Journal of Applied Mathematics and Physics, 2016, 4, 2126-2134
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

The Advection Wave-in-Secondary Saturation Movement Equation and Its Application to Concentration Tension-Driven Saturation Kinetic Flow

Tafireyi Nemaura[1,2]

[1]Department of Clinical Pharmacology, University of Zimbabwe, Harare, Zimbabwe
[2]Department of Applied Mathematics, National University of Science and Technology, Bulawayo, Zimbabwe
Email: tnemaura@gmail.com, tafireyi.nemaura@nust.ac.zw

How to cite this paper: Nemaura, T. (2016) The Advection Wave-in-Secondary Saturation Movement Equation and Its Application to Concentration Tension-Driven Saturation Kinetic Flow. *Journal of Applied Mathematics and Physics*, **4**, 2126-2134.
http://dx.doi.org/10.4236/jamp.2016.412210

Received: November 14, 2016
Accepted: December 3, 2016
Published: December 6, 2016

Abstract

The deterministic description of a wave of solution particle of efavirenz is given. Simulated pharmacokinetic data points from patients on efavirenz are used. The one dimensional wave equation is used to infer on transfer of vibrations due to tension between solution particles. The work investigates movement using wave analogy, but in a different variable space. Two important movement fluxes of a wave are derived an attracting one identified as tension conductivity and a dispersing one identified as tension diffusivity. The Wave Equation can be used to describe another spin-off movement flux formed induced by vibrations in solution particle.

Keywords

Partial Differential Equations, Wave, Movement Flux

1. Introduction

There is a rich history of quantum mechanics that investigates wave-particle duality using light [1] [2]. We derive a one dimensional wave equation in the context of particle movement [3]. This equation has extensively been studied beginning with work proposed by D'Alembert [4]. Furthermore, this work supports the hypothesis advanced by Louis de Broglie that all matter manifests a wave like nature. An idea further developed by David Bohm, who postulated that every material particle is accompanied by a field which guides the motion of the particle. This field evolves according to the Schrödinger's equation [2]. In this work we retain the conventional wave equation. It models the advective entity of a wave and proposes an extension to other components of movement flux which are passive, convective and saturation.

This work derives characterisation of a wave and shows it as a spin-off movement flux that is derived from tension in the vibrations of the solution particle. It is described as a spin-off of a solution particle or the pilot wave [2]. Furthermore, this spin-off flux is deterministic. However, other researchers consider a probabilistic view of the quantum state [2] [5]. The secondary saturation movement in the concentration-time space is used to investigate the behaviour of the wave [6]. The mathematical formulation brings to light an understanding of the wave part of a solution particle. Waves in location and time space have been studied by other researchers [1] [7]. The wave is studied in a different space (concentration-time) and analogies are proposed. A relationship of a wave is derived, with the aid of simulated pharmacokinetic data *in-vivo*.

In addition, this work shows that a wave is a system with four primary movement components of flux. The total flux at any given concentration is zero. The system formed is similar to that obtained for the one that facilitates exchange of concentration through gradient [8]. The system evolves with time. There are two forms of flux the conductivity and diffusivity.

2. Methods

Simulated projected data on secondary saturation movement, time and concentration was obtained from pharmacokinetic projections made on patients on 600 mg dose of efavirenz considered in Nemaura (2015, 2016). Partial and Ordinary Differential equations are used in the development of models that characterizes wave motion. A statistical Package R, is used to develop nonlinear regression models.

