

# A Comparative Study of Adomain Decomposition Method and He-Laplace Method

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## Abstract

In this paper, we present a comparative study between the He-Laplace and Adomain decomposition method. The study outlines the significant features of two methods. We use the two methods to solve the nonlinear Ordinary and Partial differential equations. Laplace transformation with the homotopy method is called He-Laplace method. A comparison is made among Adomain decomposition method and He-Laplace. It is shown that, in He-Laplace method, the nonlinear terms of differential equation can be easily handled by the use of He's polynomials and provides better results.

## Keywords

Adomain Decomposition Method, He-Laplace Transform Method, Homotopy Perturbation Method, Ordinary Differential Equation, Partial Differential Equations, He's Polynomials

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## 1. Introduction

This paper outlines a reliable Comparison between two powerful methods that were recently developed. The first is Adomain decomposition method (ADM) developed by Adomain in [1] [2], and used heavily in the literature in [3]-[10] and the references therein. The second is He-Laplace method, an elegant combination of the Laplace transformation, the homotopy perturbation method and He's polynomials. The use of He's polynomial in nonlinear term was first introduced by Ghorbani [11]. The proposed algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The two methods give rapidly convergent series with specific significant features for each scheme. Some of the classical analytic methods are Lyapunov's ar-

tificial small parameter method [12] perturbation techniques [13] [14] and Hiroa bilinear method [15] [16]. In recent years, many authors have paid attention to study the solution of nonlinear partial differential equation by using various methods. Variational iteration method, He's semi inverse method [17] and the differential transform method, etc. are among these. The main objective is to introduce a comparative study to nonlinear ordinary differential and partial differential equations by using adomain decomposition method and He-Laplace method.

This paper contains basic idea of homotopy perturbation method and He-Laplace method in Section 2, Adomain decomposition method in 3, Application in 4 and conclusion and discussions in 5 respectively.

## 2. Basic Idea of Homotopy Perturbation Method and He-Laplace Method

### 2.1. Homotopy Perturbation Method

Consider the following nonlinear differential equation

$$A(y) - f(r) = 0, \quad r \in \Omega \quad (1)$$

with boundary conditions of

$$B\left(y, \frac{\partial y}{\partial n}\right) = 0, \quad r \in \Gamma \quad (2)$$

where  $A$ ,  $B$ ,  $f(r)$  and  $\Gamma$  are a general differential operator, a boundary operator, a known analytic function and the boundary of the domain  $\Omega$ , respectively.

The operator  $A$  can generally be divided into a linear part  $L$  and a nonlinear part  $M$ . Equation (1) may therefore be written as:

$$L(y) + M(y) - f(r) = 0 \quad (3)$$

By the homotopy technique, we construct a homotopy  $v(r, p): \Omega \times [0, 1] \rightarrow R$  which satisfies:

$$H(v, p) = (1-p)[L(v) - L(y_0)] + p[A(v) - f(r)] = 0 \quad (4)$$

or

$$H(v, p) = L(v) - L(y_0) + p[L(y_0) + p[M(v) - f(r)]] = 0 \quad (5)$$

where  $p \in [0, 1]$  is an embedding parameter, while  $y_0$  is an initial approximation of Equation (1), which satisfies the boundary conditions. Obviously, from Equations (4) and (5), we will have:

$$H(v, 0) = L(v) - L(y_0) = 0 \quad (6)$$

$$H(v, 1) = L(v) - L(y_0) = 0 \quad (7)$$

The changing process of  $P$  from zero to unity is just that of  $v(r, p)$  from  $y_0$  to  $y(r)$ . In topology, this is called deformation, while  $L(v) - L(y_0)$  and  $A(v) - f(r)$  are called homotopy. If the embedding parameter  $p$  is considered as a small parameter, applying the classical perturbation technique, we can assume that the solution of Equations (4) and (5) can be written as a power series in  $p$ :

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \dots + \infty \quad (8)$$

Setting  $p = 1$  in Equations (8), we have

$$y = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad (9)$$

The combination of the perturbation method and the homotopy method is called the HPM, which eliminates the drawbacks of the traditional perturbation methods while keeping all its advantages. The series (9) is convergent for most cases. However, the convergent rate depends on the nonlinear operator  $A(v)$ . Moreover, He [18] made the following suggestions:

1) The second derivative of  $M(v)$  with respect to  $v$  must be small because the parameter may be relatively large, *i.e.*  $p \rightarrow 1$ .

