

Instant-Form and Light-Front Hamiltonian and Path Integral Formulations of the Conformally Gauge-Fixed Polyakov D1-Brane Action in the Presence of a Scalar Axion Field and an $U(1)$ Gauge Field*

Usha Kulshreshtha^{1#}, Daya Shankar Kulshreshtha²

¹Department of Physics, Kirori Mal College, University of Delhi, Delhi, India.

²Department of Physics and Astrophysics, University of Delhi, Delhi, India

Email: [#]ushakulsh@gmail.com, dskulsh@gmail.com

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ABSTRACT

Recently we have studied the instant-form quantization (IFQ) and the light-front quantization (LFQ) of the conformally gauge-fixed Polyakov D1 brane action using the Hamiltonian and path integral formulations. The IFQ is studied in the equal world-sheet time framework on the hyperplanes defined by the world-sheet time $\sigma^0 = \tau = \text{constant}$ and the LFQ in the equal light-cone world-sheet time framework, on the hyperplanes of the light-front defined by the light-cone world-sheet time $\sigma^+ = (\tau + \sigma) = \text{constant}$. The light-front theory is seen to be a constrained system in the sense of Dirac in contrast to the instant-form theory. However, owing to the gauge anomalous nature of these theories, both of these theories are seen to lack the usual string gauge symmetries defined by the world-sheet reparametrization invariance (WSRI) and the Weyl invariance (WI). In the present work we show that these theories when considered in the presence of background gauge fields such as the NSNS 2-form gauge field $B_{\alpha\beta}(\sigma, \tau)$ or in the presence of $U(1)$ gauge field $A_\alpha(\sigma, \tau)$ and the constant scalar axion field $C(\sigma, \tau)$, then they are seen to possess the usual string gauge symmetries (WSRI and WI). In fact, these background gauge fields are seen to behave as the Wess-Zumino or Stueckelberg fields and the terms containing these fields are seen to behave as Wess-Zumino or Stueckelberg terms for these theories.

Keywords: Lagrangian and Hamiltonian Approach; Hamiltonian Quantization; Path Integral Quantization; Light-Front Quantization; Theory of Quantized Fields; Constrained Dynamics; D-Brane Actions; Polyakov Action; Strings and Branes; String Gauge Symmetry; Gauge Field Theories

1. Introduction

Study of D-brane actions is a domain of wider interest [1-20] in string theories. Polyakov D-brane action [1-8, 12-20] does not involve any square root [1-11] and is in particular, simpler to study. Recently, we have studied the instant-form (IF) quantization (IFQ) of this action [12] for the D1 brane in the conformal gauge (CG), using the

Hamiltonian [21] and path integral [22-25] formulations in the instant-form (IF) of dynamics (on the hyperplanes defined by the world-sheet (WS) time $\sigma^0 = \tau = \text{constant}$) [26,27]. We have also studied its LFQ [13-20] using the light-front (LF) dynamics (on the hyperplanes of the LF defined by the light-cone (LC) WS time $\sigma^+ = (\tau + \sigma) = \text{constant}$) [26-33].

The LF theory [13-20] is seen to be a constrained system in the sense of Dirac [21], which is in contrast to the corresponding IF theory [12], where the theory remains unconstrained in the sense of Dirac [21]. The LF theory is seen to possess a set of twenty six second-class

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#Corresponding author.

constraints [13-20]. Further, the conformally gauge-fixed Polyakov D1 brane action (CGFPD1BA) describing a gauge-noninvariant (GNI) theory (being a gauge-fixed theory) is seen to describe a gauge-invariant (GI) theory in the presence of an antisymmetric NSNS 2-form gauge field $B_{\alpha\beta}(\tau, \sigma)$ [13-20].

Recently we have shown [13-20] that this NSNS 2-form gauge field behaves like a Wess-Zumino (WZ) field and the term involving this field behaves like a WZ term for the CGFPD1BA [13-20]. We have also studied the Hamiltonian and path integral formulations of the CGFPD1BA with and without a scalar dilaton field in the IF [12] as well as in the LF [13-18] dynamics. In both the above cases the theory is seen (as expected) to be gauge-noninvariant (GNI), possessing a set of second-class constraints in each case, owing to the conformal gauge-fixing [1-8,12-20] of the theory.

The CGFPD1BA being GNI does not respect the usual string gauge symmetries defined by the world-sheet (WS) reparametrization invariance (WSRI) and the Weyl invariance (WI) [1-8,12-20]. However, in the presence of a constant 2-form gauge field $B_{\alpha\beta}$ it is seen [13] to describe a gauge-invariant (GI) theory [13-20] respecting the usual string gauge symmetries defined by the WSRI and the WI.

The IF and the LF Hamiltonian and path integral formulations of the CGFPD1BA have been studied by us in Refs. [12-18]. The IF and the LF Hamiltonian and path integral formulations of this theory in the presence of the constant 2-form gauge field $B_{\alpha\beta}$ have been studied by us in Ref. [13].

The question of the string gauge symmetries associated with the Polyakov D1 brane action in the presence of some other background fields such as the $U(1)$ gauge field $A^\mu(\tau, \sigma)$ and the constant scalar axion field $C(\tau, \sigma)$ have been considered by us in Ref. [19].

In the present work, we study the CGFPD1BA in the presence of some other background fields such as the $U(1)$ gauge field $A^\mu(\tau, \sigma)$ and the constant scalar axion field $C(\tau, \sigma)$.

In the next section we recap the basic essentials of the CGFPD1BA in IFQ as well as in the LF quantization (LFQ) [12-18]. In Section 3, we recap the basic essentials of this theory in the presence of the constant NSNS 2-form gauge field $B_{\alpha\beta}$ [13]. In Section 4, we study the IFQ [26,27] as well as the LFQ [26-33] of the CGFPD1BA in the presence of some other background fields such as the $U(1)$ gauge field $A^\mu(\tau, \sigma)$ and the constant scalar axion field $C(\tau, \sigma)$ [19,20]. Finally the summary and discussion is given in Section 5.

2. Conformally Gauge-Fixed Polyakov D1 Brane Action (CGFPD1BA): A Recap

The Polyakov D1 brane action in a d-dimensional curved

background $h_{\alpha\beta}$ is defined by [1-8,12-20]:

$$\tilde{S} = \int \tilde{\mathcal{L}} d^2\sigma \quad (1a)$$

$$\mathcal{L} = \left[\frac{-T}{2} \sqrt{-h} h^{\alpha\beta} G_{\alpha\beta} \right] \quad (1b)$$

$$h = \det(h_{\alpha\beta}), G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \quad (1c)$$

$$\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1) \quad (1d)$$

$$\mu, \nu = 0, 1, i, i = 2, 3, \dots, (d-1), \alpha, \beta = 01(IFQ) \quad (1e)$$

$$\mu, \nu = +, -, i, i = 2, 3, \dots, (d-1), \alpha, \beta = +, -(IFQ) \quad (1f)$$

Here $\sigma^\alpha \equiv (\tau, \sigma)$ are the two parameters describing the worldsheet (WS). The overdots and primes would denote the derivatives with respect to τ and σ . T is the string tension. $G_{\alpha\beta}$ is the induced metric on the WS and $X^\mu(\tau, \sigma)$ are the maps of the WS into the d -dimensional Minkowski space and describe the strings evolution in space-time [1-8,12-20].

Here $h_{\alpha\beta}$ are the auxiliary fields (which turn out to be proportional to the metric tensor $\eta_{\alpha\beta}$ of the two-dimensional surface swept out by the string). One can think of \tilde{S} as the action describing d-massless scalar fields X^μ in two-dimensions moving on a curved background $h_{\alpha\beta}$.

Also because the metric components $h_{\alpha\beta}$ are varied in the above equation, the 2-dimensional gravitational field $h_{\alpha\beta}$ is treated not as a given background field, but rather as an adjustable quantity coupled to the scalar fields.