Derivation of Advection Kinetic Flow for the Secondary Saturation Movement Due to Tension

Consider $F(x,t)(/h)$ the secondary saturation movement at time t and concentration x-(tension-driven transportation inducing measure) in the blood. The variable $F(x,t)$, is also a measurement of tension density of movement. In addition, the variable $F(x,t)$ is taken to denote the (vertical) displacement at time t of the concentration x on the advection string component. The rate at which saturation movement crosses a solution particle is proportional to concentration, time and secondary saturation movement of solution particle [8]. The following proposition is made that the amount of tension advective conductivity flux of solution particle $\left(\tilde{\kappa}_T\left(\frac{\mu g \cdot h}{ml}\right)\right)$ is the difference between κ_{ρ_T} (that amount of tension advective conductivity flux which is generated by movement density $\rho_T(x,t)$ (obtaining in the process of solution formation)) and κ_{0_T} (an already existing constant (base) amount of tension advective conductivity flux (primarily obtaining in the independent of solute of x, solvent state)). The function of proportionality is called the amount of tension advective conductivity flux and is denoted by $\tilde{\kappa}_T = \kappa_{\rho_T} - \kappa_{0_T}$. The functions κ_{ρ_T} and κ_{0_T} are such that $\kappa_{\rho_T} \propto \rho_T(x,t)$

and $\kappa_{0_T} \propto \rho_T(y_0, t)$ where y_0 is the homogenous concentration mix in the base solvent without solute x, thus $\kappa_{\rho_T} = \tilde{k}\rho_T(x,t)$ $\kappa_{0_T} = \tilde{k}\rho_T(y_0,t)$, and where $\tilde{k}, \tilde{\kappa}$ are constants of direct proportionality associated with the tension,

$$\underline{\kappa}^A(x,t) = \frac{\tilde{k}\rho_T(x,t)}{xt} - \frac{\tilde{\kappa}\rho_T(y_0,t)}{y_0 t}. \tag{1}$$

In a homogenous mix, $\frac{\rho_T(y_0,t)}{y_0 t} = \omega_T$. Thus the tension advective conductivity flux [tension advective conductivity flux in the solvent of a solute of x, in the absence of the solute x (before mixing)] is constant and Equation (1) reduces to,

$$\underline{\kappa}^A(x,t) = \frac{\tilde{k}\rho_T(x,t)}{xt} - \tilde{\kappa}\omega_T. \tag{2}$$

Consider a small element of the string (bridge) between the two concentration points x and $x+\delta x$, ($\delta x > 0$, small). The total tension advective flux to which this string is subjected to is the tension advective flux exerted at the left end $\left(\underline{\kappa}^A(x,t)\right)$ and the advective flux exerted at the right end $\left(\underline{\kappa}^A(x+\delta x,t)\right)$ by the rest of the string.

Let $\theta(x,t)$ be the angle between $\underline{\kappa}^A(x,t)$ and $\boldsymbol{i}$ (horizontal direction of components) and $\theta(x+\delta x,t)$ be the angle between $\left(\underline{\kappa}^A(x+\delta x,t)\right)$ and $\boldsymbol{i}$ Assume $0 \le \theta \le \pi$, since we are looking at small vibrations. It follows that either θ is close to 0 or π.

The total vertical tension advective flux acting on the element is

$$\begin{aligned} \underline{T} &= \underline{\kappa}^A(x,t)\cdot \boldsymbol{j} + \underline{\kappa}^A(x+\delta x,t)\cdot \boldsymbol{j} \\ &= \underline{\kappa}^A(x,t)\left(\sin\left(\theta(x,t)\right) + \sin\left(\theta(x+\delta x,t)\right)\right). \end{aligned} \tag{3}$$

where $\boldsymbol{j}$ is the vertical direction of component. We note $\sin\theta \approx \theta \approx \tan\theta$, for θ close to zero and $\sin\theta \approx \pi - \theta \approx -\tan\theta$ for θ close to π. For a fixed t,

$$\tan\theta(x,t) = \frac{\partial F}{\partial x}(x,t), \tag{4}$$

and

$$\tan\theta(x+\delta x,t) = \frac{\partial F}{\partial x}(x+\delta x,t). \tag{5}$$

From Equations (1)-(5) and the approximation formulas for $\sin\theta$, we obtain the following approximation for the vertical flux on the element of the advective string,

$$\underline{T} = \underline{\kappa}^A(x,t)\left(\frac{\partial F}{\partial x}(x+\delta x,t) - \frac{\partial F}{\partial x}(x,t)\right). \tag{6}$$

