

# Numerical Simulation of Freak Wave Generation in Irregular Wave Train

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## Abstract

In this paper, a numerical wave tank is developed based on High Order Spectral (HOS) method considering the wave-maker boundary. The 2D irregular wave trains are simulated for a long time by using this model. The freak wave is observed in the wave train, and its generation process is analyzed via wavelet analysis. The results show that the numerical tank can accurately simulate the wave generation and propagation, even for the freak wave. From the analysis of freak wave generation, it can be found that, two wave groups with different frequency components superpose together to form a large wave group. The large wave group modulation generates the freak wave.

## Keywords

Freak Wave, High Order Spectral Method, Numerical Wave Tank

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## 1. Introduction

The freak wave (also called extreme wave, rouge wave) was defined as the wave whose height is more than twice the significant wave height, which represents the extreme oceanic conditions. It has larger wave height and stronger nonlinear, and can cause serious damage to the oceanic structures and ships. In recent years, as frequent human activities move towards the deep ocean, the harsh marine environment becomes a threat to oceanic engineering. Accurate estimation and understanding of severe extreme events are essential for human safety and for cost-effective design.

In the past years, many researchers paid attentions to the studies of freak wave [1]-[3]. Some mechanisms were presented to explain the generation of freak wave, such as, modulation instability (or Benjamin-Feir instability) [4] [5], wave focusing [6] [7], and wave-current or wave-wind interaction [8]-[10]. Based on these mechanisms, the freak wave was generated and studied through theoretical, experimental and numerical methods. The theoretical analysis was mainly based on modulation instability. Benjamin-Feir [11] found that the Stokes wave was unstable to small perturbations, and the unstable sideband components would grow exponentially. This phenomenon can be described by the nonlinear Schrödinger equation. Dysthe & Trulsen [12] and Osborne *et al.* [13] showed the breather solutions of nonlinear Schrödinger equation were considered as simple analytical models of freak waves.

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There were many physical experiments on the generation of freak wave. Onorato *et al.* [14] [15] studied freak wave generation in long-crested sea conditions initialized as a JONSWAP spectrum. Li *et al.* [16] observed the generation process of freak wave in unstable irregular wave train and analyzed the freak wave based on wavelet analysis. Baldock *et al.* [17] investigated nonlinear surface water waves undertaken in 2-dimensional wave flume by a large number of water waves were focused at one point in space and specific time to produce a large transient wave group. Chin and Yao [18] investigated the influence of current on the freak wave generation in 2D wave flume.

Numerical model was also an important approach to study the freak wave generation and its hydrodynamics. It can solve the problem that in physical experiment, the length of the wave tank cannot satisfy requirement of the long-time evolution of wave surface. The most common method is based on nonlinear Schrödinger equation. Onorato *et al.* [19] studied freak wave generation in long-crested sea conditions based on the cubic nonlinear Schrödinger equation. While, the nonlinear Schrödinger equation is weak nonlinear equation, considering that the freak wave generation is strong nonlinear process, many fully nonlinear numerical models were developed. For example, Zhao *et al.* [20] simulated the freak wave generation based on VOF method. Yan and Ma [21] simulated the interaction of freak wave and wind using finite element method.

In order to deeply understand the freak wave generation, a numerical wave model based on High Order Spectral (HOS) method (Dommermuth and Yue [22]) is developed in this paper. Long-time irregular wave trains are generated and the freak wave is observed and analyzed. The method solves the derivative terms by Fast Fourier Transform (FFT) method. Compared with other methods, the spectral method has the properties of fast convergence and low computational cost. It is convenient to the long-time simulation.

