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A Quick Method for Judging the Feasibility of Security-Constrained Unit Commitment Problems within Lagrangian Relaxation Framework*

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ABSTRACT

Generally, the procedure for Solving Security constrained unit commitment (SCUC) problems within Lagrangian Relaxation framework is partitioned into two stages: one is to obtain feasible SCUC states; the other is to solve the economic dispatch of generation power among all the generating units. The core of the two stages is how to determine the feasibility of SCUC states. The existence of ramp rate constraints and security constraints increases the difficulty of obtaining an analytical necessary and sufficient condition for determining the quasi-feasibility of SCUC states at each scheduling time. However, a necessary and sufficient numerical condition is proposed and proven rigorously based on Benders Decomposition Theorem. Testing numerical example shows the effectiveness and efficiency of the condition.

Keywords: Security Constrained Unit Commitment (SCUC); Lagrangian Relaxation; Benders Decomposition Feasibility Theorem; Ramp Rate Constraint

1. Introduction

Security-constrained Unit commitment (SCUC) is one of the most important daily functions for independent system operators (ISOs) to clear the electric power market and for generation companies (GENCOs) to analyze generation costs and determine bidding strategies [1-3]. The objective of SCUC is to minimize the total bid cost in current electric power market or generating cost in traditional power systems while satisfying the system constraints including system demand balance, system spinning reserve and related transmission security constraints, and individual unit operating limits such as minimum/maximum generation level, minimum up/down times, ramping rate constraints.

Since the SCUC is an NP-hard mixed integer-programming problem, it is extremely difficult to obtain the exact optimal solution within acceptable time [4]. Lagrangian Relaxation (LR) is one of the most successful methods for obtaining suboptimal solutions [5], where Lagrange multipliers relax the system-wide constraints such as system demand balance, system spinning reserve and DC transmission constraints. Some methods, usually heuristics are needed to modify the dual solution into a feasible one. In fact, the Lagrangian based SCUC methods are all similar but the ways to obtain feasible solutions may vary significantly.

It is clear that the core to develop an effective method for solving SCUC problems within the Lagrangian relaxation framework is how to obtain feasible solutions. First of all, a necessary or sufficient condition used for checking promptly on the feasibility of SCUC states is crucial. Our previous work [6] proposed such conditions. However, a necessary and sufficient condition for determining the feasibility of SCUC states at each scheduling time is not given. Furthermore, ramp rate constraints are not taken into consideration in those results.

A necessary and sufficient condition for determining the feasibility of SCUC states at each scheduling time is proposed and proven rigorously in this paper based on the Benders Decomposition Feasibility Theorem [7,8]. The condition is very crucial for constructing a feasible solution of a SCUC problem. Numerical test example shows that the presented condition is very efficient.

2. Problem Formulation of SCUC Problems

For the convenience of presentation, some notations are defined as follows.

T : commitment horizon in hours;

I : number of units with the index i denoting the i^{th} unit;

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$P_i(t)$: power generation by unit i at time t;

$u_i(t)$: binary variable: 1 if the i^{th} unit is turned on or kept on during the time period t, else 0;

$x_i(t)$: the number of time periods that Unit i has been up ($x_i(t) \ge 1$)or down ($x_i(t) \le -1$);

$\overline{\tau}_i$: the minimum number of time periods for which the unit i must be up;

$\underline{\tau}_i$: the minimum number of time periods for which the unit i must be down;

$C_i(P_i(t))$: fuel cost of producing power $P_i(t)$ for thermal unit i;

$S_i(x_i(t), u_i(t))$: startup/shutdown cost for unit i;

$D(t)$: total demand of the whole power system during time period t;

$P_r(t)$: the spinning reserve requirement during time period t;

$r_i(t)$: $r_i(t) = \min\{\overline{r}_i, \overline{P}_i - P_i(t)\}$ is the spinning reserve requirement during time period t, $\overline{r}_i$ is the maximum spinning reserve requirement;

$\overline{P}_{it}$: the maximum generation of unit i at scheduling time t, if unit i has no raping limit, $\overline{P}_{it} = \overline{P}_i$;

$\underline{P}_{it}$: the minimum generation of unit i at scheduling time t, if unit i has no raping limit, $\underline{P}_{it} = \underline{P}_i$;

Δ_i: the maximum ramp rate;

The objective of the unit commitment problem is to minimize the total operating cost as the following mixed integer-programming problem:

$$\min\left\{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[C_i(P_i(t))u_i(t) + S_i(x_i(t), u_i(t-1))\right]\right\}, \quad (1)$$

subject to

2.1. System Level Constraints

1) System demand constraint

$$\sum_{i=1}^{I} P_i(t)u_i(t) = D(t), \quad (2)$$

where $D_k(t)$ is the demand at bus k;

2) Spinning reserve constraint:

$$\sum_{i=1}^{I} r_i(t)u_i(t) \ge P_r(t), \quad (3)$$

where $r_i(t) = \min\{\overline{r}_i, \overline{P}_i - P_i(t)\}$ is the maximum spinning reserve requirement.

