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Creating a (Quantum?) Constraint, in Pre Planckian Space-Time Early Universe via the Einstein Cosmological Constant in a One to One and Onto Comparison between Two Action Integrals

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Abstract

We are looking at comparison of two action integrals and we identify the Lagrangian multiplier as setting up a constraint equation (on cosmological expansion). Two action integrals, one which is connected with quantum gravity is called equivalent to another action integral, and the 2^{nd} action integral has a Lagrangian multiplier in it. Using the idea of a Lagrangian multiplier as a constraint equation, we draw our conclusions in a 1 to 1 and onto assumed equivalence between the two action integrals. The viability of the 1 to 1 and onto linkage between the two action integrals is open to question, but if this procedure is legitimate, the conclusions so assumed are fundamentally important.

Keywords

Ricci Scalar, Inflaton Physics

1. Basic Idea, Can Two First Integrals Give Equivalent Information?

Our supposition is that if we wish to make an equivalence between two action integrals, *i.e.*, first integrals that we need to have a 1 to 1 and onto linkage between the integrands, in the two cases so referenced.

To do this, we are making several assumptions.

1) The two mentioned integrals are evaluated from a Pre Planckian to Planckian space-time domain, *i.e.* in the same specified integral of space-time.

2) In the process of doing so, the Universe is assumed to avoid the so called cosmic singularity. In doing so, assuming a finite "Pre Planckian to Planckian" regime of space time is similar to that given in [1] [2].

3) The integrands in the two integrals are assumed to have a 1-1 and onto relationship to one another. We will be identifying the components of the two integrands which are assumed to be proportional to each other. This idea is the foundation of our approach. The two references [1] [2] have in their own formulation specific Lagrangian formulations and a criticism our approach is that the references we are using for first integrals, namely [3] [4] are not giving action integrals identical as to [1] [2]. Our answer is that we reference [1] [2] specifically as to how to avoid the Penrose singularity theorem [5], and that not enough is known as to rule out the nonsingular starting point of the universe as having the same content for Lagrangians as given in [3] [4]. *i.e.*, for Pre Planckian space time, so long as [5] is avoided, presumably our three assumptions for comparison can be made, so long as we adhere to the "path integral" idea as represented by [6] as equivalent to what is stated in [1] [2].

2. Specifying the Particulars of the Two First Integrals in Pre-Planckian to Planckian Space-Time

Before proceeding, it is advisable to define some of the symbols which will be used in the integrals and the integrands in our document.

First of all, we have what is known as a scale factor $a(t)$. Which is nearly zero, in the Pre Planckian regime of space-time if we assume [5] does not hold, and that $a(t)$ is 1 in the present era. A good reference as to the physics behind how we set up $a(t)$ is [7]. In addition we will define, for the purpose of analysis, of the integrals, the following symbols as given in [1], for the Quantum paths sensitive first integral, with

$$\begin{aligned} &\int \mathrm{d}t \sqrt{g_{tt}} V_3(t) = V_4(t) \sim 8\pi^2 r^4 / 3 \\ &\& \ V_3(t) = 2\pi^2 a(t)^3 / 3 \\ &\& \ k_2 = 9\left(2\pi^2\right)^{2/3} \end{aligned} \tag{1}$$

These are the purported volume elements of the [3] first integral. The second first integral is using the usual *GR* inputs as defined by Padmanbhan in [4]. To review what is meant by first integrals we refer the readers to [6] [8] [9] [10].

Roughly put, according to [8] [9] [10] a Lagrangian multiplier invokes a constraint of how a "minimal surface" is obtained by constraining a physical process so as to use the idea of [8] [9] [10] which invokes the idea of minimization of a physical processes. In the case of [3], the minimization process is implicitly that, if $a(t)$ were a scale factor as defined by Roos, [7] and if g_{tt} were a time component of a metric tensor, which we will later define via [11] [12].

Here, the subscripts 3 and 4 in the volume refer to 3 and 4 dimensional spatial dimensions, and this will lead to us writing, via [3] a 1st integral as defined by [3] [3], in the form, if G is the gravitational constant, that if we have following [3], a first integral defined by

$$S_1 = \frac{1}{24\pi G} \cdot \left(\int \mathrm{d}t \sqrt{g_{tt}} \left(\frac{g_{tt} \dot{V}_3^2}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right) \right). \tag{2}$$

This should be compared against the Padmabhan 1st integral [4] of the form, with the third entry of Equation (3) having a Ricci scalar defined via [13] and usually the curvature $\aleph$ set as extremely small, with the general relativity version of

$$\begin{aligned} & S_2 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot \mathrm{d}^4 x \cdot (\Re - 2\Lambda) \\ & \& \ -g = -\det g_{uv} \\ & \& \ \Re = 6 \times \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right). \end{aligned} \tag{3}$$

Also, the variation of $\delta g_{tt} \approx a_{\min}^2 \phi$ as given by [11] [12] will have an inflaton, ϕ given by [4]

$$\begin{aligned} & a \approx a_{\min} t^{\gamma} \\ & \Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \\ & \Leftrightarrow V \approx V_0 \cdot \exp \left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}. \end{aligned} \tag{4}$$

Leading to [2]

$$\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\}. \tag{5}$$

Here, we have that $a_{\min}$ is a minimum value of the scale factor presumably given by [2] as a tiny but non zero value. Or at least a quantum bounce as given by [1].