## 2. Numerical Model

### 2.1. Governing Equations

It is assumed that the fluid is incompressible and inviscid, and the flow is irrotational. The velocity potential  $\Phi(x, z, t)$  in the 2D fluid domain satisfies Laplace's equation,

$$\nabla^2 \Phi(x, z, t) = 0. \quad (1)$$

Following Zakharov [23], the fully nonlinear free surface boundary conditions can be expressed in terms of the velocity potential at the water surface as

$$\frac{\partial \eta}{\partial t} = (1 + |\nabla_x \eta|^2) \frac{\partial \Phi}{\partial z} - \nabla_x \Phi^s \nabla_x \eta, \quad z = \eta \quad (2)$$

$$\frac{\partial \Phi^s}{\partial t} = -g\eta - \frac{1}{2} |\nabla_x \Phi^s|^2 + \frac{1}{2} (1 + |\nabla_x \eta|^2) \left( \frac{\partial \Phi}{\partial z} \right)^2, \quad z = \eta \quad (3)$$

where  $\Phi^s(x, t) = \Phi(x, \eta, t)$  is the velocity potential on the water surface  $z = \eta$ .

On the fixed boundaries (the bottom, side and end walls of the tank), the conditions can be written simply as

$$\nabla \Phi \cdot \bar{n} = 0 \quad (4)$$

where  $\bar{n}$  is a vector normal to the boundary considered.

If the wave maker boundary is assigned to the left of the wave basin, the corresponding boundary condition can be written as

$$\frac{\partial \Phi}{\partial x} = \frac{\partial X(y, t)}{\partial t} \quad \text{on } x = 0 \quad (5)$$

where  $X(t)$  is the displacement of the wave maker board.

### 2.2. Numerical Procedure

In order to solve this non-homogeneous problem, following Agnon and Bingham [24], the velocity potential can be split into the sum of a prescribed non-periodic components  $\Phi_w$  and an unknown period component  $\Phi_f$ , i.e.,

$$\Phi = \Phi_f + \Phi_w \quad (6)$$

where the prescribed non-periodic components  $\Phi_w$  satisfies the wave maker boundary condition (5) and other lateral boundary conditions (4). So, Equations (2) and (3) can be rewritten as follows:

$$\frac{\partial \eta}{\partial t} = (1 + |\nabla_x \eta|^2) \frac{\partial \Phi_f}{\partial z} + \frac{\partial \Phi_w}{\partial z} - \nabla_x (\Phi_f^s + \Phi_w) \cdot \nabla_x \eta, \quad (7)$$

$$z = \eta$$

$$\begin{aligned} \frac{\partial \Phi_f^s}{\partial t} = & -g\eta - \frac{1}{2} |\nabla_x \Phi_f^s|^2 + \frac{1}{2} (1 + |\nabla_x \eta|^2) \left( \frac{\partial \Phi_f}{\partial z} \right)^2 \\ & - \nabla_x \Phi_f^s \cdot \nabla_x \Phi_w - \frac{\partial \Phi_w}{\partial t} - \frac{1}{2} |\nabla_x \Phi_w|^2 - \frac{1}{2} \left( \frac{\partial \Phi_w}{\partial z} \right)^2, \end{aligned} \quad (8)$$

$$z = \eta$$

Therefore, determining  $\Phi_w$  becomes the first key problem in establishing the numerical model using the HOS method. Different expansions can be used, such as, Agnon and Bingham [24] used polynomials in 2D wave tank simulation. Bonnefoy *et al.* [25] employed a spectral expansion in 2D and 3D wave tank simulation. In this paper, we use Bonnefoy's method. The non-periodic components can be written as:

