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N-Summet-*k* and Its Application in the Construction of Pascal Triangle and Pascal Matrix

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Abstract

Summetor is an operator used in the mathematics to calculate the special numbers like binomial coefficients and combinations of group elements. It has many applications in algebra, matrices like calculation of pascal triangle elements and pascal matrix formation, etc. This paper explains about its functions and properties of *N*-summet-*k*. The result of variation between N and k is shown in tabulation.

Keywords

Summetor, *N*-Summet-*k*, Binomial Coefficients, Pascal's Triangle, Pascal's Matrix

1. Introduction

Summetor was firstly introduced in research article, "Jeevan-Kushalaiah Method to Find the Coefficients of Characteristic Equation of a Matrix and Introduction of Summetor" by the authors Neelam Jeevan Kumar and Neelam Kushalaiah [1]. The Summetor operator name is taken from "sum" operator. **Summetor** operation is "Sum of all positive integers form one to n". ***N*-summet-*k*:** Sum of all positive integers summeted k-times progressively.

N-summet-*k* is (**Figure 1**).

$$N_k{!} = \frac{\prod_{i=0}^{k}(N+i)}{(k+1)!} = N_{(k-1)}{!} + (N-1)_k{!} = \frac{(N+k)!}{(N-1)!(k+1)!} \quad \text{(i)}$$

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$$N_k!$$

Say "**N summet k**"

Figure 1. Representation of *N*-summet-*k*. **Symbol and explanation:** *N* is the real number or complex number. It must symbolize in Uppercase english letter only. *k* is the real number. It must symbolize in lowercase english letter only. ! is Summetor operator symbol. Tale end "+" represents "summation".

Example:

$$\begin{aligned}\text{5-summet-3} &= 5_3! = 5_2! + 4_2! + 3_2! + 2_2! + 1_2! \\ &= 5_1! + 4_1! + 3_1! + 2_1! + 1_1! + 4_1! + 3_1! + 2_1! \\ &\quad + 1_1! + 3_1! + 2_1! + 1! + 2_1! + 1_1! + 1_1! \\ &= 15 + 10 + 6 + 3 + 1 + 10 + 6 + 3 + 1 + 6 + 3 + 1 + 3 + 1 + 1 = 70\end{aligned}$$

Properties:

$$\boldsymbol{N_k}! = \frac{\prod_{i=0}^{k}(\boldsymbol{N}+\boldsymbol{i})}{(\boldsymbol{k}+\boldsymbol{1})!},\ \textbf{for } -\infty < N < \infty \textbf{ and } \boldsymbol{k} > -1 \tag{i.a}$$

$$\boldsymbol{N_k}! = \boldsymbol{N}_{(\boldsymbol{k}-\boldsymbol{1})}! + (\boldsymbol{N}-\boldsymbol{1})_{\boldsymbol{k}}!,\ \textbf{for } -\infty < N < \infty \textbf{ and } -\infty < k < \infty \tag{i.b}$$

$$\boldsymbol{N_k}! = \frac{(\boldsymbol{N}+\boldsymbol{k})!}{(\boldsymbol{N}-\boldsymbol{1})!(\boldsymbol{k}+\boldsymbol{1})!},\ \textbf{for } \boldsymbol{1} < N < \infty, -1 < k < \infty \textbf{ and } \boldsymbol{N}+\boldsymbol{k} > 0 \tag{i.c}$$

$$\boldsymbol{N_k}! = \sum_{i=0}^{n-1}(\boldsymbol{i}+\boldsymbol{1})_{(k-2)}! * (\boldsymbol{N}-\boldsymbol{i}),\ \textbf{for } -\infty < N < \infty \textbf{ and } -\infty < k < \infty \tag{i.d}$$

1) $N_k! = \begin{cases} 0, & k < -1 \\ 1, & k = -1 \\ N, & k = 0 \end{cases}$

2) N is complex number. $N = a + ib$

$N_k! = (a+ib)_k! = a_k! + ib_k!$

3) $e^n! = \prod_{i=1}^{n} e^i$

4) $P_q! = R_s!$ When $P, q, R, s > 0$, $R = q + 2, P = s + 2$ and $P + q = R + s$