2) The norm of  $L^{-1}\left(\frac{\partial M}{\partial v}\right)$  must be smaller than one so that the series converges.

## 2.2. He-Laplace Method

Consider the following nonlinear differential equation (IVP):

$$y'' + p_1 y' + p_2 y + p_3 f(y) = f(x) \quad (10)$$

$$y(0) = \alpha, \quad y'(0) = \beta \quad (11)$$

where  $p_1, p_2, p_3, \alpha, \beta$  are constant.  $f(y)$  is a nonlinear function and  $f(x)$  is the source term. Taking Laplace transformation (denoted throughout this paper by  $L$ ) on both side of Equation (10), we have

$$L[y''] + L[p_1 y'] + L[p_2 y] + L[p_3 f(y)] = L[f(x)] \quad (12)$$

By using linearity of Laplace transformation, the result is

$$L[y''] + Lp_1[y'] + Lp_2[y] + Lp_3[f(y)] = L[f(x)] \quad (13)$$

Applying the formula on Laplace transform, we obtain

$$s^2 L[y] - sy(0) - y'(0) + p_1 L\{sL[y] - y(0)\} + p_2 L[y] + p_3 L[f(y)] = L[f(x)] \quad (14)$$

Using initial conditions in Equation (14), we have

$$(s^2 + p_1 s)L[y] = \alpha s + \beta + \alpha p_1 - p_2 L(y) - p_3 L[f(y)] + L[f(x)] \quad (15)$$

Or

$$L[y] = \frac{(\alpha s + \beta + \alpha p_1)}{(s^2 + p_1 s)} - \frac{p_2}{(s^2 + p_1 s)} L[y] - \frac{p_3}{(s^2 + p_1 s)} L[f(y)] + L[f(x)] \quad (16)$$

Taking inverse Laplace transform, we have

$$y(x) = F(x) - L^{-1}\left(\frac{p_2}{s^2 + p_1 s} L[y]\right) - L^{-1}\left(\frac{p_3}{s^2 + p_1 s} L[f(y)]\right) \quad (17)$$

where  $F(x)$  represents the term arising from the source term and the prescribed initial conditions.

Now, we apply homotopy perturbation method [12],

$$y(x) = \sum_{n=0}^{\infty} p^n y_n(x) \quad (18)$$

where the term  $y_n$  are to recursively calculated and the nonlinear term  $f(y)$  can be decomposed as

$$f(y) = \sum_{n=0}^{\infty} p^n H_n(y) \quad (19)$$

for some He's polynomial  $H_n$  (see [11] [19]) that are given by

$$H_n(y_0, y_1, y_2, \dots, y_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left[ f\left(\sum_{i=0}^{\infty} p^i y_i(x)\right) \right]_{p=0} \quad n = 0, 1, 2, 3, \dots$$

Substituting Equations (18) and (19) in (17), we get

$$\sum_{n=0}^{\infty} p^n y_n(x) = F(x) - p \left( L^{-1} \left\{ \frac{p_2}{s^2 - p_1 s} L \left[ \sum_{n=0}^{\infty} p^n y_n(x) \right] \right\} + L^{-1} \left\{ \frac{p_3}{s^2 + p_1 s} L \left[ \sum_{N=0}^{\infty} p^N H_N(y) \right] \right\} \right) \quad (20)$$

which is the coupling of the Laplace transformation and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of  $p$ , the following approximations are obtained:

$$\begin{aligned} p^0 : y_0(x) &= F(x), \\ p^1 : y_1(x) &= - \left( L^{-1} \left\{ \frac{p_2}{s^2 + p_1 s} L[y_0(x)] \right\} + L^{-1} \left\{ \frac{p_3}{s^2 + p_1 s} L[H_0(y)] \right\} \right) \\ p^2 : y_2(x) &= - \left( L^{-1} \left\{ \frac{p_2}{s^2 + p_1 s} L[y_1(x)] \right\} + L^{-1} \left\{ \frac{p_3}{s^2 + p_1 s} L[H_1(y)] \right\} \right) \\ p^3 : y_3(x) &= - \left( L^{-1} \left\{ \frac{p_2}{s^2 + p_1 s} L[y_2(x)] \right\} + L^{-1} \left\{ \frac{p_3}{s^2 + p_1 s} L[H_2(y)] \right\} \right) \\ &\vdots \end{aligned} \quad (21)$$

### 3. A Domain Decomposition Method

A domain decomposition method [3] [4] define the unknown function  $u(x)$  by an infinite series

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \quad (22)$$

where the components  $u_n(x)$ , are usually determined recurrently. The nonlinear operator  $F(u)$  can be decomposed into an infinite series of polynomials given by

$$F(u) = \sum_{n=0}^{\infty} A_n \quad (23)$$

where  $A_n$  are the so-called Adomain polynomial of  $u_0, u_1 u_2, \dots, u_n$  defined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ F(\lambda^i u_i) \right]_{\lambda=0}, \quad n = 0, 1, 2, \dots \quad (24)$$

or equivalently

$$\begin{aligned} A_0 &= F(u_0), \\ A_1 &= u_1 F'(u_0), \\ A_2 &= u_2 F'(u_0) + \frac{1}{2} u_1^2 F''(u_0), \\ A_3 &= u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3} u_1^3 F'''(u_0), \\ A_4 &= u_4 F'(u_0) + \left( u_1 u_3 + \frac{1}{2} u_2^2 \right) F''(u_0) + \frac{1}{2} u_1^2 u_2 F'''(u_0) + \frac{1}{24} u_1^4 F^{(iv)}(u_0). \\ &\vdots \end{aligned} \quad (25)$$

It is now well known that these polynomials can be generated for all classes of nonlinear according to specific algorithms defined by (24). Recently, an alternative algorithm for constructing Adomain polynomials has been developed by Wazwaz [6].

This powerful technique handles both linear and nonlinear equations in unified manner without any need for the so-called Adomain polynomials. However, Adomain decomposition method provides the component of the exact solution, where these components should follow the summation given in (22), whereas ADM requires the evaluation of the Adomain polynomials that mostly require tedious algebraic work.

## 4. Applications

### 4.1. Example 1

Consider the following nonlinear PDE [20]:

$$\nabla^2 u + \left( \frac{\partial u}{\partial y} \right) = 2y + x^4 \quad (26)$$

with the following conditions:

$$\begin{aligned} u(x, 0) &= ax, & u(x, 1) &= x(x + a), \\ u(0, y) &= 0, & u(1, y) &= y + a \end{aligned} \quad (27)$$

#### 4.1.1. Using He-Laplace Method

Equation (22) can be written as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \left( \frac{\partial u}{\partial y} \right)^2 = 2y + x^4 \quad (28)$$

By applying the Laplace transform to both sides of Equation (24) subject to the initial condition, we have

$$L[u] = \frac{1}{s} \left( L[2y] + L[x^4] \right) - \frac{1}{s^2} L(u_{xx} + u_{xx}^2) \quad (29)$$

The inverse of the Laplace transform implies that

$$u(x, y) = x^2 y + \frac{x^6}{30} - pL^{-1} \left( \frac{1}{s^2} L[u_{yy}] + \frac{1}{s^2} L[u_y^2] \right) \quad (30)$$