The concentration-time amount of form movement due to tension in the solution particle bridge is $m = \frac{\rho_T(x,t)}{xt}\delta x$. Drawing analogy from Newton's Second Law to replace $\underline{T}$ in (6)

$$ma = \underline{\kappa}^A(x,t)\left(\frac{\partial F}{\partial x}(x+\delta x,t) - \frac{\partial F}{\partial x}(x,t)\right), \tag{7}$$

where,

$$a = \frac{\partial^2 F}{\partial t^2}.$$

Thus,

$$\frac{\partial^2 F}{\partial t^2} = \frac{xt}{\rho_T(x,t)} \underline{\kappa}^A(x,t) \frac{\left(\frac{\partial F}{\partial x}(x+\delta x,t) - \frac{\partial F}{\partial x}(x,t)\right)}{\delta x}. \quad (8)$$

At the limit $(\delta x \to 0)$, the Equation (8) reduces to,

$$\frac{\partial^2 F}{\partial t^2} = \frac{xt}{\rho_T(x,t)} \underline{\kappa}^A(x,t) \frac{\partial^2 F}{\partial x^2}. \quad (9)$$

The following result is immediate,

$$\frac{\partial^2 F}{\partial t^2} = \hat{\kappa}_T^A(x,t) \frac{\partial^2 F}{\partial x^2}(x,t), \quad (10)$$

where,

$$\hat{\kappa}_T^A(x,t) = \frac{xt}{\rho_T(x,t)} \underline{\kappa}^A(x,t),$$

is the *secondary saturation tension advective conductivity flux* $(/h)$ associated with a unit amount of movement in a solution-particle bridge. This reduces to,

$$\frac{\partial^2 F}{\partial t^2} = \left(N - \rho_T^{\tilde{\kappa}}(x,t)\right) \frac{\partial^2 F}{\partial x^2}, \quad (11)$$

where,

$$\rho_T^{\tilde{\kappa}}(x,t) = \frac{\tilde{\kappa}\omega_T xt}{\rho_T(x,t)} = D_T(x,t)xt, \text{ and } N = \tilde{k}.$$

The terms,

$$\rho_T^{\tilde{\kappa}}(x,t), D_T(x,t) = \frac{\tilde{\kappa}\omega_T}{\rho_T(x,t)}, \text{ and } N,$$

are *secondary saturation tension base-advection diffusivity flux* $(/h)$, *secondary saturation tension advection diffusivity* $\left(\frac{ml}{\mu g \cdot h^2}\right)$, and *secondary saturation initial or rest tension advection conductivity flux* $(/h)$ respectively.

The following relation from Equations (10) and (11) is established between tension advection conductivity and diffusivity fluxes,

$$\overbrace{\hat{\kappa}_T^A(x,t) - N}^{\text{auxilliary tension advection conductivity flux}} = -\underbrace{\rho_T^{\tilde{\kappa}}(x,t)}_{\text{tension advection diffusivity flux}} \quad (12)$$

where $N = \hat{\kappa}_T^A(0,0)$.

It is important to note that $\hat{\kappa}_T^A(x,t)$ is the tension advection conductivity flux.

3. Results

Applications to Deteministic Saturation Tension Advection Kinetic Flow of Concentration of Efavirenz and Numerical Analysis

We consider the secondary saturation $F(x,t)$ relationship to time and concentration in a patient *P*, modelled by [8],

$$F(x,t)=\begin{cases}\lambda_r\left(\mathrm{e}^{-\lambda_b t}-\mathrm{e}^{-\lambda_a t}\right), \; t\in[0,24], & (i)\\ \dfrac{ux}{v+x}, \quad x\in[0,c_{\max}], & (ii)\end{cases} \tag{13}$$

where, x is the concentration and t is time. These two variables track solution particle dynamics. Furthermore, $\lambda_r,\lambda_a,\lambda_b,u$ and v are constants to be found (Table 1 and Table 2).