5) For all –ve values of N, $-N_k! = 0$, when $|-N| \le |k|$

6) $-N_k! = -1^{k+1} * (N-k)_k! = (-1)^{k+1} * \frac{|-N|!}{(|-N|-k-1)! * (k+1)!}$ when $|-N| > |k|$

7) $\sum_{k=0}^{n} k = n!$

8) $\sum_{k=0}^{n} k^2 = n_2! + (n-1)_2!$

9) $P^2 = P! + (P-1)! = P_2! - 4_{(P-4)}!$, P is a real number, $-\infty < P < \infty$

$= P_3! - (P-1)_3! - (P-2)_3! + (P-3)_3!$

10) $P^3 = P * [P! + (P-1)!] =$

11) $P^4 = (1 + 4^{P-1})P! + (P-1)!$

2. Tabulation and Graph

2.1. Tabulation

In the given **Table 1**, *N* is taken on vertically and k is taken on horizontally. The result of *N*-summet *k* is given with variable *N* from −9 to 15 and variable *k* from −3 to 10. *N* is positive value. The tabulation is the heart of *N*-summet-*k*.

In **Table 2**, elbow arrow between 6.4 and 10 proves $N_{(k-1)}! + (N-1)_k! = N_k!$

Table 1. *N*-summet-*k* values, where $N = -9$ to 0 and $k = -3$ to 9.

−3	−2	−1	N↓/k↔	+1	+2	+3	+4	+5	+6	+7	+8	+9
0	0	1	**−9**	36	−84	126	−126	84	−36	9	−1	0
0	0	1	**−8**	28	−56	70	−56	28	−8	1	0	0
0	0	1	**−7**	21	−35	35	−21	7	−1	0	0	0
0	0	1	**−6**	15	−20	15	−6	1	0	0	0	0
0	0	1	**−5**	10	−10	5	−1	0	0	0	0	0
0	0	1	**−4**	6	−4	1	0	0	0	0	0	0
0	0	1	**−3**	3	−1	0	0	0	0	0	0	0
0	0	1	**−2**	1	0	0	0	0	0	0	0	0
0	0	1	**−1**	0	0	0	0	0	0	0	0	0
0	0	1	**0**	0	0	0	0	0	0	0	0	0

N (vertical bold numbers) and *k* (horizontal bold numbers).

Table 2. *N*-summet-*k* values, where $N = 1$ to 15 and $k = -3$ to 9.

0	0	1	**1**	1	1	1	1	1	1	1	1	1
0	0	1	**2**	3	4	5	6	7	8	9	10	11
0	0	1	**3**	6	10	15	21	28	36	45	55	66
0	0	1	**4**	10	20	35	56	84	120	165	220	286
0	0	1	**5**	15	35	70	126	210	330	495	715	1001
0	0	1	**6**	21	56	126	252	462	792	1287	2002	3003
0	0	1	**7**	28	84	210	462	924	1716	3003	5005	8008
0	0	1	**8**	36	120	330	792	1716	3432	6435	11,440	19,448
0	0	1	**9**	45	165	495	1287	3003	6435	12,870	24,310	43,758
0	0	1	**10**	55	220	715	2002	5005	11,440	24,310	48,620	92,378
0	0	1	**11**	66	286	1001	3003	8008	19,448	43,758	92,378	184,756
0	0	1	**12**	78	364	1365	4368	12,376	31,824	75,582	167,960	352,716
0	0	1	**13**	91	455	1820	6188	18,564	50,388	125,970	293,930	646,646
0	0	1	**14**	105	560	2380	8568	27,132	77,520	203,490	497,420	1,144,066
0	0	1	**15**	120	680	3060	11,628	38,760	116,280	319,770	817,190	1,961,256

But most commonly used formula to calculate *N*-summet-*k* is

$$\frac{(N+k)!}{(N-1)!(k+1)!} \text{ when } N > 0 \text{ and } K > -1$$

$$N_k! = \frac{\prod_{i=0}^{k}(N+i)}{(k+1)!}, \text{for } -\infty < N < \infty \text{ and } k > -1$$

and

$$N_{(k-1)}! + (N-1)_k! = N_k!, \text{when } -\infty < N < +\infty \text{ and } -\infty < K < +\infty$$

The dotted inclined lines shows Pascal triangle

2.2. Graph

Note: Taking $\frac{1}{(-R)!} \times x = 0 \quad = 0$, for $-1 > k$, $|-N| < k$

where $R > 0$ and x is any integer or number or equation (Figures 2-4).

