Study on Forming Shoulders with Butterfly section in Packaging Machines

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Abstract: The mathematical description of a butterfly section shoulder is presented and the equations of the bending curve, the shoulder curved surface and the fringe curve are developed. To confirm the validity of the mathematical description, a 3-D shoulder is modeled on computer, and an actual shoulder is manufactured by rapid manufacturing machine, and a practical shoulder is made by traditional method with steel sheet. The result established theoretical basis for making 'self-standing bag of four edges' on the 'form, fill and seal' machines.

Keywords: Forming shoulders of Packaging Machines, Bending curve, Shoulder Curved Surface, Developable Surface, Fringe Curve, Butterfly Section.

1. Introduction

Forming shoulder is the key part of the form, fill and seal' machines. People have studied it for decades. Mot E. and Boersma J presented the mathematical modeling for the bending curve of forming shoulders [1,2]. Along with the development of modern technologies and the ever increasing needs for packaging machines, the studying on forming shoulder is also continually getting progresses. C.J McPherson has studied the essential parameters of shoulder to the shoulder's influence in the foundation that the predecessor studied [3], and discussed a design method of forming shoulder with the section made up of the straight line and circular arcs [4]. The computer aided designing let the bending curve easer to be made, and resulted on the shoulders with multiform section been designed [5,6,7,8]. The multiform section shoulders enriched the typed of bag which 'form, fill and seal' machines could adapt, and enriched the packaging object and packaging form of this type of packaging machinery. Simultaneously, the mathematical description on forming shoulder is expended from bending curve to shoulder curved surface and fringe curve. It is for sure that the development of three-dimensional CAD software and modern manufacture technology will establish the three-dimensional mathematical modeling in the computer according to these mathematical modeling[9]. These mathematical models could process with the tools of digitally controlled machine, which changed the traditional designed method depending on two-dimensional mathematical[1,10] model and then laying out by human being to improve the quality of forming shoulders of packaging machines and then proceed to the next step to improve the quality of 'form, fill and seal' machines.

This paper concerns the design of forming Shoulders with Butterfly section. The paper beginning with the geometric parameters of forming shoulders and butterfly section, then give the mathematical models. Chapter 4 presents the mathematical model of bending curve. Chapter 5 and 6 presents the mathematical models of shoulder curved surface and fringe curve of shoulder with butterfly section. Those mathematical models can be used to set up three-dimensional digital model of shoulder on computer, an actual model is manufactured by rapid manufacturing machine directly from digital model. Chapter 7 describes the planer curves, which needed in traditional method to make shoulder with steel sheet. Finally, some important conclusions are presented.

2. The basic principle of forming shoulders

As showing in figure 1, the film is transported under tension to the shoulder along the guide roll, folds along the outside surface and goes down along the inner tube surface. When two edges are lengthways sealed the film becomes tubular. Meanwhile the dispersive material is filled into bag through the tube. Then the package is made after it has been transverse sealed. The shape of the cross-section of the vertical tube and the way of sealing determine the shape of package. When the cross-section of tube is in the shape of a butterfly and some kind of sealing is used, a 'self-standing bag of four edges' is made. The shape of the shoulder should be such that the packaging material transports without stretching or tearing. It should be a developable surface.

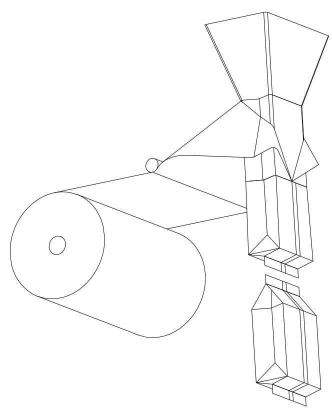

Figure1 Typical form, fill-and-seal operation with butterfly section

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3. The geometric parameters of forming shoulders and butterfly section.

In figure 2, a coordinate system, Oxyz, is chosen and the z axis is at the center of the center of the tube. The shoulder surface of forming shoulder with butterfly section is composed of a trapezoid planer (AA’B’B) and other surfaces. The intersection that shoulder surface with the wall of tube called the bending curve.