Following from Equation (13), Equation (10) assume the form of,

$$\frac{\mathrm{d}^2F}{\mathrm{d}t^2}(x,t)=\hat{\kappa}_T^A(x,t)\frac{\mathrm{d}^2F}{\mathrm{d}x^2}(x,t), \tag{14}$$

and Equation (11) becomes,

$$\frac{\mathrm{d}^2F}{\mathrm{d}t^2}(x,t)=\left(N-\rho_T^{\tilde{\kappa}}(x,t)\right)\frac{\mathrm{d}^2F}{\mathrm{d}x^2}(x,t). \tag{15}$$

The following conditions holds for secondary saturation tension advective diffusivity $D_T(x,t)$ (Figure 1) at boundary points of loss/formation of the spin-off of a solution particle (wave),

$$\lim_{(x,t)\to(0.0)} D_T(x,t)=-\infty,$$

$$\lim_{(x,t)\to(0.0)} \rho_T^{\tilde{\kappa}}(x,t)=0$$

$$\lim_{(x,t)\to(0.t)} D_T(x,t)=\infty, \quad \text{for some } t>0,$$

Table 1. Parameter estimates in modelling saturation movement rate with respect to *t* (Model 13(*i*)).

Parameters	Estimate	Std Error	t value	$\Pr(>\lvert t\rvert)$
λ_r	0.623106	0.012214	51.02	$<2\times10^{-16}$
λ_b	0.022465	0.001203	18.68	5.35×10^{-15}
λ_a	0.439258	0.017354	25.31	$<2\times10^{-16}$

Table 2. Parameter estimates in modelling saturation movement rate with respect to *x* (Model 13(*ii*)).

Parameters	Estimate	Std Error	t value	$\Pr(>\lvert t\rvert)$
u	0.801936	0.005934	135.1	$<2\times10^{-16}$
v	5.624198	0.126684	44.4	$<2\times10^{-16}$

$$\lim_{(x,t)\to(0,t)} \rho_T^{\tilde{\kappa}}(x,t) = N, \quad \text{for some } t > 0,$$

It is noted that for $\hat{\kappa}_T^A(x,t)$ the following holds at boundary points of spin-off advective conductivity flux: $\hat{\kappa}_T^A(0,0) := N$, value at rest of spin-off saturation conductivity flux.

The reference spin-off advective flux is investigated and the constituent behaviour is suggested by Equation (16) (**Table 3**),

$$\underbrace{\rho_T^{\tilde{\kappa}}(t)}_{\text{spin-off base-advection diffusivity flux}} = \overbrace{\tilde{w}te^{-\tilde{n}t}}^{\text{spin-off convection diffusivity flux}} + \underbrace{\tilde{b}\left(e^{-\tilde{d}t}-1\right)}_{\text{spin-off passive diffusivity flux}} + \overbrace{\frac{\tilde{f}t}{\tilde{g}+t}}^{\text{spin-off saturation diffusivity flux}}. \tag{16}$$