- **Proof-1:** $N < 0$, N is any non fractional real number. Assume $N = -1$.
  From Equation (i.c) we get
  $$\frac{(N+k)!}{(N-1)!(k+1)!} = \frac{(-1+k)!}{(-1-1)!(k+1)!} = \frac{(-1+k)!}{(-2)!(k+1)!} = \frac{1}{-2!} * \frac{(-1+k)!}{(k+1)!}$$
  Observe the tabulation; the N-summet-k value is 0.
- **Proof-2:** $k < -1$, let $k = -2$.
  From Equations (i.a) and (i.c) gives $\frac{1}{(-1)!}$
  Observe the tabulation, the N-summet-k value is 0.

3. Applications

3.1. Pascal's Triangle (Figure 5)

Pascal's triangle [2] is a triangular array of the binomial coefficients [3] [4]. Binomial coefficients are indexed

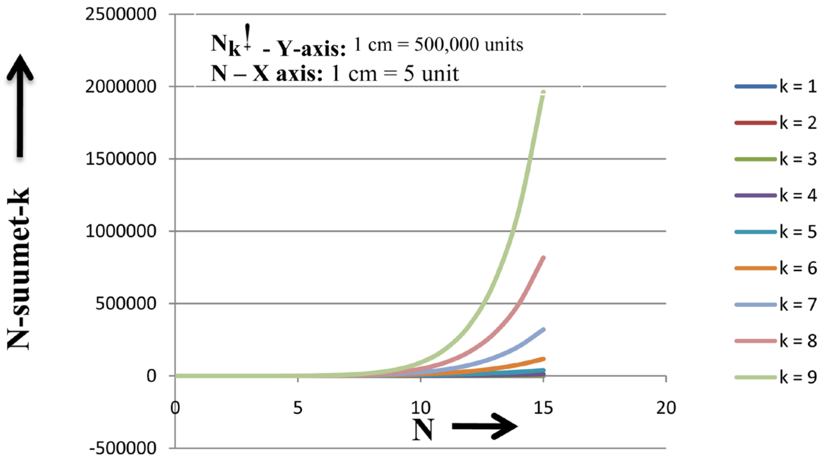

Figure 2. The range of N: $-1 < N < 16$ on X-axis and $N_k!$ on Y-axis with variable k, $0 < k < 10$.

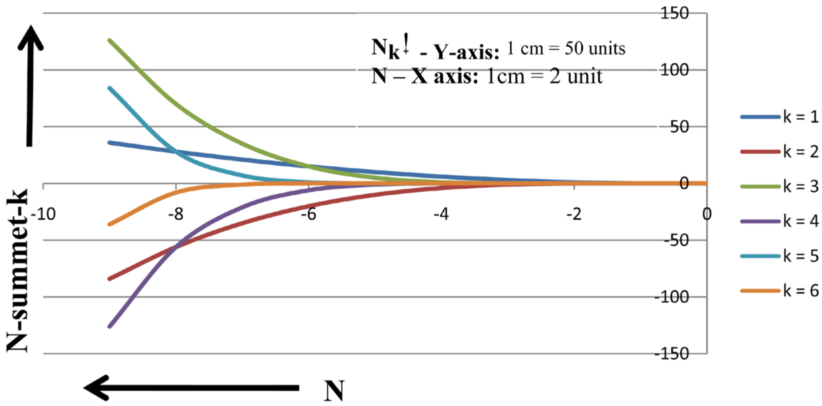

Figure 3. The range of N: $-10 < N < 1$ on X-axis and $N_k!$ on Y-axis with variable k, $0 < k < 7$.

