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Valuation of Derivatives on the Cost Variables of the Shipping Market

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Abstract

In this paper, we present stochastic differential equations related to the cost variables of the shipping market. These SDEs arise under the addition of stochastic terms on the deterministic differential equations concerning the same variables. The financial interest arises from the fact that these SDEs may be used for the valuation of derivatives on these variables, such as futures, options, and others.

Keywords

Transport Costs Per Unit of Cargo Turnover, Deadweight, Optimum Speed

1. Variables Which May Be Taken as Primary in Shipping Markets

It is obvious that ship-owners, design the ships in a way that either minimizes the costs or maximizes the earnings arising from it. The relation between the design of a ship and the costs or the earnings is mentioned in Hughes—in [1], Veenstra and Ludemaz—in [2] and in Stopford—in [3]. In the first reference what is mentioned is that ship-owners consider these costs as constant and they are interested in the evolution of the earnings arising from a ship. In the second reference, the authors examine a relation between the technical characteristics of the ship and the earnings. Finally, in the third reference, cost and revenue variables related to a ship are separated, according to the impact of them. From Chen—in [4], we take the following relations:

$$a_t = \frac{S}{Q_m},$$

where a_t denotes the transport costs per unit of cargo turnover, S denotes the transport costs per year and Q_m denotes the cargo turnover per year.

$$Q_m = \frac{(a_1 + a_2) DW \cdot T \cdot L}{\dfrac{2}{24V_s} + \dfrac{2(a_1 + a_2) DW}{M_c} + t_{p_2} + t_3}, \tag{1}$$

where a_1, a_2 denote the utilization of ship's deadweight capacity for inbound and outbound voyages, respectively, Q_m denotes the cargo turnover per year, DW denotes the deadweight of the ship, T denotes the service days per year, L denotes indicates the voyage distance, and M_c denotes the average cargo handling rate at port. Also, t_{s1} denotes the time sailing at a service speed for a round voyage, t_{s2} denotes the time of canal transit for a round voyage, t_{p1} denotes the time for cargo loading and discharging for a round voyage, t_{p_2} denotes the turn time for a round voyage, and finally the time t_3 denotes the extra time at sea and in port for a round voyage. In Chen—in [4], differentiation with respect to V_s gives

$$\frac{\mathrm{d}Q_m}{Q_m} = \frac{t_{sl}}{t} \frac{\mathrm{d}V_s}{V_s},$$

which implies

$$\frac{\mathrm{d}a_t}{a_t} = \frac{\mathrm{d}S}{S} - \frac{\mathrm{d}Q_m}{Q_m} = \frac{\mathrm{d}S}{S} - \frac{t_{s1}}{t} \frac{\mathrm{d}V_s}{V_s}.$$

Also, in Chen (2011), differentiation of (1), with respect to DW implies

$$\frac{\mathrm{d}Q_m}{Q_m} = \left(1 - \frac{t_{p1}}{t}\right) \frac{\mathrm{d}DW}{DW},$$

which leads to

$$\frac{\mathrm{d}a_t}{a_t} = \frac{\mathrm{d}S}{S} - \left(1 - \frac{t_{p1}}{t}\right) \frac{\mathrm{d}DW}{DW}.$$

2. Financial Models and Derivatives

It is obvious that a_t, DW, Q_m, S, V_s are stochastic, namely they are subject to change during the **service time-period** $\tau \in [0, T]$, due to a variety of unknown factors. We suppose that each of these variables are subject to change, according to an Itô process across the service-time period:

$$\frac{\mathrm{d}V_{s,\tau}}{V_{s,\tau}} = a(\tau, V_{s,\tau}) \mathrm{d}\tau + \sigma(\tau, V_{s,\tau}) \mathrm{d}B_\tau^1,$$

$$\frac{\mathrm{d}S_\tau}{S_\tau} = b(\tau, S_\tau) \mathrm{d}\tau + f(\tau, S_\tau) \mathrm{d}B_\tau^2,$$

$$\frac{\mathrm{d}DW_\tau}{DW_\tau} = h(\tau, DW_\tau) \mathrm{d}\tau + g(\tau, DW_\tau) \mathrm{d}B_\tau^3.$$