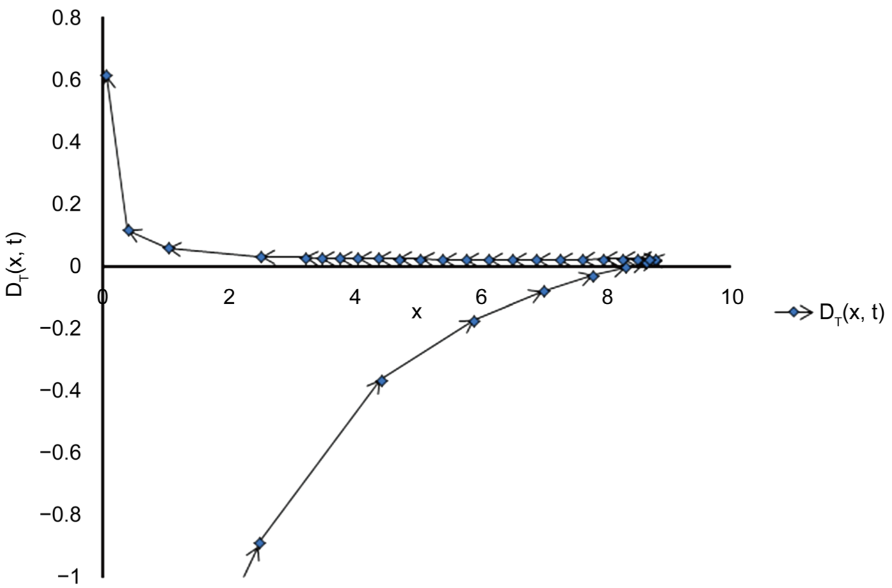

Figure 1. Time matched plot of the spin-off advection diffusivity of saturation movement in relation to concentration.

Table 3. Parameter estimates in modelling saturation movement spin-off diffusivity fluxes in Equation (16).

Parameters	Estimate	Std Error	t value	$\Pr(>\lvert t\rvert)$
$\tilde{w}$	−12.040340	0.400263	−30.081	$< 2\times10^{-16}$
$\tilde{n}$	0.428697	0.005665	75.673	$< 2\times10^{-16}$
$\tilde{b}$	19.868405	2.919178	6.806	1.69×10^{-6}
$\tilde{d}$	0.180557	0.003487	51.783	$< 2\times10^{-16}$
$\tilde{f}$	23.527008	3.210289	7.329	6.01×10^{-7}
$\tilde{g}$	1.632012	0.154663	10.552	2.20×10^{-9}

$\tilde{w}$: residence rate of the spin-off convective diffusivity flux,

$\tilde{n}$: declining rate of the spin-off convective diffusivity flux,

$\tilde{b}$: maximum spin-off passive diffusivity flux rate constant,

$\tilde{d}$: declining rate of the spin-off passive diffusivity flux,

$\tilde{f}$: maximum spin-off saturation diffusivity flux rate constant and,

$\tilde{g}$: time at which the spin-off saturation diffusivity flux was half of $\tilde{f}$.

The characterisation of the four main spin-off diffusivity flux entities are shown and magnitudes of effects (Figure 2).

$$\overbrace{N-\hat{\kappa}_T^A\left(x,t\right)}^{\text{auxilliary spin-off advection conductivity flux}} = \overbrace{-\tilde{w}te^{-\tilde{n}t}}^{\text{spin-off convection conductivity flux}} + \underbrace{-\tilde{b}(e^{-\tilde{d}t}-1)}_{\text{spin-off passive conductivity flux}} + \overbrace{\frac{-\tilde{f}t}{\tilde{g}+t}}^{\text{spin-off saturation conductivity flux}}, \quad (17)$$

where, $\tilde{w},\tilde{n},\tilde{b},\tilde{d},\tilde{f},\tilde{g}$ are constants with respect to spin-off conductivity flux as in Equation (16) and $N = 2.364914$. It is noted that auxilliary spin-off conductivity flux is translated negative spin-off conductivity flux by a constant (N). We give a summary of tension associated diffusivity, diffusivity flux, and conductivity flux in relation to concentration-time (Figure 3).

4. Conclusions

A wave in this work has been shown to be a spin-off movement flux of a solution particle. It also has the same component characterisation as a solution particle at primary level. It consists of the advective, passive, saturation and convective components [6] [8]. A wave has generally slower movement flux components relative to gradient driven