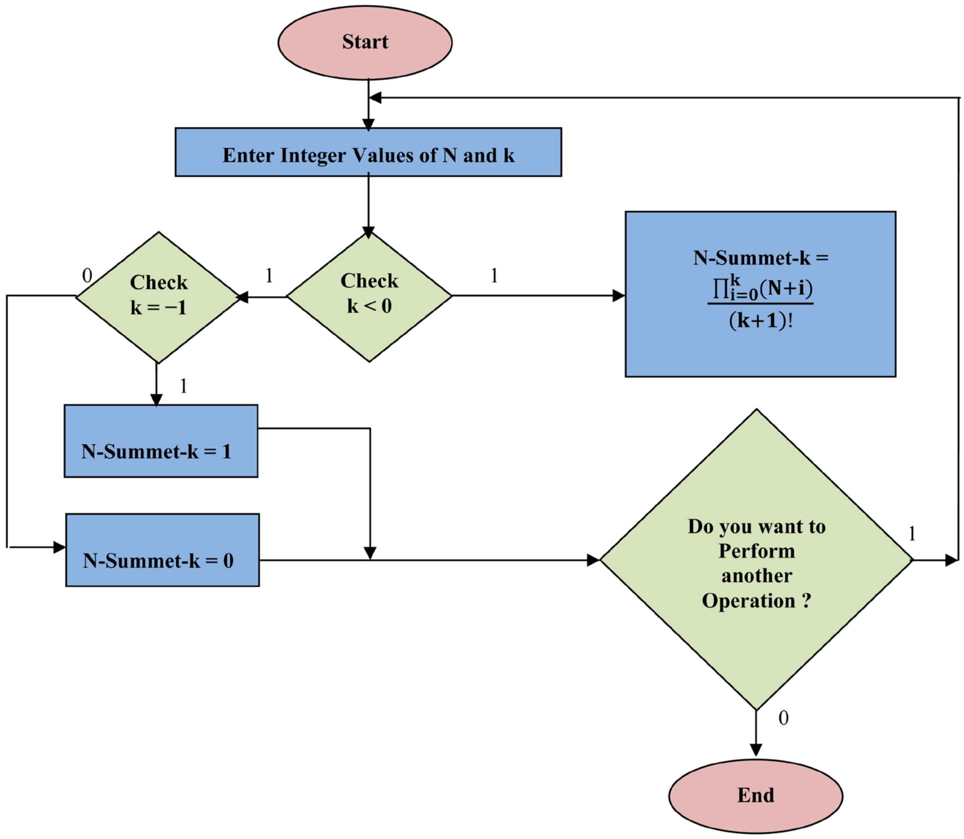

Figure 4. Flow chart for *N*-summet-*k*.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Figure 5. Pascal's triangle. Last row coefficients are 1, 5, 10, 10, 5, 1 with $n = 5$. By using Summetor, the values of binomial coefficients with $n = 5$ are $=6_{-1}!$, $5_0!$, $4_1!$, $3_2!$, $2_3!$, $1_4!$. From the properties and tabulation, we can observe that the values of corresponding *N*-summet-*k* are = 1, 5, 10, 10, 5, 1.

by two non negative integers n and k written as $\binom{n}{k}$. It is the Coefficient of X^k term in polynomial expression of $(1+x)^n$. Where *n* rises from 0 to *n*.

The coefficients are given by the expression $\frac{n!}{k!(n-k)!}$. *k* varies from 0 to *n*.

By using Summetor or *N*-summet-*k* $\frac{n!}{k!(n-k)!}$ can be written as

$$\frac{n!}{k!(n-k)!} = (n-r)r!, \quad r = k-1 \tag{ii}$$

where *k* varies from 0 to *n* in R.H.S and *r* varies from −1 to *n*−1 in L.H.S

A set, *S* has *n*-elements. The number of *k* combinations can be calculated by using Equation (ii)

Equation (ii) also gives combinations *i.e.*, ^nC_k, *k* varies from 0 to n formulated with *N*-summet-*k*.