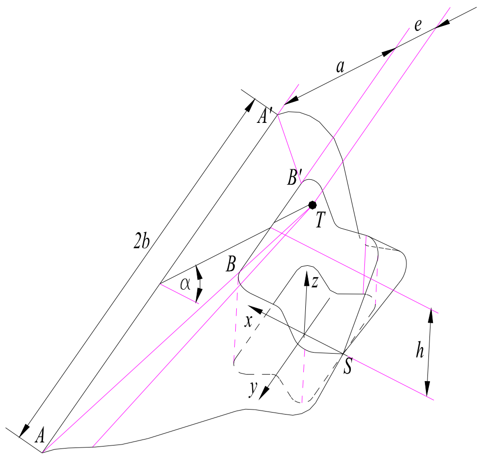

Figure 2. Forming shoulder with butterfly section.

The tube with butterfly section is composed of six planers and six column surfaces, and its bending curve is composed of thirteen parts curves accordingly when the lengthways sealing is located as show in figure 1.

Figure 3 shows the intersection of the butterfly section tube and the coordinate plane, xoy, composed of six straight lines Li (i =1,2,3,4,5,6) and six arcs Gi (i =1,2,3,4,5,6). Among those lines and arcs, L1 and L4 are in same length, L2, L3, L5 and L6 are in same length, G2 and G5 are in same radius, G1,G2,G4 and G6 are in same radius. We denote the parts of bending curve SBB’S in Figure 2 by Λi and $\Gamma i (i = 1,2,3,4,5,6)$, witch correspond to the straight lines , Liand arcs Gi in Figure 2, respectively. All lines and arcs can be determined by a set of parameters.

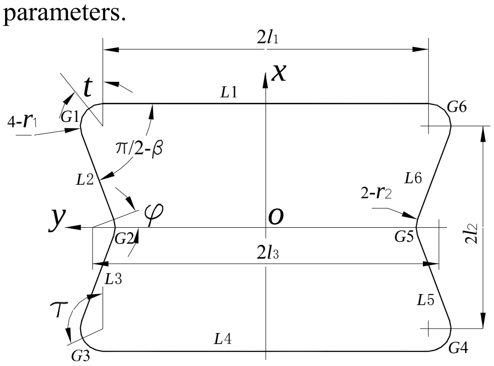

Figure 3. Butterfly Section.

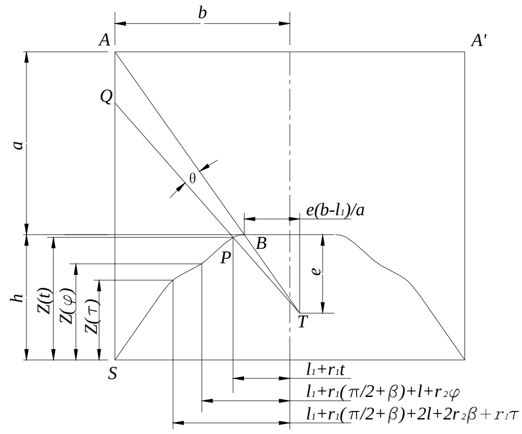

Figure 4. Developed shoulder with butterfly section.

3.1. The geometry parameter of butterfly section.

In Figure 3, by given the primordial geometry parameters of butterfly section: l_1, l_2, l_3, r_1, r_2, the shape and size of section is determined exactly. Consider one half of the shoulder due to symmetry. When the 3-D model of the shoulder is made on computer, the straight line Λi can be obtained directly because they are actually the connecting lines of space curves Γi. So, we only give the mathematical description of the space curves Γi in this paper. Consider the part of y $\geqslant$ 0 in Figure 3. Using the primordial geometry parameters, the arcs G1, G2 and G3 can be described by:

$$\textbf{G1:}\ \begin{cases} x = l_2 + r_1 \cos\ t \\ y = l_1 + r_1 \sin\ t \end{cases} \quad (0 \le t \le \frac{\pi}{2} + \beta) \quad (1)$$

Where t is the angle between the x axis and the line from the centre of the arc G1.

$$\text{G2:}\ \begin{cases} x = -r_2 \sin\ \varphi \\ y = l_3 - r_2 \cos\ \varphi \end{cases} \quad (-\beta \le \varphi \le \beta) \quad (2)$$

Where φ is the angle between the y axis and the line from the centre of the arc G2.

$$\text{G3:}\ \begin{cases} x = -l_2 + r_1 \cos\ \tau \\ y = l_1 + r_1 \sin\ \tau \end{cases} \quad (\pi/2 - \beta \le \tau \le \pi) \quad (3)$$

Where τ is the angle between the x axis and the line from the centre of the arc G3.