The above action has three local gauge symmetries given by the 2-dimensional (2D) WSRI and WI as follows:

$$X^\mu \rightarrow \tilde{X}^\mu = [X^\mu + \delta X^\mu] \quad (2a)$$

$$\delta X^\mu = [\zeta^\alpha (\partial_\alpha X^\mu)] \quad (2b)$$

$$h^{\alpha\beta} \rightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] \quad (2c)$$

$$\delta h^{\alpha\beta} = [\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma}] \quad (2d)$$

$$h_{\alpha\beta} \rightarrow [\Omega] h_{\alpha\beta}, \Omega(\tau, \sigma) = \exp(2\omega), \omega = \omega(\tau, \sigma) \quad (2e)$$

where the WSRI is defined for the two parameters $\zeta^\alpha \equiv \zeta^\alpha(\tau, \sigma)$; and the WI and is specified by a function $\Omega \equiv \Omega(\tau, \sigma)$. In the following we would, however, work in the so-called orthonormal gauge where one sets $\Omega = 1$.

Also for the CGFPD1BA one makes use of the fact that the 2D metric $h_{\alpha\beta}$ is also specified by three independent functions as it is a symmetric 2×2 metric. One can therefore use these gauge symmetries of the theory to

choose $h_{\alpha\beta}$ to be of a particular form in the IFQ (on the hyperplanes defined by $x^0 = t = \text{constant}$) as follows:

$$h_{\alpha\beta} := \eta_{\alpha\beta}, h^{\alpha\beta} := \eta^{\alpha\beta} \quad (3)$$

In the IFQ we take

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \quad (4a)$$

$$h^{\alpha\beta} = \eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \quad (4b)$$

with

$$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta})} = +1 \quad (5)$$

In LFQ we use the Light-Cone (LC) variables defined by:

$$\sigma^\pm := (\tau \pm \sigma); X^\pm := (X^0 \pm X^1)/\sqrt{2} \quad (6)$$

In the LFQ (on the hyperplanes defined by $x^+ = \tau = \text{constant}$) we take:

$$h_{\alpha\beta} := \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix} \quad (7a)$$

$$h^{\alpha\beta} := \eta^{\alpha\beta} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \quad (7b)$$

with

$$\sqrt{-h} = \sqrt{-\det h_{(\alpha\beta)}} = +\frac{1}{2} \quad (8)$$

The action \tilde{S} in the CG (in IFQ as well as in LFQ) finally becomes: reads [1-8,12-20]:

$$S^N = \int \mathcal{L}^N d^2\sigma \quad (9a)$$

$$\mathcal{L}^N = \left(\frac{-T}{2}\right) [\partial^\beta X^\mu \partial_\beta X_\mu] \quad (9b)$$

$$\beta = 0, 1, \mu = 0, 1, i, i = 2, 3, \dots, 25 (IFQ) \quad (9c)$$

$$\beta = +, -, \mu = +, -, i; i = 2, 3, \dots, 25 (LFQ) \quad (9d)$$

This is the CGFPD1BA. In the IFQ it reads:

$$S_1 = \int L_1 d^2\sigma, \mathcal{L}_1 = \frac{-T}{2} [\partial_\sigma X^\mu \partial_\sigma X_\mu - \partial_\tau X^\mu \partial_\tau X_\mu] \quad (10a)$$

$$\mathcal{L}_1 = \frac{-T}{2} [(X')^2 - (\dot{X})^2] \quad (10b)$$

$$\partial_\tau \equiv \frac{\partial}{\partial \tau}, \partial_\sigma \equiv \frac{\partial}{\partial \sigma}, \dot{X}^\mu \equiv \frac{\partial X^\mu}{\partial \tau}, X'^\mu = \frac{\partial X^\mu}{\partial \sigma} \quad (10c)$$

The IFQ of this action has been studied by us recently [12] and we recap it here very briefly. The canonical momenta conjugate to X^μ obtained from the above action are:

$$P^\mu := \frac{\partial \mathcal{L}_1}{\partial (\partial_\tau X^\mu)} = T \partial_\tau X^\mu \quad (11)$$

Here the velocities $\partial_\tau X^\mu = \frac{1}{T} P^\mu$ are expressible.

Canonical Hamiltonian density for the above theory is:

$$\mathcal{H}_1^c = [P^\mu (\partial_\tau X_\mu) - \mathcal{L}_1] = \left[\frac{1}{2T} P^2 + \frac{T}{2} (X')^2 \right]; \quad (12)$$

The quantization of the system is trivial. The nonvanishing equal WS-time (EWST) commutation relations (CR's) for this theory are obtained as:

$$[X^\mu(\sigma, \tau), P_\nu(\sigma', \tau)] = i \delta_\nu^\mu \delta(\sigma - \sigma') \quad (13)$$

where $\delta(\sigma - \sigma')$ is the one-dimensional Dirac distribution function.

It is obvious from the above considerations that the above theory is unconstrained in the sense of Dirac [21]. It may be important to emphasize here that an unconstrained system like the above theory is a gauge-noninvariant theory and it is some what akin to a gauge-fixed gauge-invariant theory which makes it a gauge-noninvariant system. In the presence of a scalar dilaton field the theory of course, becomes a constrained system in the sense of Dirac as shown in our earlier work [12]. For further details of the IFQ of this theory we refer to our earlier work of Ref. [12].

The CGFPD1BA in the LFQ reads [12-18]:

$$S_2 = \int \mathcal{L}_2 d\sigma^+ d\sigma^- \quad (14a)$$

$$\mathcal{L}_2 = \left[\frac{-T}{2} \right] \quad (14b)$$

$$\cdot [(\partial_+ X^+)(\partial_- X^-) + (\partial_+ X^-)(\partial_- X^+) + (\partial_+ X^i)(\partial_- X^i)] \quad (14c)$$

The canonical momenta P^+, P^- and P_i canonically conjugate respectively to X^-, X^+ and X^i , obtained from the above LF action are:

$$P^+ := \frac{\partial \mathcal{L}_2}{\partial (\partial_+ X^-)} = -\frac{T}{2} (\partial_- X^+) \quad (15a)$$

$$P^- := \frac{\partial \mathcal{L}_2}{\partial (\partial_+ X^+)} = -\frac{T}{2} (\partial_- X^-) \quad (15b)$$

$$P_i := \frac{\partial \mathcal{L}_2}{\partial (\partial_+ X^i)} = -\frac{T}{2} (\partial_- X^i) \quad (15c)$$

Above equations however, imply that the theory possesses 26 primary constraints:

$$\rho_i = \left(P^+ + \frac{T}{2} \partial_- X^+ \right) \approx 0 \quad (16a)$$

$$\rho_2 = \left(P^- + \frac{T}{2} \partial_- X^- \right) \approx 0 \quad (16b)$$

$$\rho_i = \left(P_i + \frac{T}{2} \partial_- X^i \right) \approx 0, i = 2, 3, \dots, 25. \quad (16c)$$

Canonical Hamiltonian density for the above theory is:

$$\mathcal{H}_2^c = \left[P^+ (\partial_+ X^-) + P^- (\partial_+ X^+) + P_i (\partial_+ X^i) - \mathcal{L}_2 \right] \approx 0 \quad (17)$$

After including the above 26 primary constraints of the theory in the canonical Hamiltonian density \mathcal{H}_2^c with the help of Lagrange multiplier fields u, v and w_i , the total Hamiltonian density \mathcal{H}_2^T could be written as

$$\mathcal{H}_2^T = \left[u \left(P^+ + \frac{T}{2} \partial_- X^+ \right) + v \left(P^- + \frac{T}{2} \partial_- X^- \right) + w_i \left(P_i + \frac{T}{2} \partial_- X^i \right) \right] \quad (18)$$

We treat u, v and w_i as dynamical. The Hamiltons equations obtained from the total Hamiltonian

$H_2^T = \int \mathcal{H}_2^T d\sigma^-$, are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that these primary constraints of the theory be preserved in the course of time one does not get any secondary constraints. The theory is thus seen to possess only 26 above constraints. Further, the matrix of the Poisson brackets of the above constraints is seen to be nonsingular, implying that the set of these constraints is second-class and consequently the theory is GNI [12-18]. The theory indeed does not possess the usual local string gauge symmetries defined by the WSRI and the WI [12-18].