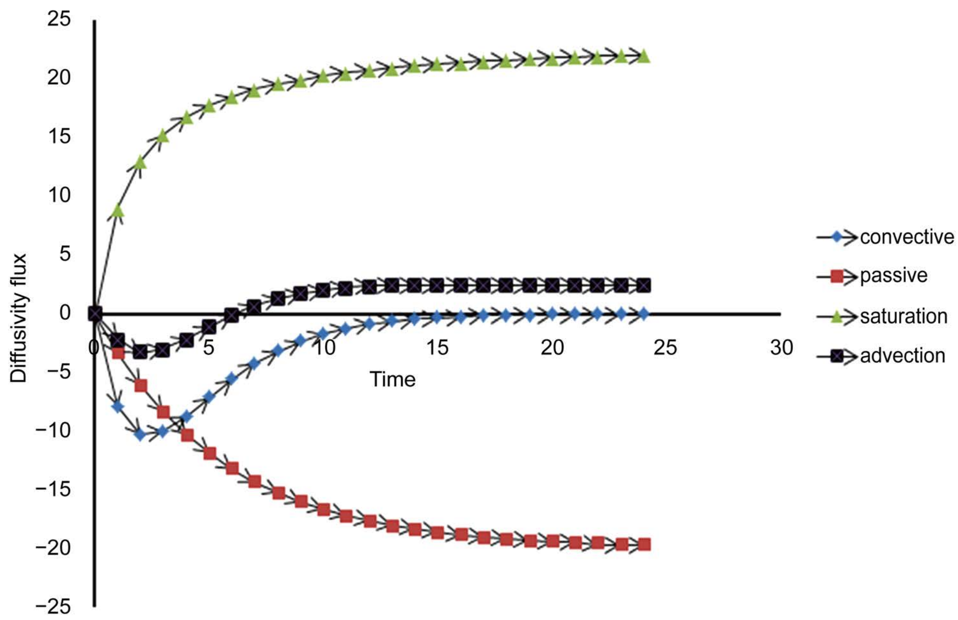

Figure 2. Components of the spin-off diffusivity flux in time.