In (1), (2) and (3), the parameter β is determined by primordial geometry parameters, it can be expressed as:

$$\tan\beta = \frac{-2l_2(l_1 - l_3) + \sqrt{(2l_2(l_1 - l_3))^2 - 4((r_1 + r_2)^2 - l_2{}^2)((r_1 + r_2)^2 - (l_1 - l_3)^2)}}{2((r_1 + r_2)^2 - l_2{}^2)} \quad (4)$$

In figure 4, width parameter b is determined by primordial geometry parameters. It will be used in the paper lately, and can be expressed as:

$$b = 2l_1 + \frac{2l_2}{\cos\ \beta} - (r_1 + r_2)\tan\ \beta + 2r_1(\pi/2 + \beta) + 2r_2\beta \quad (5)$$

3.2. The structure parameters of the forming shoulders

The section parameters are not fully identified the form of the shoulder. The shoulder has some parameters that are independent of the section geometric parameters. They are α , a e in Figure 1. α is the gradient angle of the back planer AA'B'B of the shoulder to the *xoy* plane. a is the height of the isosceles trapezoid of the planer AA'B'B . e is the distance of *B'B* to point T , which is located on the extending line of AB. Taking different positions for T, there are different values for e , and will result in somewhat different shapes of shoulder[8]. As for the utterfly section, we believe that the T point at *yoz* plane is appropriate. Extending AB to *yoz* plane to get the point T, the coordinates of T can be written as:

$$T(x.y.z) = (0, l_1 - e\frac{b - l_1}{a}, h + e\sin\alpha) \qquad (6)$$

In which:

$$e = \frac{l_2 + r_1}{\cos\alpha} \qquad (7)$$

There is a parameter h in (6). That is the height of the shoulder(see Figure 4). In Figure 2, S point is the lowest point of the bending curve that lies in x-axis. According to figure 1, based on two-point distance formula, we can get the distance between the two points as:

$$|TS| = \sqrt{(l_2 + r_1)^2 + (l_1 - e\frac{b - l_1}{a})^2 + (h + e\sin\alpha)^2} \qquad (8)$$

In figure 4, there is:

$$|TS| = \sqrt{(h - e)^2 + (b + e\frac{b - l_1}{a} - l_1)^2} \qquad (9)$$

Using (7) and (8) we obtain:

$$h = \frac{(l_2 + r_1)^2 - e^2\cos^2\alpha - (b + 2e(b - l_1)/a - 2l_1)b}{-2e(1 + \sin\alpha)} \qquad (10)$$

Such is the relationship between parameters of height h and primordial geometric parameters l_1, l_2, l_3, r_1, r_2 , and tructure parameter of the shoulder α , a and e . Of course, we can determine the height firstly in the design, but in that case, there must be one parameter in eight parameters depend on calculated.

4. Mathematical model of bending curve of butterfly section shoulder

To develop the mathematical models of space curves $\Gamma 1$, $\Gamma 2$ and $\Gamma 3$ correspond to G1, G2 and G3, analytic geometry knowledge is used. Making a line from point T to arbitrary point PGi on space curves Γi (Figure 2 and Figure 4), we can get the space distance $|TP_{Gi}|$. Note that the point T is at the same plane with the planer AA' B' B, and that the distance $|TP_{Gi}|$ is not change after the shoulder is developed.

4.1. Mathematical description of the space curve $\Gamma 1$

The coordinate of the arbitrary point PG1 in the bending curve that is corresponding to the G1 segment for:

$$(x, y, z) = [(l_2 + r_1\cos t), (l_1 + r_1\sin t), z(t)] \qquad (11)$$

Let $f(t) = TP_{G1}$, then:

$$f(t) = \sqrt{(l_2 + r_1\cos t)^2 + (r_1\sin t + e\frac{b - l_1}{a})^2 + [z(t) - (h + e\sin\alpha)]^2} \qquad (12)$$

In Figure 4, there is::

$$z(t) = h - e + \sqrt{(f(t))^2 - (e\frac{b - l_1}{a} + r_1 t)^2} \qquad (13)$$

Using (8) and (9) we obtain:

$$z(t) = \frac{-(h - e)^2 + (l_2 + r_1\cos t)^2 + (r_1\sin t + e\frac{b - l_1}{a})^2 + (h + e\sin\alpha)^2 - (e\frac{b - l_1}{a} + r_1 t)^2}{2e(1 + \sin\alpha)} \qquad (14)$$