Advantage of *N*-Summet-*k*

The computational time taken to calculate $\binom{n}{k}$ when $n \gg k$, $k > 0$ is much higher than that of $(n-k)_k!$

The computation taken to calculate $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ is t_1, $1 < k < n$ and

The computation taken to calculate $\binom{n}{k} = (n-k)_k! = \frac{\prod_{i=0}^{k}(N+i)}{(k+1)!}$ is t_2, $0 < k < n - 1$

3.2. Pascal's Matrix

The Pascal matrix [5]-[8] is an $n \times n$ dimension infinite matrix containing the binomial coefficients as its elements. The Pascal matrix generation is the matrix exponential of a special subdiagonal or superdiagonal matrix. The Three Pascal Matrices are Upper Triangular Matrix (U_n), Lower Triangular Matrix (L_n) and Symmetric Matrix (S_n). Symmetric Matrix is product of Lower Triangular Matrix and Upper Triangular Matrix. The m is the values of Subdiagonal or superdiagonal elements and lies between $-\infty$ and $+\infty$.

9×9 Pascal Matrices (U_n, L_n and S_n) represented rows as $i = n = 9$ and column as $j = n = 9$.

For positive values of diagonal elements, the Pascal matrices are

3.2.1. Upper Triangular Matrix, (U_n)

Upper triangular matrix is formatted with exponential matrix containing superdiagonal elements.

$$m_{-3} = -3,\ m_{-2} = -2,\ m_{-1} = -1,\ m_1 = 2,\ m_2 = 2,\ m_3 = 3,\ m_4 = 4$$

$$m_{\min} = -3 \text{ and } m_{\max} = 4.$$

Elements of upper triangular matrix are

$$U_{n(i,j)} = (m_{min} + i - 1)_{(-i-1+j)}! \tag{iiia}$$

$$U_9 = exp\left(\begin{bmatrix} 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -3_{-1}! & -3_0! & -3_1! & -3_2! & -3_3! & -3_4! & -3_5! & -3_6! & -3_7! \\ -2_{-2}! & -2_{-1}! & -2_0! & -2_1! & -2_2! & -2_3! & -2_4! & -2_5! & -2_6! \\ -1_{-3}! & -1_{-2}! & -1_{-1}! & -1_0! & -1_1! & -1_2! & -1_3! & -1_4! & -1_5! \\ 0_{-4}! & 0_{-3}! & 0_{-2}! & 0_{-1}! & 0_0! & 0_1! & 0_2! & 0_3! & 0_4! \\ 1_{-5}! & 1_{-4}! & 1_{-3}! & 1_{-2}! & 1_{-1}! & 1_0! & 1_1! & 1_2! & 1_3! \\ 2_{-6}! & 2_{-5}! & 2_{-4}! & 2_{-3}! & 2_{-2}! & 2_{-1}! & 2_0! & 2_1! & 2_2! \\ 3_{-7}! & 3_{-6}! & 3_{-5}! & 3_{-4}! & 3_{-3}! & 3_{-2}! & 3_{-1}! & 3_0! & 3_1! \\ 4_{-8}! & 4_{-7}! & 4_{-6}! & 4_{-5}! & 4_{-4}! & 4_{-3}! & 4_{-2}! & 4_{-1}! & 4_0! \\ 5_{-9}! & 5_{-8}! & 5_{-7}! & 5_{-6}! & 5_{-5}! & 5_{-4}! & 5_{-3}! & 5_{-2}! & 5_{-1}! \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Right Turn Elbow edged arrow, shows

$$U_{n(i,j)} = i_{(j-i-2)}! + (i-1)_{(j-i-1)}! \tag{iiib}$$

3.2.2. Lower Triangular Matrix, (L_n)

Lower triangular matrix is formatted with exponential matrix containing subdiagonal elements.