3) Transmission security constraints:

$$-\overline{F}_l \le \overline{F}_l(t) = \sum_{i=1}^{I}\Gamma_{l,i}\cdot P_i(t) - \sum_{k=1}^{K}\Gamma_{l,k}D_k(t) \le \overline{F}_l, \quad l = 1,\cdots,L, \quad (4)$$

2.2. Individual Unit Constraints

4) The minimum up/down time constraint:

$$x_i(t) \ge \overline{\tau}_i, \text{ if } u_i(t-1) = 1, u_i(t) = 0, \quad (5)$$

$$x_i(t) \le \underline{\tau}_i, \text{ if } u_i(t-1) = 0, u_i(t) = 1, \quad (6)$$

5) The relation between the unit state and unit up/down decision

$$x_i(t) = \begin{cases} x_i(t-1) + u_i(t-1), \text{if } x_i(t-1)\cdot u_i(t-1) \ge 1 \\ u_i(t), \text{if } x_i(t-1)\cdot u_i(t) \le -1 \end{cases} \quad (7)$$

6) Generation constraint

$$\begin{cases} \underline{P}_{it} \le P_i(t) \le \overline{P}_{it}, \text{ if } x_i(t) > 0, \\ P_i(t) = 0, \text{ if } x_i(t) < 0. \end{cases} \quad (8)$$

7) Ramp rate constraints: if $x_i(t-1) > 1$ and $x_i(t) > 1$ then

$$|P_i(t) - P_i(t-1)| \le \Delta_i, \quad (9)$$

8) Minimal power generation constraint at the first/last up hour:

$$P_i(t) = \underline{P}_i, r_i(x_i(t), P_i(t)) = 0, \text{ if } u_i(t-1) + u_i(t) = 1 \quad (10)$$

3. The New Necessary and Sufficient Condition for Checking the Feasibility of SCUC States

A mixed-integer programming problem can be represented as

$$\min_{y\in Y, z\in Z, g(y,z)\ge 0} f(y,z), \quad (11)$$

where $Y \subset R^n$ is assumed to be a nonempty convex set and g is concave on Y for each fixed $z \in Z \subset R^m$, y and z are continuous and discrete variables, respectively.

Definition: A vector $z_0 \in Z$ is called to be quasi-feasible if there exists a vector $y_0 \in Y$ such that $g(y_0, z_0) \ge 0$.

Benders Decomposition Feasibility Theorem [7,8]: For problem (11), Y and Z are nonempty and Y is convex, the vector function $g(y,z)$ is concave vector function for each $z \in Z$. Furthermore, the set

$$W_z = \{w \in R^m \mid G(y,z) \ge w \text{ for some } y \in Y\} \quad (12)$$

is closed for each $z \in Z$. Then $z_0 \in Z \cap V$ is quasi-feasible if and only if the following inequalities are satisfied for $\forall \lambda \in \Lambda$

$$c(\lambda) = \sup_{y\in Y} \lambda^T g(y, z_0) \ge 0, \quad (13)$$

where

$$\Lambda = \left\{\lambda \mid \lambda \ge 0, \sum_{i=1}^{m}\lambda_i = 1\right\}, \quad (14)$$

$$V = \{z : G(y,z) \ge 0, \text{for some } y \in Y\}, \quad (15)$$

Note 1: The result still holds for all $\lambda \ge 0$.