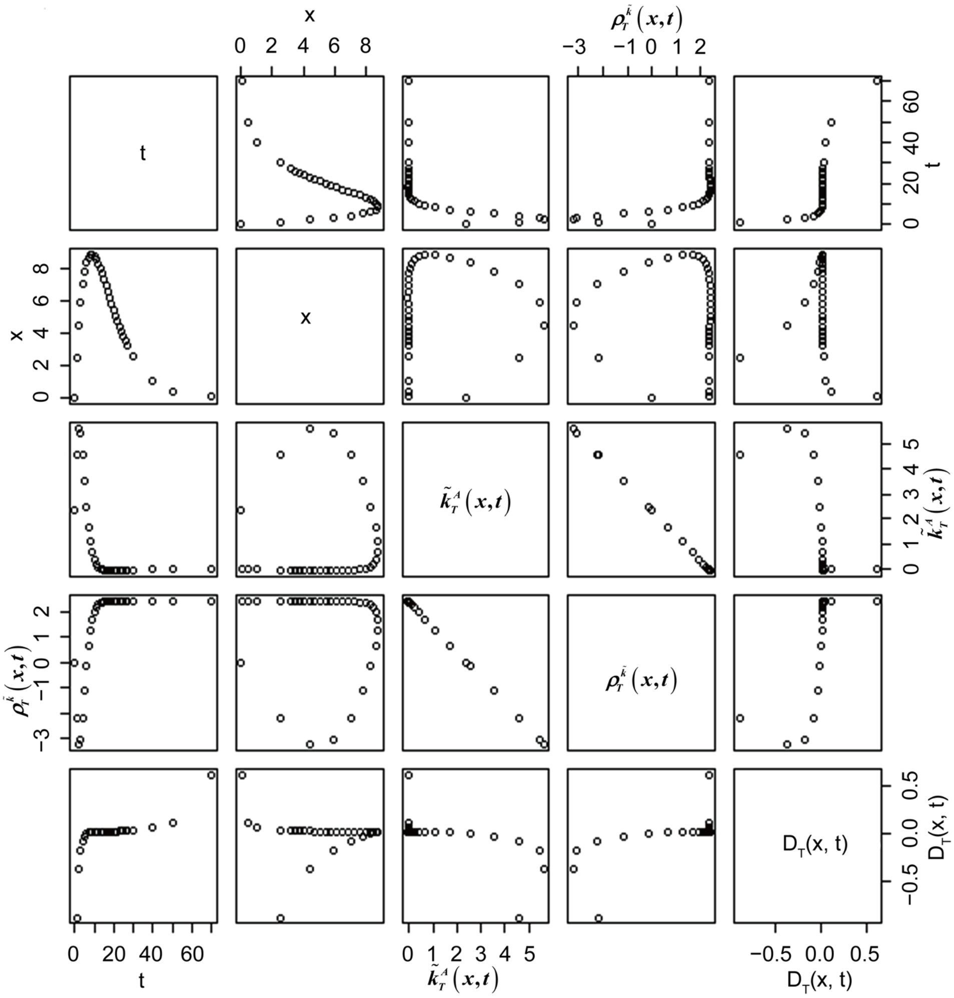

Figure 3. Summary scatter time-matched plot of the relationship between the following variables $t, x, \hat{\kappa}_T^A(x,t), \rho_T^{\tilde{k}}(x,t)$ and $D_T(x,t)$ for patient P informed by Equations (13), (14) and (15).

movement flux components [8]. There have been several experiments that support De Broglie's assertions [2]. This work removes the notion of tracking the wave using location with the aid of a probability distribution derived from Schrödinger [5]. It proposes a different space related to kinetic solubility of a solution particle to track it's wave [6].

Considering a different unique space, we obtain similar characterisation of flux of a wave to that of gradient-driven diffusion. These two are system flux movements in time. A conclusion is reached that these two forms are characterised similar pattern of movement [8]. The heat (diffusion) and wave equations are two fundamental equations [3] [9]. They describe two forms of advective movements. The heat and wave equations are derived from gradient and tension forms of movement respectively.

Acknowledgements

The author would like to thank the following; C. Nhachi, C. Masimirembwa, and G. Kadzirange, AIBST and The College of Health Sciences, University of Zimbabwe.

References

[1] Tsaparlis, G. (2001) Towards a Meaningful Introduction to the Schrödinger Equation through Historical and Heuristic Approaches. *Chemistry Education*, *Research and Practice*, **2**, 203-213. https://doi.org/10.1039/B1RP90023D

[2] Jagielski, B. (2009) Elements of the Wave-Particle Duality of Light. Master Thesis, University of Oslo, Oslo.

[3] Meziani, A. (2016) The Wave Equation. http://www2.fiu.edu/meziani/NOTE2.pdf

[4] Cannon, J.T. and Dostrovsky, S. (1981) The Evolution of Dynamics: Vibration Theory from 1687 to 1742. Springer-Verlag, New York. https://doi.org/10.1007/978-1-4613-9461-7

[5] Halliday, D., Resnik, R. and Walker, J. (2001) Fundamentals of Physics. 6th Edition, John Wiley and Sons, New York.

[6] Nemaura, T. (2015) Modeling Transportation of Efavirenz: Inference on Possibility of Mixed Modes of Transportation and Kinetic Solubility. *Frontiers in Pharmacology*, **6**, 121. https://doi.org/10.3389/fphar.2015.00121

[7] Sengupta, D.L. and Sarkar, T.K. (2003). Maxwell, Hertz, the Maxwellians, and the Early History of Electromagnetic Waves. *IEEE Antennas and Propagation Magazine*, **45**, 13-19. https://doi.org/10.1109/MAP.2003.1203114

[8] Nemaura, T. (2016) The Advection Diffusion-in-Secondary Saturation Movement Equation and Its Application to Concentration Gradient-Driven Saturation Kinetic Flow. *Journal of Applied Mathematics and Physics*, **4**, 1998-2010.

[9] Narasimhan, T.N. (1999) Fourier's Heat Conduction Equation: History, Influence, and Connections. *Reviews of Geophysics*, **37**, 151-172. https://doi.org/10.1029/1998RG900006