Thus, the of $\Gamma 1$ can be described by:

$$\begin{cases} x = l_2 + r_1\cos t \\ y = l_1 + r_1\sin t \\ z(t) = \dfrac{(l_2 + r_1\cos t)^2 + (r_1\sin t + e\frac{b - l_1}{a})^2 + (h + e\sin\alpha)^2 - (e\frac{b - l_1}{a} + r_1 t)^2 - (h - e)^2}{2e(1 + \sin\alpha)} \end{cases}$$

$$(0 \le t \le \frac{\pi}{2} + \beta) \qquad (15)$$

4.2. Mathematical description of the space curve $\Gamma 2$

The coordinates of an arbitrary point P G2 on bending curve $\Gamma 2$ that is corresponding to the G2 segment can be written as:

$$(x, y, z) = [(-r_2\sin\varphi), (l_3 - r_2\cos\varphi), z(\varphi)] \qquad (16)$$

Let $f(\varphi) = TP_{G2}$, where:

$$f(\varphi) = \sqrt{(r_2\sin\varphi)^2 + (l_3 - l_1 + e\frac{b - l_1}{a} - r_2\cos\varphi)^2 + (z(\varphi) - (h + e\sin\alpha))^2} \qquad (17)$$

In Figure 4, there is:

$$z(\varphi) = h - e + \sqrt{(f(\varphi))^2 - (r_1(\pi/2 + \beta) + l + r_2(\beta + \varphi) + e\frac{b - l_1}{a})^2}$$

$$(-\beta \le \varphi \le \beta) \qquad (18)$$

Using (16) and (17) we obtain:

$$z(\varphi) = \frac{(r_2\sin\varphi)^2 + (l_3 - l_1 + e\frac{b - l_1}{a} - r_2\cos\varphi)^2 - (r_1(\pi/2 + \beta) + l + r_2(\beta + \varphi) + e\frac{b - l_1}{a})^2 + 2he(1 + \sin\alpha) - e^2\cos^2\alpha}{2e(1 + \sin\alpha)}$$

$$(-\beta \le \varphi \le \beta) \qquad (19)$$

$$\begin{cases} x = -r_2\sin\varphi \\ y = l_3 - r_2\cos\varphi \\ z(\varphi) = \dfrac{(r_2\sin\varphi)^2 + (l_3 - l_1 + e\frac{b - l_1}{a} - r_2\cos\varphi)^2 + (h + e\sin\alpha)^2 - (r_1(\pi/2 + \beta) + l + r_2(\beta + \varphi) + e\frac{b - l_1}{a})^2 - (h - e)^2}{2e(1 + \sin\alpha)} \end{cases}$$

$$(-\beta \le \varphi \le \beta) \qquad (20)$$

4.3. Mathematical description of the space curve $\Gamma 3$

The coordinates of an arbitrary point P G3 on bending curve G3 that is corresponding to the G3 segment can be written as:

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$$(x, y, z) = [-(l_2 - r_1 \cos\tau), (l_1 + r_1 \sin\tau), z(\tau)] \quad (21)$$

Then the distance between point T and P G3 is $f(\tau)$:

$$f(\tau) = \sqrt{(-l_2 + r_1\cos\tau)^2 + (r_1\sin\tau + e\frac{b-l_1}{a})^2 + (z(\tau) - (h + e\sin\alpha))^2} \quad (22)$$

In Figure 4, there is:

$$z(\tau) = h - e - \sqrt{(f(\tau))^2 - (e\frac{b-l_1}{a} + 2l + r_1(\pi/2+\beta) + 2r_2\beta + r_1\tau)^2} \quad (23)$$