Note 2: A SCUC problem can be written as the form of Equation (11), where

$$y = \{y_{i_1}, \cdots, y_{i_n}\} \in Y = [\underline{P}_{i_1 t}, \overline{P}_{i_1 t}] \times \cdots \times [\underline{P}_{i_n t}, \overline{P}_{i_n t}], \quad (16)$$

$$Z = \overbrace{\{0,1\} \times \cdots \times \{0,1\}}^{\text{Descartes product 24 } \{0,1\}}, \quad (17)$$

and n is the number of generating units at scheduling time t, i_k are the indices of generating units, $\underline{P}_{it}$ and $\overline{P}_{it}$ are the minimal and maximal power generation of unit i level, respectively. $g(y,z)$ is a $2L+3$-dimensional vector function, of which the first and second dimensions corresponds to the system demand constraint (since such constraint can be represented as two inequalities), the third relates to the spinning reserve constraint, and the rest dimensions correspond to $2L$ transmission security constraints. $f(y,z)$ is the total cost including the fuel cost of all generating units at time t and their startup cost.

Since for SCUC state vector $z \in Z$ at each scheduling time t the power generation of units not being started up need not be optimized, the set Y is determined by z and is a convex cube of R^n. For the given SCUC state vector $z \in Z$, $g(y,z)$ is the continuous concave vector function over the closed convex set Y, and hence W_z is closed. Therefore, at each scheduling time t, the SCUC problem is a mixed-integer programming of the form (11), which satisfies the conditions of the above Benders Decomposition Feasibility Theorem.

To obtain the desired result, the units are classified into three categories at time *t*: E_{1t} is the set of units which is on the normal generating state; E_{2t} is the set of units at the first/last generating hour; E_{3t} is the set of units with ramp rate constraints. The set E_{3t} is further classified into four types, named as E_{3t}^0, E_{3t}^1, E_{3t}^2 and E_{3t}^3 respectively, as follows:

$$E_{3t}^0 = \{i \mid i \in E_{3t}, x_i(t) = 1, x_i(t) = -1, \text{first\_last}(i) = 1\}$$

$$E_{3t}^1 = \{i \mid i \in E_{3t}, \overline{P}_{it} \le \overline{P}_i - \overline{r}_i\}$$

$$E_{3t}^2 = \{i \mid i \in E_{3t}, \overline{P}_i - \overline{r}_i \le \underline{P}_{it}\}$$

$$E_{3t}^3 = \{i \mid i \in E_{3t}, \underline{P}_{it} < \overline{P}_i - \overline{r}_i < \overline{P}_{it}\}$$

Using the Benders Decomposition Feasibility Theorem, we obtain the desired necessary and sufficient condition for SCUC states to be feasible.

Theorem: SCUC states at scheduling time t is quasi-feasible if and only if the optimal value of the following nonlinear program is nonnegative:

$$\min_{\lambda \in \Lambda} c(\lambda), \quad (18)$$

where

$$\begin{aligned} c(\lambda) = & \sum_{i \in E_{1t}, \alpha_i \le 0} [\alpha_i \underline{P}_i + \lambda_3 \overline{r}_i] \\ & + \sum_{i \in E_{1t}, 0 < \alpha_i \le \mu_t} [\alpha_i \overline{P}_i + (\lambda_3 - \alpha_i)\overline{r}_i] \\ & + \sum_{i \in E_{1t}, \alpha_i > \lambda_3} \alpha_i \overline{P}_i + \sum_{i \in E_{3t}^0} \alpha_i \underline{P}_i \\ & + \sum_{i \in E_{3t}^1, \alpha_i \le 0} [\alpha_i \underline{P}_{it} + \lambda_3 \overline{r}_i] \\ & + \sum_{i \in E_{3t}^1, \alpha_i > 0} [\alpha_i \overline{P}_{it} + \lambda_3 \overline{r}_i] \\ & + \sum_{i \in E_{3t}^2, \alpha_i \le \lambda_3} [(\alpha_i - \lambda_3)\underline{P}_{it} + \lambda_3 \overline{P}_i] \\ & + \sum_{i \in E_{3t}^2, \alpha_i > \lambda_3} [(\alpha_i - \lambda_3)\overline{P}_{it} + \lambda_3 \overline{P}_i] \\ & + \sum_{i \in E_{3t}^3, \alpha_i \le 0} [\alpha_i \underline{P}_{it} + \lambda_3 \overline{r}_i] \\ & + \sum_{i \in E_{3t}^3, 0 < \alpha_i \le \lambda_3} [\alpha_i \overline{P}_i + (\lambda_3 - \alpha_i)\overline{r}_i] \\ & + \sum_{i \in E_{3t}^3, \alpha_i > \lambda_3} [(\alpha_i - \lambda_3)\overline{P}_{it} + \lambda_3 \overline{P}_i] \\ & + \sum_{i \in E_{2t}} \alpha_i \underline{P}_i - (\lambda_1 - \lambda_2) D(t) - \lambda_3 P_r(t) \\ & - \sum_{l=1}^{L} (\lambda_{3+L+l} - \lambda_{3+l}) \sum_{k=1}^{K} \Gamma_{lk} D_k(t) \\ & + \sum_{l=1}^{L} (\lambda_{3+l} + \lambda_{3+L+l}) \overline{F}_l \end{aligned}$$