Now, we apply the homotopy perturbation method, we have

$$\sum_{n=0}^{\infty} p^n u_n(x, y) = x^2 y + \frac{x^6}{30} - pL^{-1} \left( \frac{1}{s^2} \left\{ L \left[ \sum_{n=0}^{\infty} p^n u_n \right] + \frac{1}{s^2} L \left[ \sum_{n=0}^{\infty} p^n H_n(u) \right] \right\} \right) \quad (31)$$

where  $H_n(u)$  are He's polynomials. The first few components of He's polynomials are given by

$$\begin{aligned} H_0(u) &= y_{0y}^2 = x^2 \\ H_1(u) &= 2y_{0y}y_{1y} \\ H_2(u) &= y_{1y}^2 + 2y_{0y}y_{2y} = 0 \end{aligned} \quad (32)$$

Comparing the coefficient of like powers of  $p$ , we have  $p^0: u_0(x, y) = x^2 y + \frac{x^6}{30}$ , but we consider

$$u_0(x, y) = x^2 y + \frac{x^6}{30} + \alpha x$$

$$\begin{aligned} p^1: u_1(x, y) &= -L^{-1} \left[ \frac{1}{s^2} \left\{ L[y_{0yy}] + L[H_0(y)] \right\} \right] = -\frac{x^6}{30} \\ p^2: u_2(x, y) &= -L^{-1} \left[ \frac{1}{s^2} \left\{ L[y_{1yy}] + L[H_1(y)] \right\} \right] = 0 \\ p^3: u_3(x, y) &= -L^{-1} \left[ \frac{1}{s^2} \left\{ L[y_{2yy}] + L[H_2(y)] \right\} \right] = 0 \\ &\vdots \end{aligned} \quad (33)$$

So that the solution  $u(x, y)$  is given by

$$u(x, y) = u_0 + u_1 + u_2 + u_3 + \dots = x^2 y + \frac{x^6}{30} + ax - \frac{x^6}{30} + 0 + 0 + \dots = x^2 y + ax \quad (34)$$

which is the exact solution of the problem.

#### 4.1.2. Adomain Decomposition Method

We first rewrite Equation (26) in an operator  $L$  is

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \left( \frac{\partial u}{\partial y} \right)^2 &= 2y + x^4 \\ L_{xx}u + L_{yy}u + (L_y u)^2 &= 2y + x^4 \\ L_{xx}u &= 2y + x^4 - L_{yy}u - (L_y u)^2 \\ u(x, 0) &= ax, \quad u(x, 1) = x(x + a) \\ u(0, y) &= 0, \quad u(1, y) = y + a \end{aligned} \quad (35)$$

where the differential operators  $L_{xx}, L_{yy}$  &  $L_y$  are

$$L_{xx}(\cdot) = \frac{\partial^2}{\partial x^2}(\cdot), \quad L_{yy}(\cdot) = \frac{\partial^2}{\partial y^2}(\cdot) \quad \& \quad L_y(\cdot) = \frac{\partial}{\partial y}(\cdot) \quad (36)$$

The inverse  $L_{xx}^{-1}$  are assumed as an integral operator given by

$$L_{xx}^{-1}(\cdot) = \int_0^x \int_0^x (\cdot) dx dx, \quad (37)$$

Applying the inverse operator  $L_{xx}^{-1}$  on both sides of (35) and using initial condition we find

$$u(x, y) = yx^2 + \frac{1}{30}x^6 + ax - L_{xx}^{-1} \left( L_{yy}u + (L_y u)^2 \right) \quad (38)$$

Substituting (22) into the function Equation (38) gives

$$\sum_{n=0}^{\infty} u_n(x, y) = yx^2 + \frac{1}{30}x^6 + ax - L_{xx}^{-1} \left( L_{yy} \sum_{n=0}^{\infty} u_n(x, y) + \left( L_y \sum_{n=0}^{\infty} u_n(x, y) \right)^2 \right) \quad (39)$$

This can be rewrite at the form

$$\begin{aligned} u_0 + u_1 + u_2 + \dots \\ = x^2 y + \frac{1}{30}x^6 + ax - L_{xx}^{-1} \left( L_{yy} (u_0 + u_1 + u_2 + \dots) + (L_y (u_0 + u_1 + u_2 + \dots))^2 \right) \end{aligned} \quad (40)$$