By these stochastic differential equations, we make the above differential models more realistic, since we moreover assume that each of these variables has a value-but subject to a variability, because we may not forecast its exact value at a certain time-moment during the time-period $[0, T]$. Also, $\tau \in [0, T]$, and $B_\tau^1, B_\tau^2, B_\tau^3$ are usual 1-dimensional independent Brownian motions on $\mathbb{R}$,

modelling the essential factors of uncertainty. If the models for the processes $V_{s,\tau}, S_\tau, DW_\tau, \tau \in [0,T]$ are captured, by the differential form we have above we take:

Proposition 1. *The Stochastic Differential Equation for* $V_{s,\tau}$ *is*

$$\frac{dQ_\tau}{Q_\tau} = \frac{t_{sl}}{t} a(\tau, V_{s,\tau}) d\tau + \frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau}) dB_\tau^1, \tau \in [0,T].$$

Proposition 2. *The Stochastic Differential Equation for* $a_{t,\tau}$ *is*

$$\frac{da_{t,\tau}}{a_{t,\tau}} = \left(b(\tau, S_\tau) - \frac{t_{sl}}{t} a(\tau, V_{s,\tau}) \right) d\tau$$

$$+ \left(f(\tau, S_\tau), -\frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau}) \right) \cdot \left(dB_t^1, dB_t^2 \right)^{\mathrm{T}}, \tau \in [0,T],$$

where T denotes transpose matrix.

We also consider a *banking account*, namely an assetwhose evolution is

$$dB_\tau = r_\tau B_\tau d\tau, B_0 = 1, \tau \in [0,T].$$

Valuation

We consider a European call-option on the transport cost per cargo for a certain ship at the end of the service—time T at a strike—price K. The value of the underlying "asset"—which is the cost is now equal to $a_{t,T}$. The payoff of the call-option is equal to

$$\left(a_{t,T} - K \right)^+.$$

We also consider a call-option at the cargo $Q_{m,T}$ at the end of the service-time having a strike-price K_1. The payoff of the second option is

$$\left(Q_{m,T} - K_1 \right)^+.$$

The difference of the two markets is that the market of Q_m together with B is **complete**, while the market of the cost a_t together with B is **incomplete**. Hence, the second call-option may be replicated by the classical spot-market theory, see in Musiela and Rutkowski—[5], 5.1.2: The unique equivalent martingale measure for the **discounted cargo-market** as a spot market is

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left\{ \frac{r_\tau - \frac{t_{sl}}{t} a(\tau, V_{s,\tau})}{\frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau})} B_T^1 - \frac{1}{2} \left(\frac{r_\tau - \frac{t_{sl}}{t} a(\tau, V_{s,\tau})}{\frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau})} \right)^2 T \right\}, \mathbb{P} - a.s.$$

Under $\mathbb{Q}$, the price of $e^{-r_\tau} Q_{m,\tau} = Q_{m,\tau}^*, \tau \in [0,T]$ is

$$dQ_{m,\tau}^* = \frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau}) Q_{m,\tau}^* dB_\tau^*,$$

where

$$B_\tau^* = B_\tau^1 - \frac{r_\tau - \frac{t_{sl}}{t} a(\tau, V_{s,\tau})}{\frac{t_{s1}}{t} \sigma(\tau, V_{s,\tau})} \tau, \tau \in [0,T],$$

is a Brownian motion. Hence,

$$\pi_\tau\left(\left(Q_{m,T}-K_1\right)^+\right)=B_\tau\mathbb{E}_\mathbb{Q}\left(B_T^{-1}\left(Q_{m,T}-K_1\right)^+\mid\mathcal{F}_\tau\right),\tau\in[0,T],$$

denotes the unique (martingale price) of the second call-option, where the $\mathcal{F}_\tau,\tau\in[0,T]$ denotes the filtration of the last Brownian motion B^*. B_T^{-1} is the discount factor with respect to the time-period T.