and

$$\begin{aligned} & \lambda_1 = \lambda_t^1, \lambda_2 = \lambda_t^2, \lambda_3 = \mu_t \\ & \lambda_{3+l} = \rho_{lt}^1, \lambda_{3+L+l} = \rho_{lt}^2, l = 1, \cdots, L \\ & \alpha_i = \lambda_t^1 - \lambda_t^2 + \sum_{l=1}^{L} (\rho_{lt}^2 - \rho_{lt}^1) \Gamma_{li} \end{aligned}$$

Proof: The left hand side of the Benders Decomposition Feasibility Theorem is

$$\begin{aligned} \max \Bigg\{ & \Bigg\{ (\lambda_t^1 - \lambda_t^2) \Bigg[\sum_{i \in E_{1t}} P_i(t) + \sum_{i \in E_{2t}} \underline{P}_i + \sum_{i \in E_{3t}} P_i(t) - D(t) \Bigg] \Bigg\} \\ & + \mu_t \Bigg[\sum_{i \in E_{1t}} r_i(t) + \sum_{i \in E_{3t}} r_i(t) - P_r(t) \Bigg] \\ & + \sum_{l=1}^{L} \Bigg\{ (\rho_{lt}^2 - \rho_{lt}^1) \Bigg[\sum_{i \in E_{1t}} \Gamma_{li} P_i(t) + \sum_{i \in E_{2t}} \Gamma_{li} \underline{P}_i \\ & \qquad + \sum_{i \in E_{3t}} \Gamma_{li} P_i(t) - \sum_{k=1}^{K} \Gamma_{lk} D_k(t) \Bigg] \Bigg\} \\ & + \sum_{l=1}^{L} (\rho_{lt}^1 + \rho_{lt}^2) \overline{F}_l \Bigg\} \\ = \max & \sum_{i \in E_t, \underline{P}_{it} \le P_i(t) \le \overline{P}_{it}, r_i(t) = \min\{\overline{r}_i, \overline{P}_i - P_i(t)\}} f_i(P_i(t), r_i(t)) + C \end{aligned} \quad (19)$$

where

$$C=\left(\lambda_t^1-\lambda_t^2\right)\left[\sum_{i\in E_{2t}}\underline{P}_i-D(t)\right]-\mu_t P_r(t)$$
$$+\sum_{l=1}^{L}\left(\rho_{lt}^2-\rho_{lt}^1\right)\left[\sum_{i\in E_{2t}}\Gamma_{li}\underline{P}_i-\sum_{k=1}^{K}\Gamma_{lk}D_k(t)\right]$$
$$+\sum_{l=1}^{L}\left(\rho_{lt}^1+\rho_{lt}^2\right)\overline{F}_l$$

The solution of problem (19) depends on the following subproblems since the problem (19) is decomposable with respect to units:

i) Subproblem 1: $i\in E_{1t}$, we have

$$\max_{\overline{P}_i\ge P_i(t)\ge\underline{P}_i,\,r_i(t)=\min\{r_i(t),\overline{P}_i-P_i(t)\}}\left\{f_i\left(P_i(t),r_i(t)\right)=\alpha_iP_i(t)+\mu_tr_i(t)\right\}$$

$$=\begin{cases}\left\{\alpha_i\underline{P}_i+\lambda_3\overline{r}_i,\text{if }\alpha_i\le 0,P_i^*(t)=\underline{P}_i,r_i^*(t)=\overline{r}_i\right\}\\ \left\{\alpha_i\overline{P}_i+\left(\lambda_3-\alpha_i\right)\overline{r}_i,\text{if }0<\alpha_i\le\lambda_3,\right.\\ \left.P_i^*(t)=\overline{P}_i-\overline{r}_i,r_i^*(t)=\overline{r}_i\right\}\\ \alpha_i\overline{P}_i,\text{if }\alpha_i>\lambda_3,P_i^*(t)=\overline{P},r_i^*(t)=0\end{cases}$$