This action is thus seen to lack the local gauge symmetries. This is in contrast to the fact that the original action \tilde{S} had the local gauge symmetries and was therefore GI. The theory defined by the action S^N , on the other hand describes a GNI theory. This is not surprising at all because the theory defined by S^N is after-all (conformally) a gauge-fixed theory and consequently it is not expected to be GI anyway.

Infact, the IF theory defined by S^N is seen to be unconstrained [12] whereas the LF theory is seen to possess a set of 26 second-class constraints [12-18]. In both the cases theory does not respect the usual local string gauge symmetries defined by WSRI and WI [12-18].

3. CGFPD1BA in the Presence of a 2-Form Gauge Field $B_{\alpha\beta}$: A Recap

We now consider this CGFPD1BA in the presence of a constant background antisymmetric 2-form NSNS gauge field $B_{\alpha\beta}$ studied earlier by Schmidhuber, de Alwis and

Sato, Tseytlin and Abou Zeid and Hull and others defined by [1-8]. This theory has been studied by us earlier in Ref. [13]. This theory is defined by the action [1-8,13]:

$$S_B^I = \int \mathcal{L}_B^I d^2\sigma \quad (19a)$$

$$\mathcal{L}_B^I = \left[\mathcal{L}^C + \mathcal{L}^B \right] \quad (19b)$$

$$\mathcal{L}^C = \left[\lambda \mathcal{L}^N \right] = \left(\frac{-T}{2} \right) \left[\lambda \partial^\beta X^\mu \partial_\beta X_\mu \right] \quad (19c)$$

$$\mathcal{L}^B = \left(\frac{-T}{2} \right) \left[\Lambda \varepsilon^{\alpha\beta} B_{\alpha\beta} \right] \quad (19d)$$

$$\lambda = \sqrt{(1 + \Lambda^2)}, \Lambda = \text{constant}, \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (19e)$$

$$B_{\alpha\beta} := \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}, B_{\alpha\beta} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \quad (19f)$$

$$B = B_{01} = -B_{10} \text{ (IFQ)} \quad (19g)$$

$$B = B_{+-} = -B_{-+} \text{ (LFQ)} \quad (19h)$$

$$\alpha, \beta = 0, 1, \mu = 0, 1, i, i = 2, 3, \dots, 25 \text{ (IFQ)} \quad (19i)$$

$$\alpha, \beta = +, -, \mu = +, -, i, i = 2, 3, \dots, 25 \text{ (LFQ)} \quad (19j)$$

It is important to recollect here that the 2-form gauge field $B_{\alpha\beta}$ is a scalar field in the target-space whereas it is a constant anti-symmetric tensor field in the world-sheet space. The above action is seen to be GI in the IFQ as well as in the LFQ [13]. It is seen to possess only one constraint in IFQ and a set of 27 constraints in LFQ. The nature of constraints in both the cases is seen to be first-class implying that the theory is GI in both the cases [13]. In Ref. [13], we have studied the Hamiltonian and path integral formulations of this theory under appropriate gauge-fixing.

3.1. Instant-Form Quantization

In the following, we study the IFQ of this above theory using the EWST framework of dynamics on the hyperplanes defined by the WS-time $\sigma^0 = \tau = \text{constant}$ [26, 27]. In the IFQ, the above action reads as:

$$S_3 = \int \mathcal{L}_3 d\tau d\sigma, \mathcal{L}_3 = \left[-\frac{\lambda T}{2} \left[(X')^2 - (\dot{X})^2 \right] - \Lambda T B \right] \quad (20)$$

Canonical momenta are

$$P^\mu := \frac{\partial \mathcal{L}_3}{\partial (\partial_\tau X_\mu)} = \left[\lambda T \dot{X}^\mu \right], \Pi_B := \frac{\partial \mathcal{L}_3}{\partial (\partial_\tau B)} = 0 \quad (21)$$

Here P^μ and Π_B are the canonical momenta conjugate respectively to X_μ and B ($\equiv B_{01} = -B_{10}$). The above theory is thus seen to possess one primary constraint: $\Phi_1 = \Pi_B \approx 0$. Canonical Hamiltonian density of

this theory is:

$$\mathcal{H}_3^c = \left[P^\mu (\partial_\tau X_\mu) + \Pi_B (\partial_\tau B) - \mathcal{L}_3 \right] \quad (22a)$$

$$\mathcal{H}_3^c = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TB \right] \quad (22b)$$

After incorporating the primary constraint of the theory in the canonical Hamiltonian density with the help of Lagrange multiplier field $u(\tau, \sigma)$ (to be treated as dynamical) the total Hamiltonian density of the theory could be written as:

$$\mathcal{H}_3^T = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TB + u \Pi_B \right] \quad (23)$$

Also, the momenta canonically conjugate to μ is denoted by p_μ .

The Hamiltons equations obtained from the total Hamiltonian: $H_3^T = \int \mathcal{H}_3^T d\sigma$ are the equations of motion that preserve the constraints of the theory in the course of time. Demanding that the primary constraint of the theory be preserved in the course of time one does not get any further constraints. The theory is thus seen to possess the only one above constraint.

The Poisson bracket of the constraint of the theory with itself is seen to be zero implying that the constraint is first-class and that the theory is GI. It is indeed seen to possess three local gauge symmetries given by the 2D WSRI and the WI defined by:

$$X^\mu \rightarrow \tilde{X}^\mu = \left[X^\mu + \delta X^\mu \right] \quad (24a)$$

$$\delta X^\mu = \left[\zeta^\alpha (\partial_\alpha X^\mu) \right] \quad (24b)$$

$$h^{\alpha\beta} \rightarrow \tilde{h}^{\alpha\beta} = \left[h^{\alpha\beta} + \delta h^{\alpha\beta} \right] \quad (24c)$$

$$\delta h^{\alpha\beta} = \left[\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma} \right] \quad (24d)$$

$$B_{\alpha\beta} \rightarrow \tilde{B}_{\alpha\beta} = \left[B_{\alpha\beta} + \delta B_{\alpha\beta} \right] \quad (24e)$$

$$\delta B_{\alpha\beta} = \left[\zeta^\alpha \partial_\alpha B_{\alpha\beta} \right] \quad (24f)$$

$$h_{\alpha\beta} \rightarrow \Omega \left[h_{\alpha\beta} \right], \Omega = \exp(2\omega) \quad (24g)$$

It is important to recollect here that the 2-form gauge field $B_{\alpha\beta}$ is a scalar field in the target-space whereas it is a constant anti-symmetric tensor field in the world-sheet space (and consequently we have $\delta B_{\alpha\beta} = 0$).

Thus the theory is seen to be GI in IFQ as well as in LFQ. It is therefore gauge-nonanomalous possessing the three local gauge symmetries defined by the 2D WSRI and the WI. The theory could therefore be quantized under appropriate gauge-fixing.