$$m_{-3} = -3,\ m_{-2} = -2,\ m_{-1} = -1,\ m_1 = 2,\ m_2 = 2,\ m_3 = 3,\ m_4 = 4$$

$$m_{\min} = -3 \text{ and } m_{\max} = 4.$$

Elements of lower triangular matrix are

$$L_{n(i,j)} = (m_{min} + j - 1)_{(-j-1+i)}! \tag{iva}$$

$$L_9 = exp\left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -3_{-1}! & -2_{-2}! & -1_{-3}! & 0_{-4}! & 1_{-5}! & 2_{-6}! & 3_{-7}! & 4_{-8}! & 5_{-9}! \\ -3_{0}! & -2_{-1}! & -1_{-2}! & 0_{-3}! & 1_{-4}! & 2_{-5}! & 3_{-6}! & 4_{-7}! & 5_{-8}! \\ -3_{1}! & -2_{0}! & -1_{-1}! & 0_{-2}! & 1_{-3}! & 2_{-4}! & 3_{-5}! & 4_{-6}! & 5_{-7}! \\ -3_{2}! & -2_{1}! & -1_{0}! & 0_{-1}! & 1_{-2}! & 2_{-3}! & 3_{-4}! & 4_{-5}! & 5_{-6}! \\ -3_{3}! & -2_{2}! & -1_{1}! & 0_{0}! & 1_{-1}! & 2_{-2}! & 3_{-3}! & 4_{-4}! & 5_{-5}! \\ -3_{4}! & -2_{3}! & -1_{2}! & 0_{1}! & 1_{0}! & 2_{-1}! & 3_{-2}! & 4_{-3}! & 5_{-4}! \\ -3_{5}! & -2_{4}! & -1_{3}! & 0_{2}! & 1_{1}! & 2_{0}! & 3_{-1}! & 4_{-2}! & 5_{-3}! \\ -3_{6}! & -2_{5}! & -1_{4}! & 0_{3}! & 1_{2}! & 2_{1}! & 3_{0}! & 4_{-1}! & 5_{-2}! \\ -3_{7}! & -2_{6}! & -1_{5}! & 0_{4}! & 1_{3}! & 2_{2}! & 3_{1}! & 4_{0}! & 5_{-1}! \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

Down turn Elbow Edged Arrow, shows

$$L_{n(i,j)} = j_{(i-j-2)}! + (j-1)_{(i-j-1)}! \tag{ivb}$$

Lower triangular matrix is transpose of upper triangular matrix vice versa.

$$L_n = \text{Trans}(U_n)$$

3.2.3. Symmetric Matrix, (S_n)

$$S_n = L_n * U_n \tag{va}$$

Elements of S_n for positive values of subdiagonal or superdiagonal elements.

$$S_{n(i,j)} = j_{(i-2)}! = i_{(j-2)}! \tag{vb}$$

$$S_9 = \exp\left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}\right) \times \exp\left(\begin{bmatrix} 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ -3 & 10 & -11 & 4 & 0 & 0 & 0 & 0 & 0 \\ 3 & -11 & 14 & -6 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -6 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 & 3 & 6 & 10 & 15 \\ 0 & 0 & 0 & 0 & 1 & 4 & 10 & 20 & 35 \\ 0 & 0 & 0 & 0 & 1 & 5 & 15 & 35 & 70 \end{bmatrix}$$

For positive values subdiagonal or superdiagonal elements

$$S_{n_{(i,j)}} = i_{(j-i-2)}! + (i-1)_{(j-i-1)}! = j_{(i-j-2)}! + (j-1)_{(i-j-1)}! \tag{vc}$$

The (Square box) in tabulation shows 12 × 12 Pascal Symmetric Matrix for positive values and;
The in tabulation shows mirror image elements positions of 5 × 5 Pascal upper triangular matrix.

Acknowledgements

N-summet-*k* has numerous applications in mathematics and physics like simplification of laguerre polynomials [9] [10] applied in quantum mechanics, in the radial part of the solution of the Schrödinger equation for a one-electron atom and Calculation of electrical voltage distribution across high voltage suspension type string insulator [11] and grading of string insulators [12] to improve string efficiency of high voltage overhead transmission line and so on. Author found the application of *N*-summet-*k* in above three applications but the subject regarding these applications not published yet. In future, the above three applications will be published by using *N*-summet-*k*.

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