Using (22) and (23) we obtain:

$$z(\tau) = \frac{(l_2 - r_1\cos\tau)^2 + (r_1\sin\tau + e\frac{b-l_1}{a})^2 + (h+e\sin\alpha)^2 - (e\frac{b-l_1}{a} + 2l + r_1(\pi/2+\beta) + 2r_2\beta + r_1\tau)^2 - (h-e)^2}{2e(1+\sin\alpha)} \quad (24)$$

$$\begin{cases} x = -l_2 + r_1\cos\tau \\ y = l_1 + r_1\sin\tau \\ z(\tau) = \dfrac{(l_2 - r_1\cos\tau)^2 + (r_1\sin\tau + e\frac{b-l_1}{a})^2 + (h+e\sin\alpha)^2 - (e\frac{b-l_1}{a} + 2l + r_1(\pi/2+\beta) + 2r_2\beta + r_1\tau)^2 - (h-e)^2}{2e(1+\sin\alpha)} \end{cases}$$

$$(\beta \le \tau \le \pi) \quad (25)$$

5. Mathematical model of shoulder curved surface with butterfly section

The shoulder curved surface must be a developable surface according to the principle of shoulder design. When it is developed, its edges must match the edges of the developed tube. Differential geometry[12] prescribes that the cone is developable, it can be generated by a line moving continually along the directrixe and through a fixed point.

Note that the distance from the space point T to the point P on the bending curve is the length $|TP_{Gi}|$ f , which isn't change when it unfold to the plane. When the surface made by the lines from the space point T to the point P on the bending curve BS is developed, it is the plane BTS with edge BS. Now we extend the surface along TB and TS to cover the area ABS. According to the definition of a cone[12,13] and the bending curve equations obtained above, we can make certain that the shoulder with butterfly section curved surface at y ≥ 0 is composed of seven cone pieces, with T as the apex and seven bending curves as directrixes. Because L1、L2、L3 and L4 are all lines, so in these seven cones, four cones with L1、L2、L3 andL4 as directrixes are planar.

The equation for the plane is simple, we now study only the equation of three cones with G1、G2 and G3 as directrixes, which is shown in (15),(20) and (25)

Let F(X, Y, Z) be an arbitrary point on the shoulder curved surface, the cone with G1 as directrix can be described by:

$$\overrightarrow{OF} = \overrightarrow{OT} + \overrightarrow{TF} = \overrightarrow{OT} + u\overrightarrow{TP} \qquad (u \geq 1) \qquad (26)$$

The point P is on the bending curve G1, and its position is determined by (12). Rewriting (26), we obtain the equation of the cone with G1 as directrix as:

$$\begin{cases} X = [l_2 + r_1\cos t]u \\ Y = l_1 - e\frac{b-l_1}{a} + [r_1\sin t + e\frac{b-l_1}{a}]u \\ Z = h + e\sin\alpha + [z(t) - h - e\sin\alpha]u \end{cases} \quad (0 \le t \le \frac{\pi}{2} + \beta)$$

$$(u \ge 1) \quad (27)$$

Where z(t) is described by (14). Using the same method, we can obtain the equation of the fringe cone with G2 as directrix:

$$\begin{cases} X = [-r_2\sin\varphi]w \\ Y = l_1 - e\frac{b-l_1}{a} + [l_3 - r_2\cos\varphi - l_1 + e\frac{b-l_1}{a}]w \\ Z = h + e\sin\alpha + [z(\varphi) - h - e\sin\alpha]w \end{cases} \quad (-\beta \le \varphi \le \beta) \quad (28)$$

where $z(\varphi)$ is described by (19). Also we can obtain the equation of the fringe cone with G3 as directrix:

$$\begin{cases} X = [-l_2 + r_1\cos\tau]v \\ Y = l_1 - e\frac{b-l_1}{a} + [r_1\sin\tau + e\frac{b-l_1}{a}]v \\ Z = h + e\sin\alpha + [z(\tau) - h - e\sin\alpha]v \end{cases} \quad (\beta \le \tau \le \pi)\ (v \ge 1) \quad (29)$$

Where $z(\tau)$ is expressed in (24)

6. Mathematical model of Fringe curves

Cones can be extended infinitely: any real shoulder is a part of some cones. To build the model of a forming shoulder on a computer we must obtain the conical surface of the boundary curve. Here, we also need only find the boundary curve of the shoulder surface that directrix is not a straight line.