Hamiltonian and path integral formulations of this theory could be studied under appropriate gauge-fixing e.g., under the gauge: $\Phi = B \approx 0$. Corresponding to this

choice of gauge the total set of constraints of the theory under which the quantization of the theory could e.g., be studied becomes: $\Phi_1 = \Pi_B \approx 0$ and $\Phi_2 = \Phi = B \approx 0$. The matrix of the Poisson brackets of these constraints is seen to be nonsingular implying that the corresponding set of constraints is second-class. Following the Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing EWST CR's of the theory under the above gauge are obtained as:

$$\left[X^\mu(\sigma, \tau), P^\nu(\sigma', \tau) \right] = (i) \eta^{\mu\nu} \delta(\sigma - \sigma') \quad (25)$$

3.2. Light-Front Quantization

In LFQ, using the ELCWST framework of dynamics on the hyperplanes defined by the LC WS-time

$\sigma^+ = (\tau + \sigma) = \text{constant}$, the action of the theory reads:

$$S_4 = \int \mathcal{L}_4 d\sigma^+ d\sigma^- \quad (26a)$$

$$\mathcal{L}_4 = \left[-\frac{\lambda T}{2} \right] \left[(\partial_+ X^+) (\partial_- X^-) + (\partial_+ X^-) (\partial_- X^+) \right. \\ \left. + (\partial_+ X^i) (\partial_- X^i) - \Lambda TB \right] \quad (26b)$$

Canonical momenta Π_B, P^+, P^- , and P_i conjugate respectively to B, X^-, X^+ , and X_i are:

$$\Pi_B := \frac{\partial \mathcal{L}_4}{\partial (\partial_+ B)} = 0 \quad (27a)$$

$$P^+ := \frac{\partial \mathcal{L}_4}{\partial (\partial_+ X^-)} = -\frac{\lambda T}{2} (\partial_- X^+) \quad (27b)$$

$$P^- := \frac{\partial \mathcal{L}_4}{\partial (\partial_+ X^+)} = -\frac{\lambda T}{2} (\partial_- X^-) \quad (27c)$$

$$P_i := \frac{\partial \mathcal{L}_4}{\partial (\partial_+ X^i)} = -\frac{\lambda T}{2} (\partial_- X^i), i = 2, 3, \dots, 25 \quad (27d)$$

Above equations however, imply that the theory possesses the following 27 primary constraints:

$$\psi_1 = \Pi_B \approx 0 \quad (28a)$$

$$\psi_2 = \left[P^+ + \frac{\lambda T}{2} (\partial_- X^+) \right] \approx 0 \quad (28b)$$

$$\psi_3 = \left[P^- + \frac{\lambda T}{2} (\partial_- X^-) \right] \approx 0 \quad (28c)$$

$$\psi_i = \left[P_i + \frac{\lambda T}{2} (\partial_- X^i) \right] \approx 0, i = 2, 3, \dots, 25 \quad (28d)$$

Canonical Hamiltonian density corresponding to \mathcal{L}_4 is

$$\mathcal{H}_4^c = \left[\Pi_B (\partial_+ B) + P^+ (\partial_+ X^-) \right. \\ \left. + P^- (\partial_+ X^+) + P_i (\partial_+ X^i) - \mathcal{L}_4 \right] \quad (29a)$$

$$\mathcal{H}_4^c = [\Lambda TB] \quad (29b)$$

After including the primary constraints of the theory in the canonical Hamiltonian density \mathcal{H}_4^c with the help of Lagrange multiplier fields $s(\sigma^+, \sigma^-)$, $u(\sigma^+, \sigma^-)$, $v(\sigma^+, \sigma^-)$ and $w_i(\sigma^+, \sigma^-)$ (which we treat as dynamical), the total Hamiltonian density of the theory could be written as:

$$\mathcal{H}_4^T = [\mathcal{H}_2^c + s\psi_1 + u\psi_2 + v\psi_3 + w_i\psi_i] \quad (30a)$$

$$\begin{aligned} \mathcal{H}_4^T = & \left[\Lambda TB + s\Pi_B + u \left[P^+ + \left(\frac{\lambda T}{2} \right) (\partial_- X^+) \right] \right. \\ & + v \left[P^- + \left(\frac{\lambda T}{2} \right) (\partial_- X^-) \right] \\ & \left. + w_i \left[P_i + \left(\frac{\lambda T}{2} \right) (\partial_- X^i) \right] \right] \end{aligned} \quad (30b)$$

Hamilton's equations obtained from the total Hamiltonian $H_4^T = \int \mathcal{H}_4^T d\sigma^-$ are the equations of motion that preserve the constraints of the theory in the course of time. The matrix of the Poisson brackets of these above constraints is seen to be singular implying that the corresponding set of constraints is first-class and that the corresponding theory is GI [13].

The above theory is indeed seen to possess three local gauge symmetries given by the 2D WSRI and the WI [13] and the theory could be quantized under appropriate gauge-fixing. To study the Hamiltonian and path integral formulations of the theory under gauge-fixing, we could e.g., choose the gauge: $\eta = B \approx 0$. corresponding to this gauge choice the total set of 28 constraints of the theory becomes [13]:

$$\phi_1 = \eta = B \approx 0 \quad (31a)$$

$$\phi_2 = \psi_1 = \Pi_B \approx 0 \quad (31b)$$

$$\phi_3 = \psi_2 = \left[P^+ + \frac{\lambda T}{2} (\partial_- X^+) \right] \approx 0 \quad (31c)$$

$$\phi_4 = \psi_3 = \left[P^- + \frac{\lambda T}{2} (\partial_- X^-) \right] \approx 0 \quad (31d)$$

$$\phi_i = \psi_i = \left[P_i + \frac{\lambda T}{2} (\partial_- X^i) \right] \approx 0, i = 2, 3, \dots, 25 \quad (31e)$$

The matrix of the Poisson brackets of the above 28 constraints is seen to be nonsingular implying that the corresponding set of constraints is second-class. Now following the Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing ELCWST CR's of the theory under the gauge-fixing $\eta = B \approx 0$ are obtained as [13]:

$$[X^+, P^-] = [X^-, P^+] = [X^i, P_i] = \frac{i}{2} \delta(\sigma - \sigma') \quad (32a)$$

$$[X^-, X^+] = [X^i, X^i] = \frac{i}{2\lambda T} \epsilon(\sigma - \sigma') \quad (32b)$$

$$[P^+, P^-] = [P_i, P_i] = \frac{-iT\lambda}{4} \partial_- \delta(\sigma - \sigma') \quad (32c)$$

For further details of the Hamiltonian and path integral formulations of the above theory, we refer to our earlier work [13].

4. CGFPD1BA in the Presence of a Scalar Axion Field C and an $U(1)$ Gauge Field A^μ

In this section, we study the IFQ and LFQ of the CGFPD1BA in the presence of a $U(1)$ gauge field $A_\alpha (\equiv A_\alpha(\tau, \sigma))$ and a constant scalar axion field $C (\equiv C(\tau, \sigma))$ [19,20]. We find that the CGFPD1BA describing a GNI theory (being a gauge-fixed theory) is seen to describe a GI theory when considered in the presence of above background fields.

We also find that the $U(1)$ gauge field $A_\alpha(\tau, \sigma)$ and the constant scalar axion field $C(\tau, \sigma)$ are both seen to behave like the Wess-Zumino (WZ) fields [19,20] and the term involving these fields in the action is seen to behave like a WZ term for the CGFPD1BA [13,19,20]. Here the field A_α is a scalar field in the target space and a vector field in the WS space and the axion field C is a constant scalar field in both the target space as well as in the WS space [19,20].

We find that the resulting theory obtained in the above manner describes a GI system respecting the usual string gauge symmetries defined by the 2D WSRI and the WI. It is seen that the axion field C and the $U(1)$ gauge field A_α , in the resulting theory behave like the WZ fields and the term involving these fields behaves like a WZ term for the CGFPD1BA [19].

The situation in the present case is seen to be exactly analogous to a theory where one considers the CGFPD1BA in the presence of a 2-form gauge field $B_{\alpha\beta}$ as studied by us in our earlier work [13], where the field $B_{\alpha\beta}$ behaves like a WZ field and the term involving this field behaves like a WZ term for the CGFPD1BA [13].