ii) Subproblem 2: $i\in E_{3t}^0$, we have

$$\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}f_i\left(P_i(t),r_i(t)\right)$$
$$=\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}\alpha_iP_i(t)+\lambda_3r_i(t)$$
$$=\alpha_i\underline{P}_i,\text{if }P_i^*(t)=\underline{P}_i,r_i^*(t)=0$$

iii) Subproblem 3: $i\in E_{3t}^1$, we have

$$\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}f_i\left(P_i(t),r_i(t)\right)$$
$$=\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}\alpha_iP_i(t)+\lambda_3r_i(t)$$
$$=\begin{cases}\alpha_i\underline{P}_{it}+\lambda_3\overline{r}_i,\text{if }\alpha_i\le 0,P_i^*(t)=\underline{P}_{it},r_i^*(t)=\overline{r}_i\\ \alpha_i\overline{P}_{it}+\lambda_3\overline{r}_i,\text{if }\alpha_i>0,P_i^*(t)=\overline{P}_{it},r_i^*(t)=\overline{r}_i\end{cases}$$

iv) Subproblem 4: $i\in E_{3t}^2$, we have

$$\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}f_i\left(P_i(t),r_i(t)\right)$$
$$=\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}\alpha_iP_i(t)+\lambda_3r_i(t)$$
$$=\begin{cases}\left\{\alpha_i\underline{P}_{it}+\lambda_3\left(\overline{P}_i-\underline{P}_{it}\right),\text{if }\alpha_i\le\lambda_3,\right.\\ \left.P_i^*(t)=\underline{P}_{it},r_i^*(t)=\overline{P}_i-\underline{P}_{it}\right\}\\ \left\{\alpha_i\overline{P}_{it}+\lambda_3\left(\overline{P}_i-\overline{P}_{it}\right),\text{if }\alpha_i>\lambda_3,\right.\\ \left.P_i^*(t)=\overline{P}_{it},r_i^*(t)=\overline{P}_i-\overline{P}_{it}\right\}\end{cases}$$

v) Subproblem 5: $i\in E_{3t}^3$, we have

$$\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}f_i\left(P_i(t),r_i(t)\right)$$
$$=\max_{\overline{P}_{it}\ge P_i(t)\ge\underline{P}_{it},\,r_i(t)=\min\{\overline{r}_i,\overline{P}_{it}-P_i(t)\}}\alpha_iP_i(t)+\lambda_3r_i(t)$$
$$=\begin{cases}\left\{\alpha_i\underline{P}_{it}+\lambda_3\overline{r}_i,\text{if }\alpha_i\le 0,P_i^*(t)=\underline{P}_{it},r_i^*(t)=\overline{r}_i\right\}\\ \left\{\alpha_i\left(\overline{P}_i-\overline{r}\right)_i+\lambda_3\overline{r}_i,\text{if }0<\alpha_i\le\lambda_3,\right.\\ \left.P_i^*(t)=\overline{P}_i-\overline{r}_i,r_i^*(t)=\overline{r}_i\right\}\\ \left\{\alpha_i\overline{P}_{it}+\lambda_3\left(\overline{P}_i-\overline{P}_{it}\right),\text{if }\alpha_i>\lambda_3,\right.\\ \left.P_i^*(t)=\overline{P}_{it},r_i^*(t)=\overline{P}_i-\overline{P}_{it}\right\}\end{cases}$$

By Benders Decomposition Feasibility Theorem, we have the desired result. **Q.E.D.**

Note 3: The all subproblems above are linear programming problems with simple constraints. Thus, by comparing the values of all extreme points, the optimal solutions and corresponding optimal values can be obtained easily.

Note 4: It should be noted that the theorem still hold for $\lambda\ge 0$.

4. The Numerical Solution of the Problem

Consider the SCUC problem at scheduling time t with 0 being the objective

$$\begin{cases}\max 0,\\ g\left(y,z_0\right)\ge 0,\\ y\in Y,\end{cases}\tag{20}$$

where y, Y and Z are defined in Equations (16)-(17). The dual problem of the problem (20) is

$$\min_{\lambda\ge 0}c(\lambda),\tag{21}$$

By the theorem and the note 4, z_0 is quasi-feasible if and only if and only if the optimal value of $c(\lambda)$ is nonnegative over the positive orthant $\lambda\ge 0$. While the problem (21) can be solved by using subgradient method [9], and the value $c(\lambda)$ can be obtained by Equation (19) for the given Lagrange multiplier vector λ, the Lagrange multiplier can then be updated by subgradient method. Since the best multiplier vector in the dual iteration of the SCUC problem is taken as the initial multiplier λ_0, the rate of convergence of λ is quite promptly.