In view of (39), the following recursive relation

$$\begin{aligned} u_0 &= x^2 y + \frac{1}{30}x^6 + ax \\ u_{k+1}(x, y) &= -L_{xx}^{-1} \left( L_{yy} u_k(x, y) + (L_y u_k(x, y))^2 \right), \quad k \geq 0 \end{aligned} \quad (41)$$

follows immediately. Consequently, we obtain

$$\begin{aligned}
 u_0 &= x^2 y + \frac{1}{30} x^6 + ax \\
 u_1 &= -L_{xx}^{-1} \left( L_{yy} u_0(x, y) + (L_y u_0(x, y))^2 \right) \\
 u_2 &= -L_{xx}^{-1} \left( L_{yy} u_2(x, y) + (L_y u_2(x, y))^2 \right) \\
 &\vdots
 \end{aligned} \tag{42}$$

According to Adomain [19], and approximate solution can be obtained [12].

$$u(x, y) = x(xy + a) + \left(\frac{1}{2}\right) \left\{ y(y-1) \left( \frac{x^4}{2} + \frac{y+1}{3} \right) + \frac{x}{30} (x^5 - 1) \right\} \tag{43}$$

the exact solution is given by  $u(x, y) = x(xy + a)$ .

## 4.2. Example 2

Consider the following non-homogeneous nonlinear PDE [20]:

$$\frac{\partial u}{\partial t} = x^2 - \left(\frac{1}{4}\right) \left(\frac{\partial u}{\partial x}\right)^2 \tag{44}$$

with the following condition:

$$u(x, 0) = 0 \tag{45}$$

### 4.2.1. Using He-Laplace Method

By applying the Laplace transform method subject to the initial condition, we have

$$y(x, s) = \frac{L[x^2]}{s} - \frac{1}{4s} L[u_x^2] \tag{46}$$

The inverse of the Laplace transform implies that

$$u(x, t) = x^2 t - L^{-1} \left[ \frac{1}{4s} L[u_x^2] \right] \tag{47}$$

Now, we apply the homotopy perturbation method, we have

$$\sum_{n=0}^{\infty} p^n u_n(x, t) = x^2 t - p \left( L^{-1} \left[ \frac{1}{4s} \left\{ L \left[ \sum_{n=0}^{\infty} p^n u_n(u) \right] \right\} \right] \right) \tag{48}$$

where  $H_n(u)$  are He's polynomials. The first few components of He's polynomials are given by

$$\begin{aligned}
 H_0(u) &= u_{0x}^2 = 4x^2 t^2 \\
 H_1(u) &= 2u_{0x} u_{1x} = -\frac{8x^2 t^4}{3} \\
 H_2(u) &= u_{1x}^2 + 2u_{0x} u_{2x} = \frac{2x^2 t^3}{9} + \frac{16x^2 t^6}{15}
 \end{aligned} \tag{49}$$

Comparing the coefficient of like powers of  $p$  we have

$$\begin{aligned}
 p^0 : u_0(x, t) &= x^2 t \\
 p^1 : u_1(x, t) &= -L^{-1} \left[ \frac{1}{4s} \{ L[H_0(u)] \} \right] = \frac{x^2 t^3}{3} \\
 p^2 : u_2(x, t) &= -L^{-1} \left[ \frac{1}{4s} \{ L[H_1(u)] \} \right] = \frac{2x^2 t^5}{15}
 \end{aligned} \tag{50}$$

Proceeding in a similar manner, we have

$$p^3 : u_3(x, t) = -\frac{17x^2t^7}{315}$$

$$\vdots$$

So that the solution  $u(x, t)$  is given by

$$u(x, t) = u_0 + u_1 + u_2 + u_3 + \dots = x^2t - \frac{x^2t^3}{3} + \frac{2x^2t^5}{15} - \frac{17x^2t^7}{315} + \dots \quad (51)$$