The CGFPD1BA in the presence of a constant background scalar axion field C and an $U(1)$ gauge field A_α is defined by [1-8,19,20]:

$$S_A^I = \int \mathcal{L}_A^I d^2\sigma \quad (33a)$$

$$\mathcal{L}_A^I = (\mathcal{L}^C + \mathcal{L}^A) \quad (33b)$$

$$\mathcal{L}^C = (\lambda \mathcal{L}^N) = \left(\frac{-T}{2} \right) (\lambda \partial^\beta X^\mu \partial_\beta X_\mu) \quad (33c)$$

$$\mathcal{L}^A = \left(\frac{-T}{2} \right) (-\Lambda C \varepsilon^{\alpha\beta} F_{\alpha\beta}) \quad (33d)$$

$$\lambda = \sqrt{(1 + \Lambda^2)}, \Lambda = \text{constant} \quad (33e)$$

$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \quad (33f)$$

$$f = F_{01} = -F_{01}(IFQ) \quad (33g)$$

$$f = F_{+-} = -F_{+-}(LFQ) \quad (33h)$$

$$\alpha, \beta = 0, 1, \mu = 0, 1, i, i = 2, 3, \dots, 25 (IFQ) \quad (33i)$$

$$\alpha, \beta = +, -, \mu = +, -, i, i = 2, 3, \dots, 25 (LFQ) \quad (33j)$$

4.1. Instant-Form Quantization

In this section, we study the IF Hamiltonian and path integral quantization of the above theory using the EWST framework, on the hyperplanes defined by the WS-time $\sigma^0 = \tau = \text{constant}$. The IF action reads:

$$S_5 = \int \mathcal{L}_5 d\tau d\sigma, \mathcal{L}_5 = \left[-\frac{\lambda T}{2} \left((X')^2 - (\dot{X})^2 \right) + \Lambda TCf \right] \quad (34)$$

Overdots and primes denote derivatives with respect to τ and σ respectively. The canonical momenta obtained are:

$$P^\mu := \frac{\partial \mathcal{L}_5}{\partial (\partial_\tau X_\mu)} = [\lambda T \dot{X}^\mu] \quad (35a)$$

$$\Pi_c := \frac{\partial \mathcal{L}_5}{\partial (\partial_\tau C)} = 0 \quad (35b)$$

$$\Pi^0 := \frac{\partial \mathcal{L}_5}{\partial (\partial_\tau A_0)} = 0 \quad (35c)$$

$$E (\equiv \Pi^1) := \frac{\partial \mathcal{L}_5}{\partial (\partial_\tau A_1)} = \Lambda TC \quad (35d)$$

where $P^\mu, \Pi^0, E (\equiv \Pi^1)$ and Π_c are the canonical momenta conjugate respectively to X_μ, A_0, A_1 and C . The theory is thus seen to possess three primary constraints:

$$\Psi_1 = \Pi^0 \approx 0 \quad (36a)$$

$$\Psi_2 = (E - \Lambda TC) \approx 0 \quad (36b)$$

$$\Psi_3 = \Pi_c \approx 0 \quad (36c)$$

Canonical Hamiltonian density corresponding to above Lagrangian density is:

$$\mathcal{H}_5^c = \left[P^\mu (\partial_\tau X_\mu) + \Pi^0 (\partial_\tau A_0) + E (\partial_\tau A_1) + \Pi_c (\partial_\tau C) - \mathcal{L}_5 \right] \quad (37a)$$

$$\mathcal{H}_5^c = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TC A_0' \right] \quad (37b)$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density with the help of

Lagrange multiplier fields $u(\tau, \sigma), v(\tau, \sigma)$ and $w(\tau, \sigma)$ (treated as dynamical) the total Hamiltonian density of the theory becomes:

$$\mathcal{H}_5^T = \left[\mathcal{H}_5^c + u\Psi_1 + v\Psi_2 + w\Psi_3 \right] \quad (38a)$$

$$\mathcal{H}_5^T = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TC A_0' + \Pi^0 u + (E - \Lambda TC)v + \Pi_c w \right] \quad (38b)$$

Momenta canonically conjugate to u, v and w are denoted respectively by p_u, p_v and p_w . Hamiltons equations obtained from the total Hamiltonian $H_5^T = \int \mathcal{H}_5^T d\sigma$, for the closed string with periodic boundary conditions (BC's) e.g., are:

$$\partial_\tau X^\mu = \frac{\partial H_5^T}{\partial P_\mu} = \left[\frac{1}{\lambda T} \right] P^\mu, \quad (39a)$$

$$-\partial_\tau P^\mu = \frac{\partial H_5^T}{\partial X_\mu} = (-\lambda T) X^{\mu\prime}$$

$$\partial_\tau C = \frac{\partial H_5^T}{\partial \Pi_c} = w, -\partial_\tau \Pi_c = \frac{\partial H_5^T}{\partial C} = \Lambda T (A_0' - v) \quad (39b)$$

$$\partial_\tau A_0 = \frac{\partial H_5^T}{\partial \Pi^0} = u, -\partial_\tau \Pi^0 = \frac{\partial H_5^T}{\partial A_0} = 0 \quad (39c)$$

$$\partial_\tau A_1 = \frac{\partial H_5^T}{\partial E} = v, -\partial_\tau E = \frac{\partial H_5^T}{\partial A_1} = 0 \quad (39d)$$

$$\partial_\tau u = \frac{\partial H_5^T}{\partial p_u} = 0, -\partial_\tau p_u = \frac{\partial H_5^T}{\partial u} = \Pi^0 \quad (39e)$$

$$\partial_\tau v = \frac{\partial H_5^T}{\partial p_v} = 0, -\partial_\tau p_v = \frac{\partial H_5^T}{\partial v} = (E - \Lambda TC) \quad (39f)$$

$$\partial_\tau w = \frac{\partial H_5^T}{\partial p_w} = 0, -\partial_\tau p_w = \frac{\partial H_5^T}{\partial w} = \Pi_c \quad (39g)$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraints of the theory be preserved in the course of time one does not get any further constraints. The theory is thus seen to possess only three constraint Ψ_1, Ψ_2 and Ψ_3 .

Matrix of the Poission brackets of these constraints is seen to be singular implying that the constraints form a set of first-class constraints and that the theory is GI (and consequently gauge-nonanomalous). It is indeed seen to possess three local gauge symmetries given by the 2D WSRI and the WI defined by:

$$X^\mu \rightarrow \tilde{X}^\mu = \left[X^\mu + \delta X^\mu \right] \quad (40a)$$

$$\delta X^\mu = \left[\zeta^\alpha (\partial_\alpha X^\mu) \right] \tag{40b}$$

$$h^{\alpha\beta} \rightarrow \tilde{h}^{\alpha\beta} = \left[h^{\alpha\beta} + \delta h^{\alpha\beta} \right] \tag{40c}$$

$$\delta h^{\alpha\beta} = \left[\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma} \right] \tag{40d}$$

$$A_\beta \rightarrow \tilde{A}_\beta = \left[A_\beta + \delta A_\beta \right] \tag{40e}$$

$$\delta A_\beta = \left[\zeta^\alpha \partial_\alpha A_\beta \right] \tag{40f}$$

$$C \rightarrow \tilde{C}_\alpha = \left[C + \delta C \right] \tag{40g}$$

$$\delta C = \left[\zeta^\alpha \partial_\alpha C \right] \tag{40h}$$

$$h_{\alpha\beta} \rightarrow \left[\Omega h_{\alpha\beta} \right], \Omega = \exp(2\omega) \tag{40i}$$

The first order Lagrangian density of the theory is:

$$\begin{aligned} \mathcal{L}_5^{LO} = & \left[P^\mu (\partial_\tau X_\mu) + \Pi^0 (\partial_\tau A_0) + E (\partial_\tau A_1) + \Pi_c (\partial_\tau c) \right. \\ & \left. + p_u (\partial_\tau u) + p_v (\partial_\tau v) + p_w (\partial_\tau w) - \mathcal{H}_5^T \right] \end{aligned} \tag{41a}$$

$$\mathcal{L}_5^{LO} = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TCf \right] \tag{41b}$$