5. Numerical Testing Result

The standard IEEE example [5] tests the effectiveness and efficiency of the proposed method (**Figure 1**), which has 16 units, 43 transmission lines, 31 buses (of which 11 is load bus). The fuel cost function of unit i is

$$C_i\left(P_i\left(t\right)\right)=a_iP_i^2\left(t\right)+b_iP_i\left(t\right)$$

The data of units, the system reserve requirement $P_r\left(t\right)$, the system demand $D\left(t\right)$ at each scheduling time t, the maximal value of DC power flow $\overline{F}_l$ on each transmission line l and the amount of electric power on each load bus are given in **Tables 1-5**, respectively. Units 1 and 4 have minimal power generation constraint at the first/last up hour.

The CPU-time is within 2.3 second for checking SCUC states for 24 scheduling period on a DELL Computer with 2G RAM using MATLAB 7.01. **Table 6** gives a feasible SCUC obtained within Lagrangian framework. **Figure 2** presented the tendency of $c\left(\lambda^{(k)}\right)$ with $\lambda^{(k)}$ at $t=8$. Testing example shows that the numerical method for determining the feasibility of a SCUC is effective and efficient.

6. Conclusions

The key of solving SCUC problems is to determine whether a SCUC is quasi-feasible or not. The existence of ramp rate constraints and transmission security constraints increases the difficulty of obtaining an analytical condition. However, a numerical necessary and sufficient condition for checking on the feasibility of SCUC states at each scheduling time is proposed and proved rigorously based on Benders Decomposition Feasibility

Table 1. Generation level and its coefficient of fuel cost function of each unit.

Unit i	$\underline{P}_i$ (MW)	$\overline{P}_i$ (MW)	$\overline{r}_i$ (MW)	a_i (k$/MW2)	b_i (k$/MW)
1	300	1350	1000	0.0015	8.752
2	360	1620	1200	0.0016	7.654
3	360	1620	1200	0.0016	7.654
4	360	1620	1200	0.0016	7.654
5	300	1875	1500	0.0013	6.052
6	300	1875	1500	0.0013	6.052
7	240	1080	800	0.0015	9.072
8	150	675	500	0.0015	8.752
9	100	625	500	0.0015	8.752
10	45	202.5	150	0.0019	12.54
11	90	405	300	0.0018	11.62
12	120	750	600	0.0017	9.543
13	150	937.5	750	0.0015	8.352
14	52	235.7	175	0.0019	13.00
15	60	270	200	0.0018	14.62
16	120	750	600	0.0017	9.543