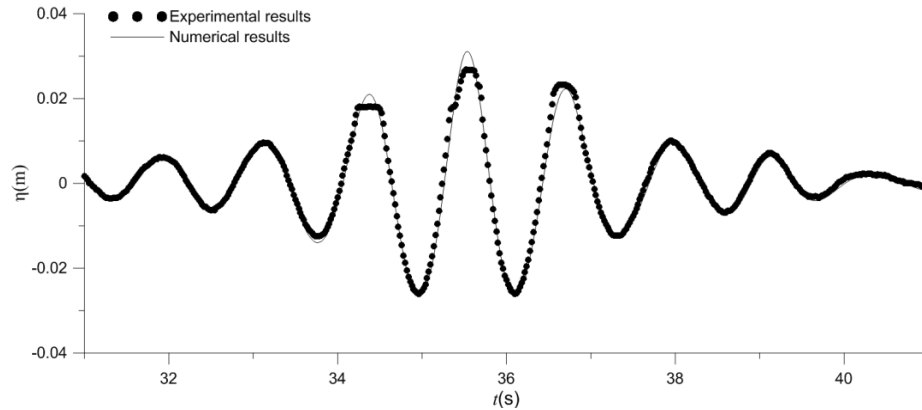
$$\Phi_w = \sum_{p=0}^{N_s} \frac{\cosh[k_p(L_x - x)]}{\cosh[k_p L_x]} \cos[k_p(z + h)], \quad (9)$$

in which  $k_p = (2p - 1)\pi/4h$ ,  $h$  is water depth.

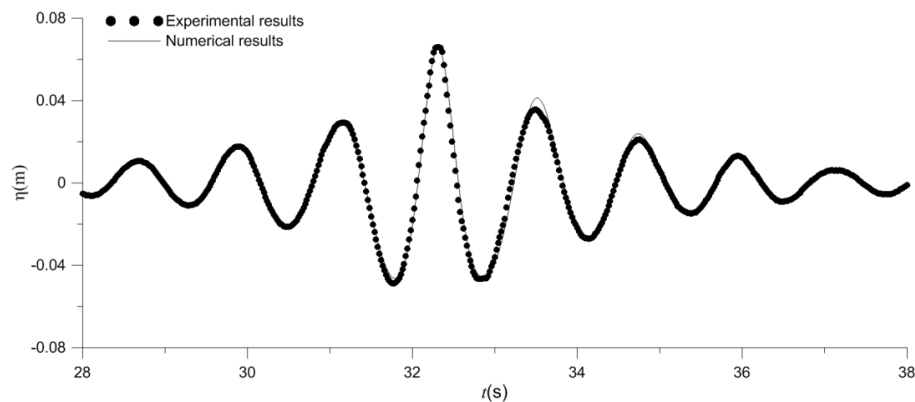
The unknown period component  $\Phi_f$  satisfies the free surface Equations (2), (3) and lateral boundary condition (4). It can be solved by HOS method proposed by Dommermuth and Yue [22].

### 2.3. Verification of the Numerical Model

In order to verify the accuracy of the numerical model, the two-dimensional experimental focused wave data was used. The focused wave experiment was conducted in a two-dimensional wave flume. The focusing method was introduced in Li and Liu [26]. The two cases wave, with  $A = 0.03$  m,  $f = 0.5 - 1.16$  Hz,  $f_c = 0.83$  Hz and  $A = 0.06$  m,  $f = 0.5 - 1.36$  Hz,  $f_c = 0.83$  Hz, were adopted. The corresponding wave steepness  $k_c A$  were 0.09 and 0.18, respectively. The waves were assumed to be focused at a distance 11.4m from the wave-maker. The NJS amplitude distribution was used, and the water depth was 0.5 m ( $k_c h = 1.54$ ). The numerical wave tank is 40 m long. **Figure 1** and **Figure 2** present the comparisons of the numerical and experimental time histories of the wave at the focusing point. The figures show that the numerically simulated focused wave surfaces agree well with the experimental data. However, the numerical and experimental values of the focused wave amplitude are larger than the assumed amplitude because of the nonlinear wave evolution.



**Figure 1.** Comparison between experimental data (dotted) and numerical results (solid lines) for two-dimensional focused waves for the case with  $A = 0.03$  m,  $f = 0.50$  Hz - 1.16 Hz,  $f_c = 0.83$  Hz.



