Simulation Study on the Shock Properties of the Double-Degree-of-Freedom Cushioning Packaging System

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Abstract: According to the mechanics model of the double-degree-of-freedom cushioning packaging system, the paper establishes a mathematics differential equation. It sets up the linear and nonlinear simulation models of shock properties for the double-degree of freedom cushioning packaging system with the principle of Matlab/Simulink. It gives the model parameter values, then plots the acceleration response curves of product and vulnerable parts, and does a comparative analysis on the acceleration response curves of linear and nonlinear systems. The conclusion is that simplifying the nonlinear system to a linear system will induce inadequate packaging.

Keywords: double-degree-of-freedom; cushioning packaging; shock properties; simulink simulation

Products suffer from different forms of damage in the process of logistics. The damage caused by impact and vibration is the most common, and the loss is the most serious, therefore there is a need to study cushioning packaging of products to reduce the force caused by impact and vibration. There is also a need to combine lots of experimentation into researching on cushioning packaging, or it will induce inadequate or over packaging. The products will be damaged easily and the intactness rate will decrease in the case of inadequate packaging, while part of the wrapper will be wasted and the packaging costs will increase when the products are over packaged. As experimentation on real products is usually destructive and expensive, simulation technique can be used to save time, labor and money.

1. Present Research

In the process of logistics, it is hard to describe the process of the shock and the curve of the shock wave with mathematics formula. Therefore there is a need to study the shock properties of products and the more vulnerable parts, such as acceleration, velocity and displacement. At present, there are many simulation studies on the shock properties of cushioning packaging systems.

Yang Hong et al. [1] did simulation study on single-degree-of-freedom cubic nonlinear cushioning system, and plotted the dropping impact curves. Li Xiaoli et al. [2] established the simulation diagram of cushioning packaging system at impact conditions of dropping, plotted the displacement and velocity response curves of linear and cubic nonlinear cushioning systems, and then compared the two types of systems. Liu Xia et al. [3] analyzed the effects of airdrop velocity, cargo materials, and cargo thickness on peak acceleration and pulse duration, and then obtained the experiential formula between the correlative variables from the simulation data. The above researches studied single-degree-of- freedom systems, but the cushioning systems in practice are usually double-degree-of-freedom or multi-degree-of-freedom nonlinear systems, so if the conclusions reached in these studies are applied to real life practice, it will induce error. Wang Chunyan et al. [4] did simulation research on natural frequency of the reasonable distribution of the buffering cushion and the vulnerable parts using Matlab with the solution method of non-decoupling, and obtained two natural frequency optional distribution scopes. But his conclusion can only be applied when the mass of the vulnerable part is much less than that of the product. Sek MA et al. [5] analyzed enhancement of cushioning performance with paperboard crumple inserts. But it failed to do adequate research on the nonlinear systems. Moreover, there is lots of research on the double-degree-of-freedom linear cushioning systems.

Therefore, the cushioning packaging models based on the present research are mostly single-degree-of-freedom models, but the cushioning systems in practice are usually double-degree-of-freedom or multi-degree-of-freedom nonlinear systems. Up to now, there is not any research on the shock proprieties of multi-degree-of-freedom cushioning systems, so this paper will study double-degree-of-freedom cushioning systems.

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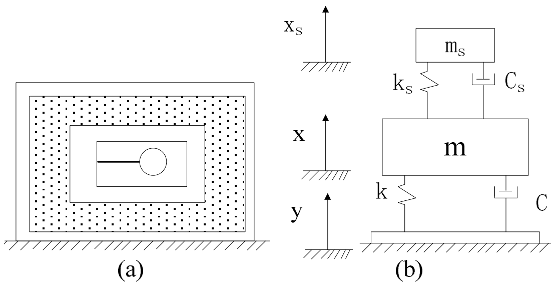

Figure 1. The mechanics model of package system.

2. The Mechanics Model of Cushioning Package Systems

The cushioning package system is the abstract of diversified cushioning package parts, including container used for outer packing, and cushioning pad used as the medium. The theory of cushioning packaging doesn't research on concrete products or packages, but the mechanics model.