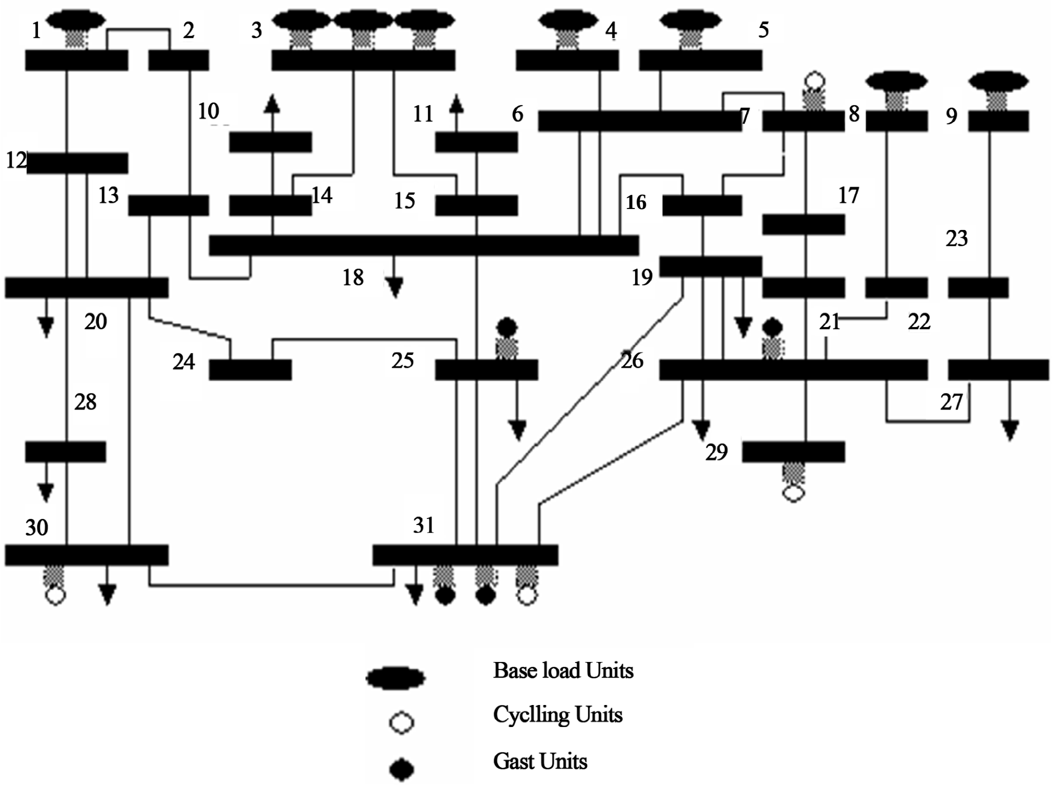

Figure 1. The one-line diagram for the 31-bus test system.

Table 2. Length of up/down time, initial states and coefficient of startup cost function of each unit.

Unit i	$\overline{\tau}_i$	$\underline{\tau}_i$	x_{i0}	P_{i0}	Δ_i	S_i^h	S_i^c	τ_i^c
1	5	6	5	500	-	2120.8	2597.6	12
2	5	6	5	370	-	1572.9	1912.4	12
3	5	6	5	400	-	2277.5	2765.8	12
4	5	6	5	400	200	2350.7	2862.3	12
5	5	6	5	350	180	2227.5	2715.8	12
6	5	6	5	350	180	2077.5	2565.8	12
7	5	6	5	300	200	2403.9	2938.7	12
8	3	4	3	200	-	1416.0	1734.6	8
9	3	4	3	150	200	1295.9	1570.3	8
10	1	1	−1	0	-	36.3	38.6	2
11	1	1	−1	0	-	27.8	33.6	2
12	3	4	−4	0	-	1370.9	1645.3	8
13	3	4	−4	0	-	1624.6	2019.9	8
14	1	1	−1	0	-	45.8	51.6	2
15	1	1	−1	0	-	24.6	29.3	2
16	3	4	−4	0	-	1198.5	1477.6	8

Table 3. System load and system reserve requirement for 24 scheduling hours.

Hour t	$D(t)$ (MW)	$P_r(t)$ (MW)	Hour t	$D(t)$ (MW)	$P_r(t)$ (MW)
1	2502	250.2	13	7995	799.5
2	2441	244.1	14	7201	720.1
3	2197	219.7	15	6591	659.1
4	2075	207.5	16	6225	622.5
5	2502	250.2	17	6652	665.2
6	3418	341.8	18	7812	781.2
7	4809	480.9	19	8056	805.6
8	5859	585.9	20	7079	707.9
9	6957	695.7	21	5188	518.8
10	7690	769	22	4028	402.8
11	8056	805.6	23	3174	317.4
12	8300	830	24	2807	280.7

Table 4. Limits of DC power flow $\overline{F}_l$ on each transmission line.

Line: $i > j$	Capacity (MW)	Reactance (p.u.)	Line: $i > j$	Capacity (MW)	Reactance (p.u.)
1 - 2	1000	0.025	16 - 18	1200	0.01
1 - 12	1000	0.008	16 - 19	800	0.01
2 - 13	1000	0.054	17 - 21	1200	0.015
3 - 14	2000	0.01	18 - 25	2500	0.0005
3 - 15	2000	0.01	19 - 26 (Double lines)	250	0.045
4 - 6	1500	0.01	19 - 31	200	0.04
5 - 6	1500	0.01	20 - 24	1000	0.03
6 - 7	1200	0.015	20 - 28	1000	0.025
6 - 18 (Double lines)	1200	0.046	20 - 30	1000	0.05
7 - 16	1200	0.025	21 - 26	900	0.01
7 - 17	1200	0.015	22 - 26	1250	0.01
8 - 22	1000	0.01	23 - 27	1250	0.01
9 - 23	1000	0.01	24 - 25	1000	0.012
10 - 14	1000	0.0035	25 - 31 (Double lines)	250	0.045
11 - 15	1000	0.0035	26 - 27	1200	0.025
12 - 20 (Double lines)	1000	0.054	26 - 29	800	0.01
13 - 18	1000	0.03	26 - 31	600	0.0333
13 - 20	1000	0.01	28 - 30	1000	0.025
14 - 18	1780	0.00815	30 - 31	700	0.022
15 - 18	1780	0.00815			

Table 5. Percent of system load drawn by load bus.