**Figure 2.** Comparison between experimental data (dotted) and numerical results (solid lines) for two-dimensional focused waves for the case with  $A = 0.06$  m,  $f = 0.50$  Hz - 1.16 Hz,  $f_c = 0.83$  Hz.

### 3. The Simulation of Freak Wave Generation in Random Trains

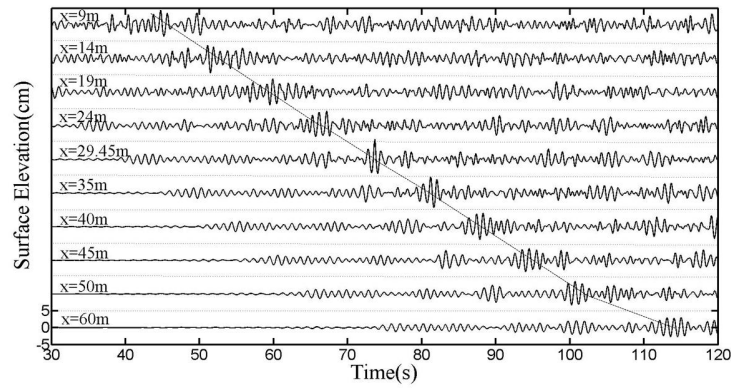
In this paper, long-time irregular wave trains are simulated to generate and observe the freak wave. The incident irregular waves are modeled by a superposition of regular waves with different frequencies with random phases. The frequency spectrum adopts the JONSWAP spectrum with the peak enhancement factor  $\gamma = 3.3$ . The peak frequency is  $f_p = 1.0$  Hz. In order to avoid the numerical calculation instability owing to the wave breaking, the significant wave height of the wave train was adopted 0.037 m. The corresponding BFI value is 0.58. Although the value of BFI is moderate, the freak wave can be generated during long time simulation (see Li *et al.* [16]). The wave tank is 100 m long, and the water depth is 1.2 m.

From long time simulation, the freak wave occurrence can be observed. **Figure 3** gives short segments of time series of wave surface elevation at different locations. The freak generation process can be observed from the figure. At the location near the wave-maker, there are no signs of freak wave. With the wave propagation, some large wave generates. At 29.45 m from the wave maker, owing to the modulation of wave group, these large waves focus together and form a freak wave. The freak wave is similar to the typical form of “New Year Wave”. After the location 29.45 m, the freak wave disappears, these waves are dispersed and the number of wave in group increases. Comparing to physical experiment, the long distance evolution of freak wave can be conveniently observed in the numerical model simulation.