The products in the container are called packages, as shown in Fig 1(a). When the products in the container move downwards, upward elasticity will be caused by the cushioning pad below the products. When the products in the container move upwards, downward elasticity will be caused by the cushioning pad above the products. So the function of the cushioning pad is similar to that of a spring. As to those fixed packages, vibration generated by the carrier will be transferred to the products via the container and the cushioning pad, so the function of the container is similar to the bearing force. According to the above analysis, the cushioning pad can be abstracted to the spring, and the container can be abstracted to the bearing of the products-cushioning pad system, and the function of diversified resistance caused by the vibration of the products can be abstracted to the damper, so the mechanics model of cushioning package systems can be obtained, as shown in Fig. 1(b) [6].

In Fig.1, *m* denotes the quality of products; *k* denotes the elasticity constant of the cushioning pad; *C* denotes the resistance coefficient of products-cushioning pad systems, especially the inner resistance of cushioning pad; m_s denotes the quality of vulnerable parts; k_s denotes the elasticity constant of vulnerable parts; C_s denotes the resistance coefficient of vulnerable parts. The balance point of the package is defined as the origin; the package is defined as the y axis; the product is defined as the x axis; the vulnerable part is defined as the x_s axis. The movement of the container is the same with that of the carrier when the vibration of package is analyzed. The *y* axis is the quantity known, and the space point of package can be known when the coordinates of the product and vulnerable parts are got, so the mechanics model of cushioning package systems is the double-degree-of-freedom system.

3. Simulation Analysis on the Shock Properties of Double-Degree-of-Freedom Linear Cushioning Packaging Systems

To analyze the shock properties, the first step is to establish the mathematical model of double-degree-of-freedom liner cushioning packaging systems.

When the cushioning packaging system falls down from the height *h*, its function is equal to the free falling movement from the same height. At the moment of touching the ground, the system is on the same velocity. And the initialization condition is:

$$x(0) = x_s(0) = 0, \dot{x}(0) = \dot{x}_s(0) = \sqrt{2gh} \tag{1}$$

The mathematics equation of package at impact conditions of dropping is:

$$\begin{cases} m\ddot{x} + C(\dot{x} - \dot{y}) + k(x - y) + C_s(\dot{x} - \dot{x}_s) + k_s(x - x_s) = 0 \\ m_s\ddot{x}_s + C_s(\dot{x}_s - \dot{x}) + k_s(x_s - x) = 0 \end{cases} \tag{2}$$

It is obvious that there are force and counterforce between products *m* and vulnerable part m_s, so it is hard to solve the equation in the mathematical method because it is in coupled states, and the equation even can't be solved. Now the equation in coupled states is solved by two-stage estimation method, which simplify the double-degree-of-freedom transport packaging system to two independent single-degree-of-freedom systems, and it will induce error. But it could be solved directly by Matlab/Simulink simulation, and the result will be more accurate. The ground is considered as a concrete body, so $y \equiv 0, \dot{y} \equiv 0$. Then the shock properties of double-degree-of-freedom cushioning packaging systems will be studied by Matlab/Simulink.

Equation (2) is transformed as the following formula in order to establish the Simulink simulation models.

$$\begin{cases} \ddot{x} = -\dfrac{C + C_s}{m}\dot{x} + \dfrac{C_s}{m}\dot{x}_s - \dfrac{k + k_s}{m}x + \dfrac{k_s}{m}x_s \\ \ddot{x}_s = -\dfrac{C_s}{m_s}\dot{x}_s + \dfrac{C_s}{m_s}\dot{x} - \dfrac{k_s}{m_s}x_s + \dfrac{k_s}{m_s}x \end{cases} \tag{3}$$

The simulation models of double-degree-of-freedom cushioning packaging systems are established with the principle of Matlab/Simulink, which could get the shock properties, as shown in Fig. 2.

In Fig. 2, instructe the central simulation module to make the simulation module to be read easily. The model parameter values are given as the following: $m = 80$ kg, $C = 165$ Ns/m, $k = 20000$ N/m, $m_s = 2$ kg, $C_s = 20$ Ns/m, $k_s = 3000$ N/m. And the height h =0.8m.