The innovation we will be looking at will be in comparing a 1-1 and onto equivalence, *i.e.* an information based isomorphism between 1st integrals with a nod to [14]

$$S_1 \cong S_2. \tag{6}$$

We will be making a simple equivalence between the two first integrals via Equation (6) assuming that even in the Pre Planck-Planck regime that curvature $\aleph$ will be a very small part of Ricci scalar $\Re$ and that to first approximation even in the Plank time regime, that to first order [13] has a value altered to be

$$\Re = 6 \times \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right) \sim 6 \times \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right). \tag{7}$$

This last approximation will make a statement as to applying Equation (6) far easier may not be defensible, but we will use it for the time being.

2.1. Comparison between Equations ((2) and (3) with (5)-(7))

In order to obtain maximum results, we will be stating that the following will be assumed to be equivalent.

$$\sqrt{g_{tt}}\left(\frac{g_{tt}\dot{V}_3^2}{V_3(t)}+k_2V_3^{1/3}(t)-\lambda V_3(t)\right)\sim\frac{1}{2\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x\cdot\left(6\times\left(\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2\right)-2\Lambda\right) \quad (8)$$

i.e.

$$\sqrt{g_{tt}}\left(\frac{g_{tt}\dot{V}_3^2}{V_3(t)}+k_2V_3^{1/3}(t)\right)\sim\frac{1}{2\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x\cdot\left(6\times\left(\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2\right)\right) \quad (9)$$

And

$$\sqrt{g_{tt}}\left(\lambda V_3(t)\right)\sim\frac{1}{2\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x\cdot(2\Lambda). \quad (10)$$

If the term Λ is indeed a constant (*i.e.* we avoid Quinessence, and the vacuum energy is invariant), then Equation (10) puts a profound restriction upon g_{tt} which will be elaborated upon in the next section. *i.e.* for the sake of Argument we will make the following assumptions which may be debatable, *i.e.*

$$\sqrt{-g} \text{ is approximately a constant.} \quad (11)$$

For extremely small time intervals (in the boundary between Pre Planckian to Planckian physics boundary regime).

$$g_{tt}\sim\delta g_{tt}\approx a_{\min}^2\phi \quad (12)$$

The next section will be investigating the physical implications of such assumptions.

2.2. What We Can Extract in Physics, If Equations (9)-(12) Hold?

Simply put a relationship of the Lagrangian multiplier giving us the following:

$$\lambda\sim\frac{1}{\kappa}\sqrt{\frac{-g}{\left(\delta g_{tt}\approx a_{\min}^2\phi\right)}}\cdot\Lambda. \quad (13)$$

If the following is true, *i.e.* in a Pre Plankian to Planckian regime of space-time

$$\sqrt{\frac{-g}{\left(\delta g_{tt}\approx a_{\min}^2\phi\right)}}\approx\text{constant}. \quad (14)$$

Then what has been done is to conflate the Lagrangian as equivalent to Λ which if Λ is also a constant is implying that the cosmological constant is obtaining for us the consomological constant value chosen as a precursor for (DE?) expansion of the universe, as given in the scale factors as of Equation (9) and Equation (8). *i.e.* what we are inferring then is similar to a result assumed by Padmanabhan, in [15].

3. Conclusions

But what is noticeable is that the inflaton equation as given by Padmanabhan [4] hopefully will not be incommensurate with the physics of the Corda Criteria given in the Gravity's breath document [16]. Keep in mind the importance of the result from reference [17] below which forms the core of Equation (15) below

$$N_{e-\text{foldings}} = -\frac{8\pi}{m_{\text{Planck}}^2} \cdot \int_{\phi 1}^{\phi 2} \frac{V(\phi)}{\left(\frac{\partial V(\phi)}{\partial \phi}\right)} \mathrm{d}\phi \geq 65. \tag{15}$$

Furthermore, we should keep in mind the physics incorporated in [18] [19], *i.e.* as to the work of LIGO. *i.e.* it is important to keep in mind that in addition, [20] has confirmed that a subsequent analysis of the event GW150914 by the LSC constrained the graviton Compton wavelength of those alternative theories of gravity in which the graviton is massive and placed a level of 90% confidence on the lower bound of 10^{13} km for a Compton wavelength of the graviton. Doing this sort of vetting protocols in line with being consistent with investigation as to a real investigation as to the fundamental nature of gravity. This is a way of confirming and showing via experimental data sets if general relativity is the final theory of gravitation. *i.e.*, if massive gravity is confirmed, as given in [21], then GR is perhaps to be replaced by a scalar-tensor theory, as has been shown by Corda.

We can say though if we do confirm Equation (13) and Equation (14) that such observations may enable a more precise rendering of settling the issues brought up by references [16], and [21], as well as the appropriate use of the structures, algebraically given in [22] [23] for our comparison of the first integrals.

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