In the area on the developed shoulder in area $(0 \le \varphi \le \delta)$, extending line TPG1 to the point Q on the boundary AS, we denote the angle between line TQ and TA by $\theta(t)$, and let $|BP| = s(t)$, $|TQ| = m(t)$ .From the geometrical relations we obtain the following equations:

$$s(t) = \sqrt{(h - z(t))^2 + (r_1 t)^2} \quad (30)$$

$$\theta(t) = \arccos[\frac{f(t)^2 + e^2(1 + ((b-l_1)/a)^2) - (h - z(t)) - (r_1 t)^2}{2e\sqrt{1 + ((b-l_1)/a)^2}\, f(t)}] \quad (31)$$

Where $f(t)$ is expressed in (12) and $z(t)$ is expressed in (14), The length of $|TQ|$ can be written as

$$m(t) = \frac{b - l_1 + e(b - l_1)/a}{\sin[\arctan((b - l_1)/a) + \theta(t)]} \quad (32)$$

Denoting the coordinates of the point Q by (ξ, η, ς), which satisfies (27), we obtain:

$$\begin{cases} \xi = [l_2 + r_1\cos t]u \\ \eta = l_1 - e\frac{b-l_1}{a} + [r_1\sin t + e\frac{b-l_1}{a}]u \\ \varsigma = h + e\sin\alpha + [z(t) - h - e\sin\alpha]u \end{cases}$$

$$(0 \le t \le \frac{\pi}{2} + \beta)\ (u \ge 1) \quad (33)$$

The distance between the two points Q and T leads the following equation:

$$m(t)=\left|\overline{QT}\right|=\sqrt{\xi^2+(\eta-l_1+e\frac{b-l_1}{a})^2+[\zeta-(e\sin\alpha+h)]^2} \quad (34)$$

From (32)、(33) and (34) we obtain:

$$u=\frac{b-l_1+e(b-l_1)/a}{\sin(\arctan((b-l_1)/a)+\theta(t))f(t)} \quad (35)$$

Rewriting u in (35) as u(t), then according to (33), we can obtain the mathematical model of fringe cone with curve $\Gamma 1$ as directrix:

$$\begin{cases}\xi=[l_2+r_1\cos t]u(t)\\ \eta=l_1+e\dfrac{b-l_1}{a}+[r_1\sin t+e\dfrac{b-l_1}{a}]u(t)\\ \zeta=h+e\sin\alpha+[z(t)-h-e\sin\alpha]u(t)\end{cases}$$

$$(0\le t\le\frac{\pi}{2}+\beta) \quad (36)$$

Using the same method, the equation of the cone with G2 and G3 as directrix can be obtained. The difference is that the angle $\theta(t)$ is to be $\theta(\varphi)$ and $\theta(\tau)$ corresponding in G2 and the G3.

For G2 segment:

$$s(\varphi)=\sqrt{(h-z(\varphi))^2+(r_1(\pi/2+\beta)+l+r_2\varphi)^2} \quad (37)$$

$$m(\varphi)=\frac{b-l_1+e(b-l_1)/a}{\sin[\arctan((b-l_1)/a)+\theta(t)]} \quad (38)$$

$$\begin{cases}\xi=[-r_2\sin\varphi]w(\varphi)\\ \eta=l_1-e\dfrac{b-l_1}{a}+[l_3-r_2\cos\varphi-l_1+e\dfrac{b-l_1}{a}]w(\varphi)\\ \zeta=h+e\sin\alpha+[z(\varphi)-h-e\sin\alpha]w(\varphi)\end{cases}$$

$$(-\beta\le\varphi\le\beta) \quad (39)$$

$$m(\varphi)=\left|\overline{QT}\right|=\sqrt{\xi^2+(\eta-l_1+e\frac{b-l_1}{a})^2+[\zeta-(e\sin\alpha+h)]^2} \quad (40)$$

Where:

$$w(\varphi)=\frac{b-l_1+e}{\sin[\arctan((b-l_1)/a)+\theta(\varphi)]} \quad (41)$$

Where $f(\varphi)$ is expressed in (17) and $z(\varphi)$ is expressed in (19), For G3 segment:

$$s(\tau)=\sqrt{[h-z(\tau)]^2+[r_1(\pi/2+\beta)+2l+2r_2\beta+r_2\tau)]^2} \quad (42)$$

$$\theta(\tau)=\arccos[\frac{(e^2+l_1{}^2)-e(h-z(\tau))+l_1r(\pi/2+\tau)+l_1\sqrt{(l_1-l_3)^2+(2l_2)^2}}{\sqrt{e^2+l_1{}^2}f(\tau)}] \quad (43)$$

$$m(\tau)=\left|\overline{RT}\right|=\sqrt{\xi^2+(\eta-l_1+e\frac{b-l_1}{a}0)^2+[\zeta-(e\sin\alpha+h)]^2} \quad (44)$$