The theory could be quantized under appropriate gauge-fixing. To study the Hamiltonian and path integral formulations of this theory under gauge-fixing, we could choose e.g., the gauge:

$$\zeta = A_0 \approx 0 \tag{42}$$

Corresponding to this choice of gauge the total set of constraints of the theory under which the quantization of the theory could e.g., be studied becomes:

$$\xi_1 = \Psi_1 = \Pi^0 \approx 0 \tag{43a}$$

$$\xi_2 = \Psi_2 = (E - \Lambda TC) \approx 0 \tag{43b}$$

$$\xi_3 = \Psi_3 = \Pi_c \approx 0 \tag{43c}$$

$$\xi_4 = \zeta = A_0 \approx 0 \tag{43d}$$

We now calculate the matrix $M_{\alpha\beta} \left(:= \{ \Psi_\alpha, \Psi_\beta \}_{PB} \right)$ of the Poisson brackets of the constraints Ψ_i . The non-vanishing elements of the matrix $M_{\alpha\beta}$ are obtained as:

$$\Lambda TM_{14} = -\Lambda TM_{41} = M_{23} = -M_{32} = [-\Lambda T] \delta(\sigma - \sigma') \tag{44}$$

The matrix $M_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of constraints is a set of second-class constraints. The determinant of the matrix $M_{\alpha\beta}$ is given by:

$$\left[\left| \det(M_{\alpha\beta}) \right| \right]^{1/2} = \Lambda T \delta^2(\sigma - \sigma') \tag{45}$$

and the nonvanishing elements of the inverse of the matrix $M_{\alpha\beta}$ (i.e., the elements of the matrix $(M^{-1})_{\alpha\beta}$) are obtained as:

$$\begin{aligned} (M^{-1})_{14} &= -(M^{-1})_{41} = \Lambda T (M^{-1})_{23} \\ &= -\Lambda T (M^{-1})_{32} = \delta(\sigma - \sigma') \end{aligned} \tag{46a}$$

$$\int M(\sigma, \sigma'') M^{-1}(\sigma'', \sigma') d\sigma'' = \mathbf{1}_{4 \times 4} \delta(\sigma - \sigma') \tag{46b}$$

Following the Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing EWST CR's of the theory under the gauge $\zeta = A_0 \approx 0$ (with the arguments being suppressed) are obtained as:

$$\left[X^\mu(\sigma, \tau), P_\mu(\sigma', \tau) \right] = i\eta^{\mu\nu} \delta(\sigma - \sigma') \tag{47}$$

In the path integral formulation, the transition to the quantum theory, is, however, made by writing the vacuum to vacuum transition amplitude called the generating functional $Z_1[J_i]$ of the theory under GFC ζ in the presence of external sources J_i as follows:

$$\begin{aligned} Z_1[J_i] := & \int [d\mu] \exp \left[i \int d^2\sigma \left[J_i \Phi^i \right. \right. \\ & \left. \left. + \frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} (X')^2 + \Lambda TCf \right] \right] \end{aligned} \tag{48}$$

where the phase space variables of the theory are $\Phi^i \equiv (X^\mu, A_0, A_1, C, u, v, w)$ with the corresponding respective canonical conjugate momenta:

$$\Pi_i \equiv (P_\mu, \Pi^0, E, \Pi_c, p_u, p_v, p_w).$$

The functional measure $[d\mu]$ of the generating functional $Z_1[J_i]$ under the GFC ζ is obtained as:

$$\begin{aligned} [d\mu] = & \left[\Lambda T \delta^2(\sigma - \sigma') \right] \left[dX^\mu \right] \left[dA_0 \right] \left[dA_1 \right] \\ & \cdot [dC] [du] [dv] [dw] \left[dP_\mu \right] \left[d\Pi^0 \right] [dE] \\ & \cdot [d\Pi_c] [dp_u] [dp_v] [dp_w] \\ & \times \delta \left[(\Pi^0) \approx 0 \right] \delta \left[(\Pi_c) \approx 0 \right] \delta \left[(A_0) \approx 0 \right] \\ & \cdot \delta \left[(E - \Lambda TC) \approx 0 \right] \end{aligned} \tag{49}$$

The Hamiltonian and path integral quantization of the above theory under the GFC $\zeta = A_0 \approx 0$ is now complete.

4.2. Light-Front Quantization

In LFQ, the action of the theory reads:

$$S_6 = \int \mathcal{L}_6 d\sigma^+ d\sigma^- \tag{50a}$$

$$\begin{aligned} \mathcal{L}_6 = & \left[\frac{-\lambda T}{2} \right] \left[(\partial_+ X^+) (\partial_- X^-) \right. \\ & \left. + (\partial_+ X^-) (\partial_- X^+) + (\partial_+ X^i) (\partial_- X^i) + \Lambda TCf \right] \end{aligned} \tag{50b}$$

In the following we study the LF Hamiltonian and path integral formulations of the above action. The canonical

momenta $P^+, P^-, P_i, \Pi_c, \Pi^+$ and Π^- conjugate respectively to X^-, X^+, X_i, C, A^- and A^+ are obtained as:

$$\Pi^+ = \frac{\partial \mathcal{L}_6}{\partial(\partial_+ A^-)} = 0 \tag{51a}$$

$$\Pi^- = \frac{\partial \mathcal{L}_6}{\partial(\partial_+ A^+)} = \Lambda TC \tag{51b}$$

$$\Pi_c := \frac{\partial \mathcal{L}_6}{\partial(\partial_+ C)} = 0 \tag{51c}$$

$$P^+ := \frac{\partial \mathcal{L}_6}{\partial(\partial_+ X^-)} = -\frac{\lambda T}{2}(\partial_- X^+) \tag{51d}$$

$$P^- := \frac{\partial \mathcal{L}_6}{\partial(\partial_+ X^+)} = -\frac{\lambda T}{2}(\partial_- X^-) \tag{51e}$$

$$P_i := \frac{\partial \mathcal{L}_6}{\partial(\partial_+ X^i)} = -\frac{\lambda T}{2}(\partial_- X^i), \tag{51f}$$

$$i = 2, 3, \dots, 25$$

Above equations however, imply that the theory possesses 29 primary constraints:

$$\chi_1 = \Pi^+ \approx 0 \tag{52a}$$

$$\chi_2 = (\Pi^- - \Lambda TC) \approx 0 \tag{52b}$$

$$\chi_3 = \Pi_c \approx 0 \tag{52c}$$

$$\chi_4 = \left[P^+ + \frac{\lambda T}{2}(\partial_- X^+) \right] \approx 0 \tag{52d}$$

$$\chi_5 = \left[P^- + \frac{\lambda T}{2}(\partial_- X^-) \right] \approx 0 \tag{52e}$$

$$\chi_i = \left[P_i + \frac{\lambda T}{2}(\partial_- X^i) \right] \approx 0, \tag{52f}$$

$$i = 2, 3, \dots, 25$$

Canonical Hamiltonian density of this theory is:

$$\mathcal{H}_6^c = \left[P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P_i(\partial_+ X^i) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) + \Pi_c(\partial_+ C) - \mathcal{L}_6 \right] \tag{53a}$$

$$\mathcal{H}_6^c = \left[\Lambda TC(\partial_- A^-) \right] \tag{53b}$$

After including the above 29 primary constraints in the canonical Hamiltonian density \mathcal{H}_6^c with the help of Lagrange multiplier fields $v_1(\sigma^+, \sigma^-)$, $v_2(\sigma^+, \sigma^-)$, $v_3(\sigma^+, \sigma^-)$, $v_4(\sigma^+, \sigma^-)$, $v_5(\sigma^+, \sigma^-)$ and $v_i(\sigma^+, \sigma^-)$ (which we treat as dynamical), the total Hamiltonian density of the theory could be written as:

$$\mathcal{H}_6^T = \left[\mathcal{H}_6^c + s_1 \chi_1 + s_2 \chi_2 + s_3 \chi_3 + u \chi_4 + v \chi_5 + w_i \chi_i \right] \tag{54a}$$

$$\begin{aligned} \mathcal{H}_6^T = & \left[\Lambda TC(\partial_- A^-) + u \left[P^+ + \left(\frac{\lambda T}{2} \right) (\partial_- X^+) \right] \right. \\ & + v \left[P^- + \left(\frac{\lambda T}{2} \right) (\partial_- X^-) \right] + w_i \left[P_i + \left(\frac{\lambda T}{2} \right) (\partial_- X^i) \right] \\ & \left. + s_1 \Pi^+ + s_2 (\Pi^- - \Lambda TC) + s_3 \Pi_c \right] \end{aligned} \tag{54b}$$

