary condition on the flux function is modified so that we have:

$$\psi(b,\theta) = \text{const} + \psi_v(b,\theta), \tag{7}$$

where $\psi_v(r,\theta) = R_0 B_v r\cos\theta$, is the flux function due to external vertical field coils and therefore the full toroidal correction to ψ is:

$$\psi_1(\text{total}) = \psi_{1T}(r)\cos\theta = \left[\psi_1(r) + \left[\frac{bR_0B_v}{B_{\theta 1}(b)}\right]B_{\theta 1}(r)\right]\cos\theta \tag{8}$$

The shift of the plasma column center from the geometrical center of vacuum chamber is given by [29]:

$$\Delta R = -\frac{\psi_{1T}(a)}{\psi_0'(a)} = -\frac{\psi_1(a)}{\psi_0'(a)} - \Delta R_v = -\frac{\psi_1(a)}{\psi_0'(a)} - \frac{bB_v}{B_{\theta 1}(b)} \tag{9}$$

where $B_{\theta 1}(b) = \dfrac{\mu_0 I_p}{2\pi b}$.

Therefore, the first relation for plasma position [29] is

$$\Delta R_{\text{Analytical}} = \frac{b^2}{2R_0}\times\left[\left(\beta_p + \frac{l_i - 1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right] - \frac{bB_v}{B_{\theta 1}(b)} \tag{10}$$

where β_p is the poloidal beta, l_i is the internal inductance of the plasma, and B_v is the average vertical magnetic field over the vacuum chamber. We can find B_v from saddle sine coil [22] and the sum of the poloidal beta and half the plasma internal inductance expression $\Lambda = \beta_p + \frac{l_i}{2} - 1$, where ($\Lambda$ is Shafranov parameter) from magnetic coils measurement [22-25,30] using following equations

$$\Lambda = \beta_p + \frac{l_i}{2} - 1 = \ln\frac{a}{b} + \frac{\pi R_0}{\mu_0 I_0}\left(\langle B_\theta\rangle + \langle B_n\rangle\right), \tag{11}$$

where

$$\langle B_\theta\rangle = B_\theta(\theta = 0) - B_\theta(\theta = \pi), \quad \langle B_n\rangle = B_n\left(\theta = \frac{\pi}{2}\right) - B_n\left(\theta = \frac{3\pi}{2}\right), \tag{12}$$

We measured these local magnetic fields with magnetic probes [22] at above angles.

Therefore the effective edge safety factor at the plasma edge is given by [31]:

$$q_{eff} = \frac{2\pi a^2 B_\varphi}{\mu_0 R_0 I_p}\left(1 + \varepsilon^2\left(1 + \frac{(\Lambda + 1)^2}{2}\right)\right) \tag{13}$$

According to this relation, and also the value of Shafranov parameter which measured with magnetic probe we plotted time history of the effective edge safety factor in target shot for circular cross section HT-7 tokamak, as shown in **Figure 1**.

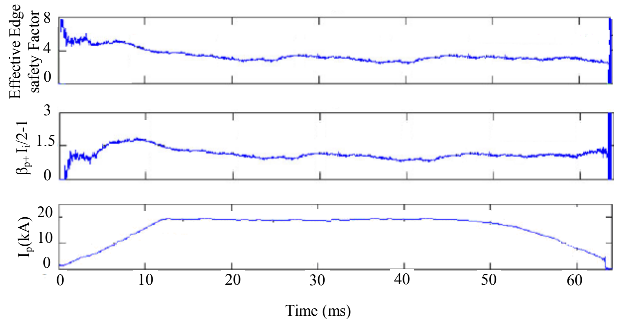

Figure 1. Edge safety factor obtained for circular cross section HT-7 tokamak by the GSE solution and magnetic probes.

4. Conclusion

We have determined the effective edge safety factor using analytical solution of the Grad-Shafranov equation based on expansion of free functions of first order and magnetic probes for circular cross section HT-7 tokamak.

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