$$\begin{cases}\xi=[-l_2+r_1\cos\tau]v(\tau)\\ \eta=l_1+r_1\sin\tau+[r_1\sin\tau+e\dfrac{b-l_1}{a}]v(\tau)\\ \zeta=h+e\sin\alpha+[z(\tau)-h-e\sin\alpha]v(\tau)\end{cases}$$

$$(\beta\le\tau\le\pi) \quad (v\ge1) \quad (45)$$

Where:

$$v(\tau)=\frac{b}{\sin(\arctan((b-l_1)/a)+\theta(\tau))f(\tau)} \quad (46)$$

Where $f(\tau)$ is expressed in (22) and $z(\tau)$ is expressed in (24), According to the mathematical model above, a 3-D model of shoulder with butterfly section is presented in the computer, furthermore, the actual shoulder can be manufactured by digitally controlled machine tools or rapid manufacturing machine. Figure 5 shows a typical 3-D shoulder with butterfly section on a computer, and Figure 6 shows an actual shoulder manufactured by rapid manufacturing machine according to the digital model in Figure 5.

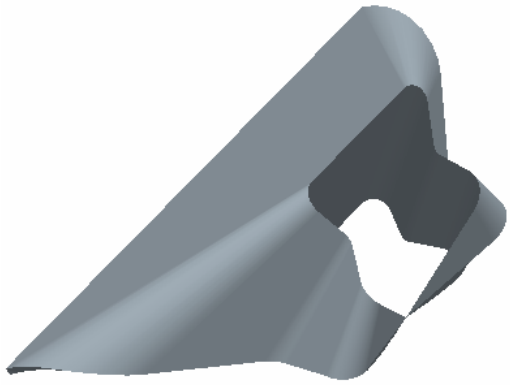

Figure 5. 3-D Model of shoulder with butterfly section.
$(l1=15, l2=8, l3=16, r1=3, r2=2, \alpha=50°, a=60)$

7. The mathematical model of the planar curve

If making forming shoulders by the traditional way that using steel sheet to draw curve then bending, we only need to draw the developed bending curve on steel sheet. Figure 4 is the developed sheet. The curve part $Z(t)$, $Z(\varphi)$, $Z(\tau)$ are confirmed by the corresponding formulary of (15), (20) and (25) respectively. Figure 6 is a shoulder with butterfly section made by this way.

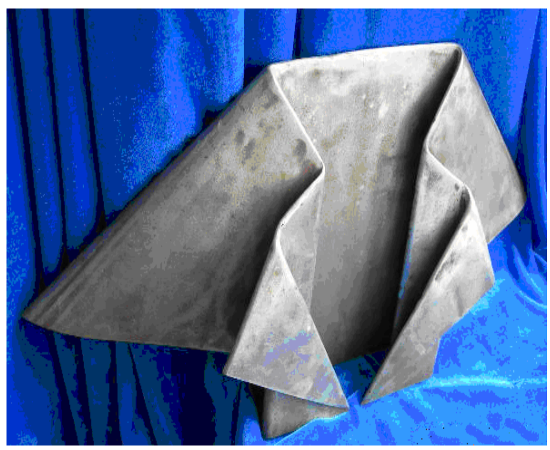

Figure 6. A shoulder with butterfly section made by steel sheet.
$(l1=77, l2=17, l3=42, r1=3, r2=2, \alpha=75°, a=160)$

8. Conclusion

1) The bending curve of forming shoulder with butterfly section is made up of thirteen parts different space curve, the six parts is space curve, the seven parts is spatial straight line.

2) The shoulder curved surface of forming shoulder

with butterfly section is a cone section that takes T as the tip, and takes bending curve as the cone section which is made up of thirteen different parts, while seven parts are space plane.

3) Bending curve, shoulder curved surface and fringe curve could all show with mathematical equation. So we can establish its three-dimensional model in the computer, then we can manufacture the shoulder using modern manufacture technology.

4) The size and form of forming shoulder with butterfly section is decided by section primordial geometrical parameters l_1, l_2, l_3, r_1, r_2 and structure parameters of shoulder α , a , e and. Among these nine parameters, eight parameters decide the else.

5) The position of tip point T is located on the plane yoz in this paper. That is only an especial position. It can be located on other positions. Of cause the equations in this paper will somewhat different accordingly. If the position is best to has been not

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