#### 4.2.2. Adomain Decomposition Method

We first rewrite Equation (44) in an operator  $L$  is

$$L_t u = x^2 - \frac{1}{4}(L_x u)^2 \quad (52)$$

$$u(x, 0) = 0$$

where the differential operators are define as;

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot), \quad L_x(\cdot) = \frac{\partial}{\partial x}(\cdot)$$

And the inverse operator  $L_t^{-1}$ , provided that it exists, is defined as:

$$L_t^{-1}(\cdot) = \int_0^t (\cdot) dt \quad (53)$$

Applying the inverse operator on both the sides of (52) and using the initial condition, yields:

$$L_t^{-1}(L_t u) = L_t^{-1}(x^2) - \frac{1}{4} L_t^{-1}(L_x u)^2$$

$$u(x, t) = x^2t - \frac{1}{4} L_t^{-1}(L_x u)^2 \quad (54)$$

Now, we decompose the unknown function  $u(x, t)$  as a sum of components defined by the series (22):

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) \quad (55)$$

where  $u_0$  is identified as  $u(x; 0)$ . The components  $u_n(x, t)$  are obtained by the recursive formula:

$$\sum_{n=0}^{\infty} u_n(x, t) = x^2t - \frac{1}{4} L_t^{-1} \left( L_x \left( \sum_{n=0}^{\infty} u_n(x, t) \right) \right)^2 \quad (56)$$

Or

$$u_0(x, t) = x^2t \quad (57)$$

$$u_{k+1}(x, t) = -\frac{1}{4} L_t^{-1} \left( L_x (u_k(x, t)) \right)^2, \quad k \geq 0 \quad (58)$$

We note that the recursive relationship is constructed on the basis that the component  $u_0(x, t)$  is defined by all terms that arise from the initial condition and from integrating the source term. The remaining components  $u_k(x, t)$ , can be completely determined recursively.

Accordingly, considering the first few terms, Equations (14) and (15) give:



$$\begin{aligned}
u_0 &= x^2 t \\
u_1 &= -\frac{1}{4} L_t^{-1} \left( L_x \left( u_0(x, t) \right) \right)^2 = -\frac{1}{4} L_t^{-1} \left( L_x \left( x^2 t \right) \right) = -\frac{x^2 t^2}{3} \\
u_2 &= -\frac{1}{4} L_t^{-1} \left( L_x \left( u_2(x, t) \right) \right)^2 = -\frac{x^2 t^7}{63} \\
&\vdots
\end{aligned}$$

Finally, using (55) we obtain the solution in series form:

$$u(x, t) = u_0 + u_1 + u_2 + u_3 + \dots \quad (59)$$

That is:

$$u(x, t) = x^2 t - \frac{x^2 t^3}{3} - \frac{x^2 t^7}{63} + \dots \quad (60)$$

### 4.3. Example 3

Consider the following first order nonlinear differential equation [19]

$$y' + y^2 = 0, \quad y \geq 0 \quad (61)$$

With the following condition:

$$y(0) = 1 \quad (62)$$

#### 4.3.1. Using He-Laplace Method

By applying the aforesaid method subject to the initial condition, we have

$$L[y(x)] = \frac{1}{s} - \frac{1}{s} L[y^2] \quad (63)$$

The inverse of Laplace transform implies that

$$y(x) = 1 - L^{-1} \left[ \frac{1}{s} L[y^2] \right] \quad (64)$$

Now we apply the homotopy perturbation method, we have

$$\sum_{n=0}^{\infty} p^n y_n(x) = 1 - p \left( L^{-1} \left[ \frac{1}{s} L \left[ \sum_{n=0}^{\infty} p^n H_n(y) \right] \right] \right) \quad (65)$$

where  $H_n(y)$  are He's polynomials. The first few components of He's polynomials are given by

$$\begin{aligned}
H_0(y) &= y_0^2 = 1 \\
H_1(y) &= 2y_0 y_1 = -2x \\
H_2(y) &= y_1^2 + 2y_0 y_2 = 3x^2
\end{aligned} \quad (66)$$