The Hamiltons equations of motion of the theory that preserve the constraints of the theory in the course of time obtained from the total Hamiltonian:

$$H_6^T = \int \mathcal{H}_6^T d\sigma^- \tag{55}$$

e.g., for the closed strings with periodic BC's are obtained as:

$$\partial_+ X^- = \frac{\partial H_6^T}{\partial P^+} = u, -\partial_+ P^+ = \frac{\partial H_6^T}{\partial X^-} = \left[-\frac{\lambda T}{2} \right] (\partial_- v) \tag{56a}$$

$$\partial_+ X^+ = \frac{\partial H_6^T}{\partial P^-} = v, -\partial_+ P^- = \frac{\partial H_6^T}{\partial X^+} = \left[-\frac{\lambda T}{2} \right] (\partial_- u) \tag{56b}$$

$$\partial_+ X^i = \frac{\partial H_6^T}{\partial P_i} = w_i, -\partial_+ P_i = \frac{\partial H_6^T}{\partial X^i} = \left[-\frac{\lambda T}{2} \right] (\partial_- w_i) \tag{56c}$$

$$\partial_+ u = \frac{\partial H_6^T}{\partial p_u} = 0, -\partial_+ p_u = \frac{\partial H_6^T}{\partial u} = \left(P^+ + \frac{\lambda T}{2} \partial_- X^+ \right) \tag{56d}$$

$$\partial_+ v = \frac{\partial H_6^T}{\partial p_v} = 0, -\partial_+ p_v = \frac{\partial H_6^T}{\partial v} = \left(P^- + \frac{\lambda T}{2} \partial_- X^- \right) \tag{56e}$$

$$\partial_+ w_i = \frac{\partial H_6^T}{\partial p_{w_i}} = 0, -\partial_+ p_{w_i} = \frac{\partial H_6^T}{\partial w_i} = \left(P_i + \frac{\lambda T}{2} \partial_- X^i \right) \tag{56f}$$

$$\partial_+ C = \frac{\partial H_6^T}{\partial \Pi_c} = s_3, -\partial_+ \Pi_c = \frac{\partial H_6^T}{\partial C} = \Lambda T (\partial_- A^- - s_2) \tag{56g}$$

$$\partial_+ A^+ = \frac{\partial H_6^T}{\partial \Pi^-} = s_2, -\partial_+ \Pi^- = \frac{\partial H_6^T}{\partial A^+} = 0 \tag{56h}$$

$$\partial_+ A^- = \frac{\partial H_6^T}{\partial \Pi^+} = s_1, -\partial_+ \Pi^+ = \frac{\partial H_6^T}{\partial A^-} = 0 \tag{56i}$$

$$\partial_+ s_1 = \frac{\partial H_6^T}{\partial p_{s_1}} = 0, -\partial_+ p_{s_1} = \frac{\partial H_6^T}{\partial s_1} = \Pi^+ \tag{56j}$$

$$\partial_+ s_2 = \frac{\partial H_6^T}{\partial p_{s_2}} = 0, -\partial_+ p_{s_2} = \frac{\partial H_6^T}{\partial s_2} = (\Pi^- - \Lambda TC) \tag{56k}$$

$$\partial_+ s_3 = \frac{\partial H_6^T}{\partial p_{s_3}} = 0, -\partial_+ p_{s_3} = \frac{\partial H_6^T}{\partial s_3} = \Pi_c \tag{56l}$$

Demanding that the primary constraints of the theory be preserved in the course of time one does not get any secondary constraints. The theory is thus seen to possess

only 29 constraints: $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ and χ_i . Further the matrix of the Poisson brackets of these 29 constraints among themselves is easily seen to be singular, implying that the set of these 29 constraints is first-class. This in turn implies that the theory is GI (and consequently gauge anomalous). The theory is indeed seen to possess three local gauge symmetries given by the 2D WSRI and the WI defined by Equation (40). The theory could now be quantized under appropriate gauge-fixing.

The first-order Lagrangian density of the theory is:

$$\begin{aligned} \mathcal{L}_6^{I0} = & \left[\Pi_c (\partial_+ C) + P^+ (\partial_+ X^-) + P^- (\partial_+ X^+) \right. \\ & + P_i (\partial_+ X^i) + \Pi^+ (\partial_+ A^-) + \Pi^- (\partial_+ A^+) \\ & + p_{s_1} (\partial_+ s_1) + p_{s_2} (\partial_+ s_2) + p_{s_3} (\partial_+ s_3) \\ & \left. + p_u (\partial_+ u) + p_v (\partial_+ v) + p_{w_i} (\partial_+ w_i) - \mathcal{H}_6^T \right] \end{aligned} \quad (57a)$$

$$\begin{aligned} \mathcal{L}_6^{I0} = & \left[\Lambda TC (s_2 - \partial_- A^-) \right. \\ & \left. + \frac{\lambda T}{2} \left[u (\partial_- X^+) + v (\partial_- X^-) + w_i (\partial_- X^i) \right] \right] \end{aligned} \quad (57b)$$

To study the Hamiltonian and path integral formulations of the theory under gauge-fixing, we could e.g., choose the gauge:

$$\theta = A^- \approx 0 \quad (58)$$

corresponding to this gauge choice, the total set of constraints of the theory under which the quantization of the theory could be studied becomes:

$$\eta_1 = \chi_1 = \Pi^+ \approx 0 \quad (59a)$$

$$\eta_2 = \chi_2 = (\Pi^- - \Lambda TC) \approx 0 \quad (59b)$$

$$\eta_3 = \chi_3 = \Pi_c \approx 0 \quad (59c)$$

$$\eta_4 = \theta = A^- \approx 0 \quad (59d)$$

$$\eta_5 = \chi_4 = \left[P^+ + \frac{\lambda T}{2} (\partial_- X^+) \right] \approx 0 \quad (59e)$$

$$\eta_6 = \chi_5 = \left[P^- + \frac{\lambda T}{2} (\partial_- X^-) \right] \approx 0 \quad (59f)$$

$$\eta_i = \chi_i = \left[P_i + \frac{\lambda T}{2} (\partial_- X^i) \right] \approx 0, i = 2, 3, \dots, 25 \quad (59g)$$

We now calculate the matrix

$$R_{\alpha\beta}(\sigma, \sigma') := \{ \eta_\alpha(\sigma), \eta_\beta(\sigma') \}_{\text{PB}}$$

of the Poisson brackets of these above 30 constraints. The nonvanishing elements of the matrix $R_{\alpha\beta}$ are obtained as:

$$R_{56} = +R_{65} = R_{ii} = (\lambda T) \partial_- \delta(\sigma - \sigma'), i = 2, 3, \dots, 25 \quad (60a)$$

$$\Lambda TR_{14} = -\Lambda TR_{41} = R_{23} = -R_{32} = (-\Lambda T) \delta(\sigma - \sigma') \quad (60b)$$

The matrix $R_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of these 30 constraints is second-class. The determinant of the matrix $R_{\alpha\beta}$ is given by

$$\begin{aligned} & \left[\left\| \det(R_{\alpha\beta}) \right\| \right]^{1/2} \\ & = \left[(\Lambda T \delta^2(\sigma - \sigma')) (\lambda T \partial_- \delta(\sigma - \sigma'))^{13} \right] \end{aligned} \quad (61)$$

Nonvanishing elements of the inverse of this matrix $R_{\alpha\beta}$ (i.e. the elements of the matrix $(R^{-1})_{\alpha\beta}$) are:

$$\begin{aligned} (R^{-1})_{56} = (R^{-1})_{65} = (R^{-1})_{ii} = & \frac{1}{2\lambda T} \epsilon(\sigma - \sigma'), \\ i = & 2, 3, \dots, 25 \end{aligned} \quad (62a)$$

$$\begin{aligned} (R^{-1})_{14} = - (R^{-1})_{41} = \Lambda T (R^{-1})_{23} \\ = -\Lambda T (R^{-1})_{32} = \delta(\sigma - \sigma') \end{aligned} \quad (62b)$$

with

$$\int R(\sigma, \sigma'') R^{-1}(\sigma'', \sigma') d\sigma'' = \mathbf{1}_{30 \times 30} \delta(\sigma - \sigma') \quad (63)$$