On the other hand, for the valuation of the call-option

$$\left(a_{t,T}-K\right)^+,$$

since the market of

$$\frac{\mathrm{d}a_{t,\tau}}{a_{t,\tau}}=\left(b\left(\tau,S_\tau\right)-\frac{t_{sl}}{t}a\left(\tau,V_{s,\tau}\right)\right)\mathrm{d}\tau$$

$$+\left(f\left(\tau,S_\tau\right),-\frac{t_{s1}}{t}\sigma\left(\tau,V_{s,\tau}\right)\right)\cdot\left(\mathrm{d}B_t^1,\mathrm{d}B_t^2\right)^\mathrm{T},\tau\in[0,T],$$

and

$$\mathrm{d}B_\tau=r_\tau B_\tau\mathrm{d}\tau,B_0=1,\tau\in[0,T]$$

is incomplete, in order to find the non-arbitrage prices for the $\left(a_{t,T}-K\right)^+$ we formulate the process equation

$$\left(f\left(\tau,S_\tau\right),-\frac{t_{s1}}{t}\sigma\left(\tau,V_{s,\tau}\right)\right)\cdot\left(u_\tau^1,u_\tau^2\right)^\mathrm{T}$$

$$=\left(b\left(\tau,S_\tau\right)-\frac{t_{sl}}{t}a\left(\tau,V_{s,\tau}\right)\right),$$

which implies the existence of the scalar processes $u_\tau^1,u_\tau^2,\tau\in[0,T]$, such that

$$f\left(\tau,S_\tau\right)u_\tau^1-\frac{t_{s1}}{t}\sigma\left(\tau,V_{s,\tau}\right)u_\tau^2=b\left(\tau,S_\tau\right)-\frac{t_{sl}}{t}a\left(\tau,V_{s,\tau}\right),\tau\in[0,T],\qquad(2)$$

and consider a solution $u_\tau=\left(u_\tau^1,u_\tau^2\right),\tau\in[0,T]$, of the above stochastic equation (2) that satisfies the condition

$$\mathbb{E}\left(\exp\left\{\frac{1}{2}\int_0^T\|u_\tau\|_2^2\,\mathrm{d}\tau\right\}\right)<\infty.$$

We also consider

$$\frac{\mathrm{d}\mathbb{Q}_u}{\mathrm{d}\mathbb{P}}=\exp\left\{-\int_0^T u_\tau\mathrm{d}B_\tau-\frac{1}{2}\int_0^T\|u_\tau\|_2^2\,\mathrm{d}\tau\right\},$$

and $\mathrm{d}B_\tau=\left(\mathrm{d}B_\tau^1,\mathrm{d}B_\tau^2\right)^\mathrm{T},\tau\in[0,T]$. Also, the process

$$B_\tau^*=\int_0^\tau u_s\mathrm{d}s+B_\tau,$$

is a $\mathbb{Q}_u$-Brownian, where $\int_0^\tau u_s\mathrm{d}s$ denotes component-wise integration. Hence,

$$\frac{\mathrm{d}a_{t,\tau}}{a_{t,\tau}}=\left(f\left(\tau,S_\tau\right),-\frac{t_{s1}}{t}\sigma\left(\tau,V_{s,\tau}\right)\right)\mathrm{d}B_\tau^*,\tau\in[0,T],$$

is a $\mathbb{Q}_u$-martingale (we assume that the filtration is considered to be the same) —is Øksendal in [6], Chapter 12.

Uder the above considerations, the following Theorem is obvious:

Theorem 3. *The non-arbitrage prices of the call-option are—at time period* 0:

$$\left(\inf_{\mathbb{Q}_u} \mathbb{E}_{\mathbb{Q}_u} B_T^{-1}\left(\left(a_{t,T}-K\right)^+\right), \sup_{\mathbb{Q}_u} \mathbb{E}_{\mathbb{Q}_u} B_T^{-1}\left(\left(a_{t,T}-K\right)^+\right)\right),$$

where B_T^{-1} is the discount factor with respect to the time-period T.

3. Conclusion

The conclusion of this brief paper is that we may discriminate the primary cost variables in the shipping market and the dependence of other variables of this market, by the formulation of Stochastic Differential Equations. These SDE imply the valuation of European options written on the primary variables.

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