Comparing the coefficient of like powers of  $p$ , we have

$$\begin{aligned}
p^0 : y_0(x) &= 1 \\
p^1 : y_1(x) &= -L^{-1} \left[ \frac{1}{s} \{ L[H_0(y)] \} \right] = -x \\
p^2 : y_2(x) &= -L^{-1} \left[ \frac{1}{s} \{ L[H_1(y)] \} \right] = x^2 \\
p^3 : y_3(x) &= -L^{-1} \left[ \frac{1}{s} \{ L[H_2(y)] \} \right] = -x^3 \\
&\vdots
\end{aligned} \quad (67)$$

So that the solution  $y(x)$  is given by

$$\begin{aligned} y(x) &= y_0 + y_1 + y_2 + y_3 + \cdots \\ y(x) &= 1 - x + x^2 - x^3 + \cdots \end{aligned} \quad (68)$$

which is converging to  $\left(\frac{1}{1+x}\right)$  i.e. exact solution.

#### 4.3.2. Adomain Decomposition Method

We first rewrite Equation (61) in an operator  $L$  is

$$\begin{aligned} Ly &= -y^2 \\ y(0) &= 1 \end{aligned} \quad (69)$$

where the differential operators are define as;

$$L(\cdot) = \frac{\partial}{\partial t}(\cdot) \quad (70)$$

And the inverse operator  $L^{-1}$  provided that it exists, is defined as

$$L^{-1}(\cdot) = \int_0^t (\cdot) dt \quad (71)$$

Applying the inverse operator on both the sides of (69) and using the initial condition yields:

$$\begin{aligned} L^{-1}(Ly) &= L^{-1}(-y^2) \\ y(x) &= 1 - L^{-1}(y^2) \end{aligned} \quad (72)$$

Now, we decompose the unknown function  $y(t)$  as a sum of components defined by the series (22):

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \quad (73)$$

where  $y_0$  is identified as  $y(0)$ . The components  $y_k(t)$  are obtained by the recursive formula:

$$\sum_{n=0}^{\infty} y_n(t) = 1 - L^{-1} \left( \left( \sum_{n=0}^{\infty} y_n^2 \right) \right) \quad (74)$$

Or

$$\begin{aligned} y_0 &= 1 \\ y_{k+1}(t) &= -L^{-1}(y_k^2) \end{aligned} \quad (75)$$

We note that the recursive relationship is constructed on the basis that the component  $y_0(t)$  is defined by all terms that arise from the initial condition and from integrating the source term. The remaining components  $y_k(t)$ , can be completely determined recursively.

Accordingly, considering the first few terms, Equations (72) and (73) give:

$$\begin{aligned} y_0 &= 1 \\ y_1(t) &= -L^{-1}(y_0^2) = -L^{-1}(1) = -t \\ y_2(t) &= -L^{-1}(y_1^2) = -L^{-1}(t^2) = -\frac{t^3}{3} \\ y_3(t) &= -L^{-1}(y_2^2) = -L^{-1}\left(\frac{t^6}{9}\right) = -\frac{t^7}{63} \\ &\vdots \end{aligned}$$

Finally, using (55) we obtain the solution in series form:

$$y(x) = y_0 + y_1 + y_2 + y_3 + \dots \quad (76)$$

That is:

$$y(x) = 1 - t - \frac{t^3}{3} - \frac{t^7}{63}$$

## 5. Discussions

The main goal of this work is to conduct a comparative study between Adomian decomposition method and the He-Laplace method. The two methods are powerful and efficient methods that both give approximations of higher accuracy and closed form solutions if existing.

An important conclusion can be made here. Adomian decomposition method for solving nonlinear ordinary and partial differential equations, the same problems are solved by He-Laplace method. Adomian decomposition method provides the components of exact solution, where these components should follow the summation given in (22). However, He-Laplace is an elegant combination of the Laplace transformation, the homotopy perturbation method and He's polynomials. Moreover, the ADM requires the evaluation of the Adomian polynomial that mostly require tedious algebraic calculations. The ADM provides the solution in successive components that will be added to get the series solution.

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