Finally, following the Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing ELCWST commutation relations of this theory under the gauge $\theta = A^- \approx 0$ (with the arguments being suppressed again) are obtained as:

$$[X^+, P^-] = [X^-, P^+] = [X^i, P_i] = [i/2] \delta(\sigma - \sigma') \quad (64a)$$

$$[X^-, X^+] = [X^i, X^i] = [i/(2\lambda T)] \epsilon(\sigma - \sigma') \quad (64b)$$

$$[P^+, P^-] = [P_i, P_i] = [-iT\lambda/4] \partial_- \delta(\sigma - \sigma') \quad (64c)$$

$$[A^+, \Pi^-] = \Lambda T [A^+, C] = (i) \delta(\sigma - \sigma') \quad (64d)$$

In the path integral formulation, the transition to the quantum theory, is, however, made by writing the vacuum to vacuum transition amplitude called the generating functional $Z_2[J_i]$ of the theory in the presence of external sources J_i as follows:

$$\begin{aligned} & Z_2[J_i] \\ & := \int [d\mu] \exp \left[i \int d\sigma^+ d\sigma^- \left[J_i \Phi^i + \Lambda TC (s_2 - \partial_- A^-) \right. \right. \\ & \left. \left. + \left(\frac{\lambda T}{2} \right) \left[u (\partial_- X^+) + v (\partial_- X^-) + w_i (\partial_- X^i) \right] \right] \right] \end{aligned} \quad (65)$$

where the phase space variables of the theory are $\Phi^i \equiv (C, X^+, X^-, X^i, A^+, A^-, v_1, v_2, v_3, v_4, v_5, v_6)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_c, P^-, P^+, P_i, p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}, p_{v_5}, p_{v_6})$. The functional measure $[d\mu]$ of the generating functional

$Z_2[J_i]$ is obtained as:

$$\begin{aligned}
 [d\mu] = & (\Lambda T \delta^2(\sigma - \sigma')) (\lambda T \partial_- \delta(\sigma - \sigma'))^{13} \\
 & \cdot [dC] [dX^+] [dX^-] [dX^i] [dA^+] [dA^-] \\
 & \times [dv_1] [dv_2] [dv_3] [dv_4] [dv_5] [dv_6] [d\Pi_c] \\
 & \cdot [dP^-] [dP^+] [dP_i] [d\Pi^+] [d\Pi^-] \\
 & \times [dp_{v_1}] [dp_{v_2}] [dp_{v_3}] [dp_{v_4}] [dp_{v_5}] [dp_{v_6}] \\
 & \cdot \delta \left[\left(P^+ + \frac{\lambda T}{2} \partial_- X^+ \right) \approx 0 \right] \\
 & \times \delta \left[\left(P^- + \frac{\lambda T}{2} \partial_- X^- \right) \approx 0 \right] \\
 & \cdot \Pi_i \left(\delta \left[\left(P_i + \frac{\lambda T}{2} \partial_- X^i \right) \approx 0 \right] \right) \\
 & \times \delta \left[(\Pi^+) \approx 0 \right] \delta \left[(\Pi^- - \Lambda T C) \approx 0 \right] \\
 & \cdot \delta \left[(\Pi_c) \approx 0 \right] \delta \left[(A^-) \approx 0 \right]
 \end{aligned} \tag{66}$$

where Π_i denotes the product of similar expressions for all $i = 2, 3, \dots, 25$. The LF Hamiltonian and path integral quantization of the above theory is now complete.

5. Summary and Discussion

The Polyakov D1 brane action in a d-dimensional curved background $h_{\alpha\beta}$ defined by (Equation (1)) is GI and it possesses the well-known three local gauge symmetries given by the 2D WSRI and the WI defined by Equation (2).

However, when we study this action under the conformal gauge-fixing defined by Equation (3) to obtain the CGFPD1BA defined by the action S^N , we find that the CGFPD1BA given by S^N is no longer GI and it describes a gauge anomalous (and GNI) theory and it also does not possess the usual local gauge symmetries defined by Equation (2) being a gauge-fixed theory. However, this GNI theory when considered in the presence of a constant antisymmetric 2-form gauge field $B_{\alpha\beta}$ is seen to become a GI theory possessing the three local gauge symmetries defined by the 2D WSRI and the WI defined by Equation (24).

The 2-form gauge field $B_{\alpha\beta}$ in this case is seen to behave like a WZ field and the term involving this field is seen to behave like a WZ term for the CGFPD1BA which, in the absence of this term, is seen to possess a set of second-class constraints and consequently describe a GNI theory which does not respect the local gauge symmetries defined by the WSRI and WI given by Equation (2).

In our earlier work (cf. Section 3) [13], we have studied the IF and LF Hamiltonian and path integral formula-

tions of this GI theory describing the CGFPD1BA in the presence of the constant antisymmetric 2-form gauge field have been studied under appropriate gauge choices in the absence of BC's. The BC's could however be taken into account either by imposing them directly in the usual way for the open and closed strings separately in an appropriate manner or by considering them as the Dirac primary constraints [13-20,34] and study them accordingly.

In the present work [20], we have studied the IF and LF Hamiltonian and path integral formulations of the CGFPD1BA in the presence of a constant scalar axion field C and an $U(1)$ gauge field A_α (cf. Section 4). We find that the scalar axion field C and the $U(1)$ gauge field A_α are seen to behave like the WZ field and the term involving these fields is seen to behave like a WZ term for the CGFPD1BA, which in the absence of this term is seen to possess a set of second-class constraints and consequently describe a GNI theory which does not respect the local gauge symmetries defined by the WSRI and WI.

The situation in the present case, as pointed out in the foregoing, is analogous to the theory where one considers the CGFPD1BA in the presence of the constant 2-form gauge field $B_{\alpha\beta}$, where $B_{\alpha\beta}$ which is a scalar in the target space and an antisymmetric tensor in the WS space, behaves like a WZ field and the term involving this field behaves like a WZ term for the CGFPD1BA [13]. The later theory has been studied by the present authors in Ref. [13] to which we refer the reader for further details [3].

The IF and FF Hamiltonian and path integral formulations of the GI theory describing the CGFPD1BA in the presence of the constant scalar axion field C and the $U(1)$ gauge field A_α have been studied in this work under appropriate gauge choices in the absence of BC's [34]. The BC's, however, could be taken into account in the usual manner either by imposing them directly in the usual way for the open and closed strings separately [1-8] or by considering them as the Dirac primary constraints, and study them accordingly [13-20,34].

In conclusion, the Polyakov D1 brane action in a d-dimensional curved background $h_{\alpha\beta}$ defined by \tilde{S} is GI and it possesses the well-known three local string gauge symmetries. However, under conformal gauge-fixing, the CGFPD1BA is no longer GI as expected and it also does not possess the local string gauge symmetries being a gauge-fixed theory. However, this GNI theory when considered in the presence of a constant background scalar axion field C and an $U(1)$ gauge field A_α it is seen to become a GI theory possessing the three local string gauge symmetries.

The scalar axion field C and the $U(1)$ gauge field A_α are seen to behave like the WZ fields and the term

(\mathcal{L}^4) involving these fields is seen to behave like a WZ term for the CGFPD1BA, which in the absence of this term is seen to possess a set of second-class constraints and consequently describes a GNI theory which does not respect the local string gauge symmetries. The situation in the present case is analogous to a theory where one considers the CGFPD1BA in the presence of a constant 2-form gauge field $B_{\alpha\beta}$ which behaves like a WZ field and the term involving this field behaves like a WZ term for the CGFPD1BA.

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