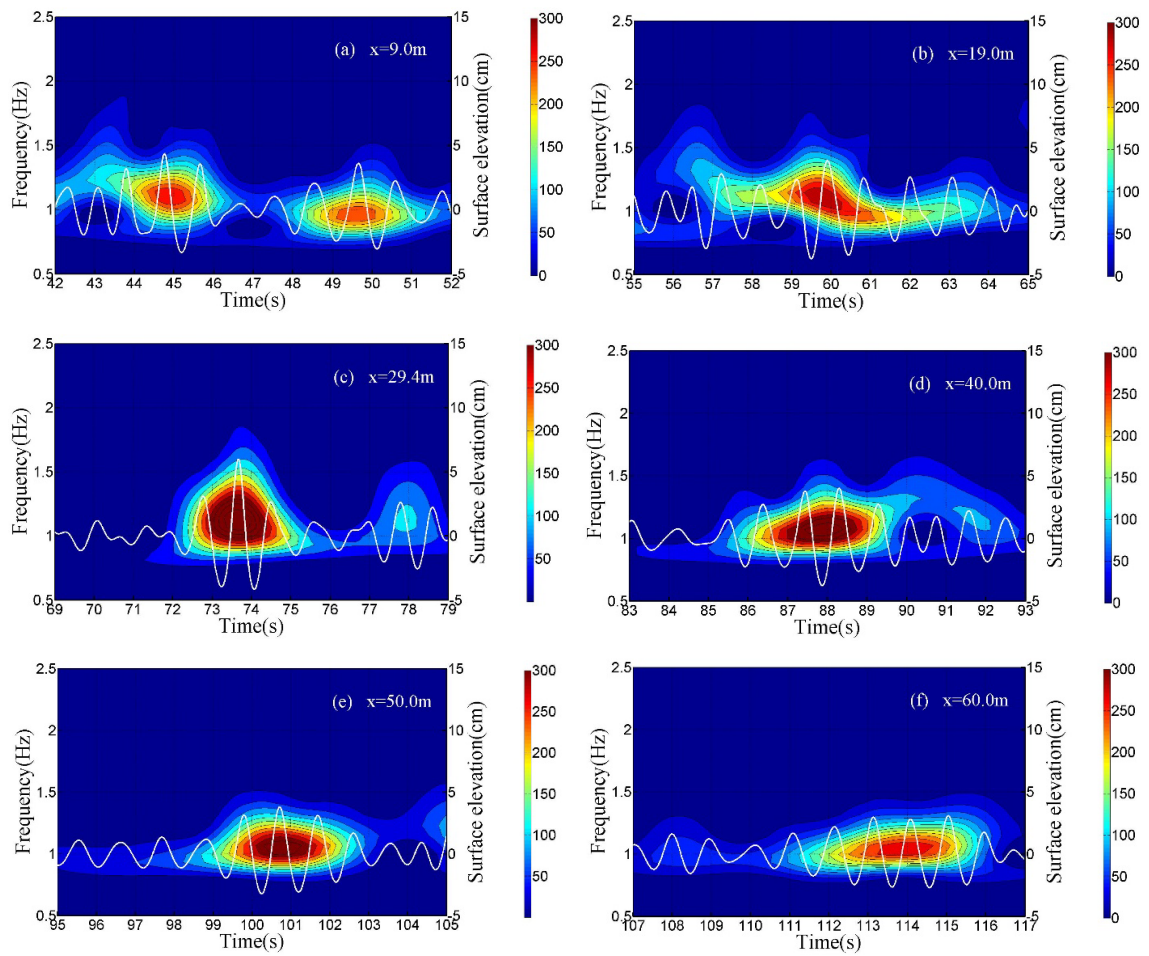
In order to analyze the process of freak wave generation, the evolution of the wavelet energy spectra at selected positions from the wave-maker is shown in **Figure 4**. In the figure, the surface elevation of the waves is shown by the white curves superimposed on the energy isolines. Two successive wave groups are seen at 9.0 m from the wave-maker. A group with comparatively low-frequency components is lagging behind the group with high-frequency components. Then, at the 19 m location, the group with low-frequency components, travelling at greater speed, is merging with the higher-frequency group. At 29.45 m from the wave-maker, owing to the modulation of wave group, the freak wave occurs, the wavelet spectrum isolines show an almost symmetrical triangular form and higher-frequency components and higher energy values are evident, indicative of stronger nonlinear interactions at the time of the freak wave generation. Then, at 50 m to 60 m from the wave-maker, the surfaces begin the group demodulation, the wavelet spectra show that the low-frequency component is at the front of the wave group.

### 4. Conclusion

In order to deeply understand the freak wave generation, a 2D numerical wave tank is developed based on high order spectral method considering the non-periodic boundary condition in this paper. Long time irregular wave simulation is conducted. The freak wave generation process is analyzed by wavelet analysis. The simulation results show that the numerical model is valid to simulate the freak waves. From the process of freak wave generation in the simulation, it can be found that, two wave groups with different frequency components superpose together to form a large wave group. The large wave group modulation generates the freak wave.



**Figure 3.** The time series of wave surface elevation at different locations.



**Figure 4.** Wavelet energy spectra at different distances.

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