Bus	Percent	Bus	Percent
1	0.024	7	0.265
2	0.024	8	0.062
3	0.361	9	0.024
4	0.036	10	0.048
5	0.012	11	0.12
6	0.024		

Table 6. A Feasible SCUC obtained within Lagrangian relaxation framework.

Units	The Unit States (0 Denotes Downstate, 1 upstate) Hour 1 -----------------s------------------Hour 24
1	1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
2	0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
3	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
5	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
6	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
7	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
8	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
9	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
10	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
11	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
12	0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0
13	0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
14	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
16	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

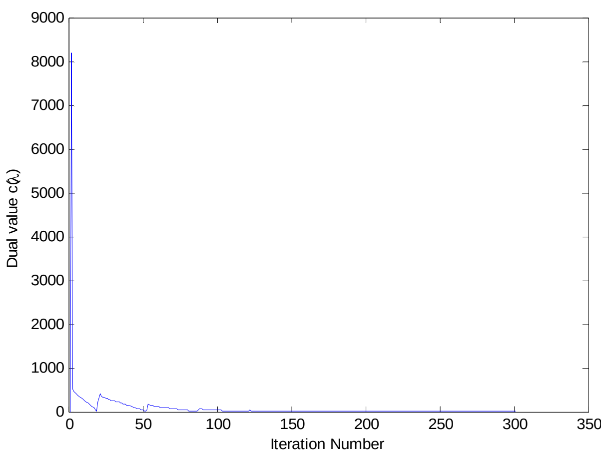

Figure 2. The tendency of $c\left(\lambda^{(k)}\right)$ with $\lambda^{(k)}$ at t = 8 in the first dual iteration of the SCUC problem. The SCUC at t = 8 is feasible.

Theorem. The condition is very crucial for constructing a feasible solution of a SCUC problem. Numerical testing example shows that the proposed condition is very effective and efficient.

REFERENCES

[1] B. F. Hobbs, M. H. Rothhopf, R. P. Oneill and H. Chao, "The Next Generation of Electric Power Unit Commitment Models," Kluwer Academic Publishers, London, 1999.

[2] J. M. Crepo, J. Usao la and J. L. Fernandez, "Security-constrained Optimal Generation Scheduling in Large-Scale Power Systems," *IEEE Transactions on Power Systems*, Vol. 21, No. 1, 2006, pp. 321-332. doi:10.1109/TPWRS.2005.860942

[3] Y. Fu and M. Shahidepour, "Fast SCUC for Large-Scale Power Systems," *IEEE Transactions on Power Systems*, Vol. 22, No. 4, 2007, pp. 2144-2151. doi:10.1109/TPWRS.2007.907444

[4] X. Guan, Q. Zhai and A. Papalexpoulos, "Optimization Based Methods for Unit Commitment: Lagrangian Relaxation versus General Mixed Integer Programming," 2003 *IEEE Power Engineering Society General Meeting*, Ontario, 13-17 July 2003, pp. 1095-1100.

[5] J. J. Shaw, "A Direct Method for Security-Constrained Unit Commitment," *IEEE Transactions on Power Systems*, Vol. 10, No. 3, 1995, pp. 1329-1342. doi:10.1109/59.466520

[6] X. Guan, S. G. Guo and Q. Z. Zhai, "Conditions for Obtaining Feasible Solutions to Security Constrained Unit Commitment Problems," *IEEE Transactions on Power System*, Vol. 20, No. 4, 2005, pp. 1746-1756. doi:10.1109/TPWRS.2005.857399

[7] J. F. Benders, "Partition Procedures for Solving Mixed-Variables Programming Problems," *Computational Management Science*, Vol. 2, No. 1, 2005, pp. 3-19.

[8] A. M. Geoffrion, "Generalized Benders Decomposition," *Journal of Optimization Theory and Applications*, Vol. 10, No. 4, 1972, pp. 237-260. doi:10.1007/BF00934810

[9] D. P. Bertsekas, "Nonlinear Programming," Athena Scientific, 1995.