The ode45 of Runge-Kutta method has lots of advan-

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tages, such as high precision and little error, and it is used commonly, so it is chosen as the arithmetic in the simulation. The simulation time is 40 s, for the system will not change obviously and it will stabilize after 40s. Run the emulators, and then the variables tout and yout of the Matlab workspace are given the simulation results by the output modules of the simulation models during the initialized time, so the results are saved in the Matlab workspace. The acceleration response curves of product and vulnerable parts could be got by the inner function of Matlab. And the acceleration response curves of product and vulnerable parts are plotted together in one chart to compare the shock properties of the product and vulnerable parts easily, which can be seen in Fig. 3.

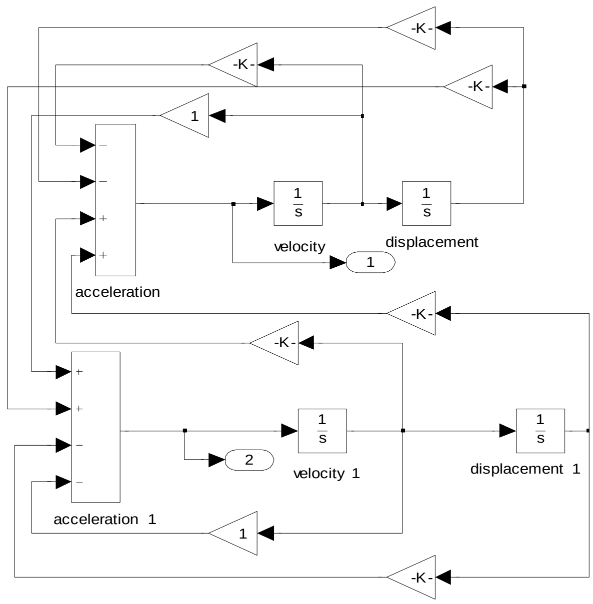

Figure 2. The simulation diagram of shock properties of linear system.

It can be seen from the simulation result that the system stabilizes after 30s and it is periodic obviously. It can be seen from Fig. 3 that it takes 0.3248s for both the acceleration of product and vulnerable parts to reach the maximum, and the maximum of the vulnerable parts acceleration is 29.2042 m/s^2, while that of the product is only 18.7358 m/s^2. So the products always break away from the vulnerable parts, which is coincidence with the conclusion about the product fragility in literature [7].

4. Simulation Analysis on the Shock Properties of Double-Degree-of-Freedom Nonlinear Cushioning Packaging Systems.

The cushioning packaging systems in practice are usually nonlinear systems. The nonlinear characteristic is mainly caused by the nonlinear elasticity of cushioning material or the configuration by packing. The damp of the cushioning material will result in the nonlinear characteristic, but the nonlinear effect caused by damp can always be ignored.

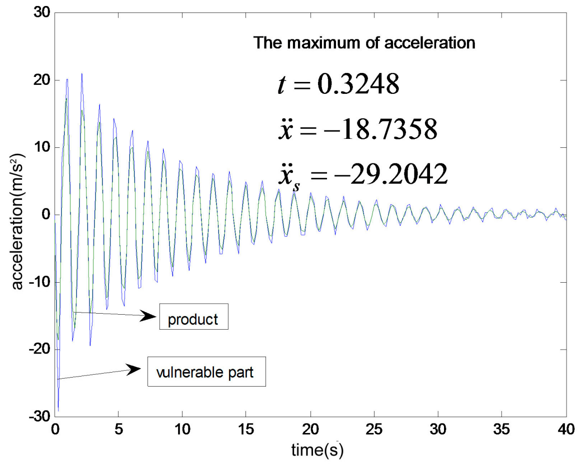

Figure 3. The acceleration response curves of product and vulnerable part （linear system).

The cushioning material is considered as cubic material, so the following formula can be concluded according to the relation between force and deformation.

$$F = kx + rx^3 \tag{4}$$

So the mathematics equation of double-degree-of-freedom cubic cushioning packaging systems is:

$$\begin{cases} m\ddot{x} + C\dot{x} + kx + rx^3 + C_s(\dot{x} - \dot{x}_s) + k_s(x - x_s) = 0 \\ m_s\ddot{x}_s + C_s(\dot{x}_s - \dot{x}) + k_s(x_s - x) = 0 \end{cases} \tag{5}$$

The initialization conditions is:

$$x(0) = x_s(0) = 0, \dot{x}(0) = \dot{x}_s(0) = \sqrt{2gh} \tag{6}$$

The above formula is transformed as the following in order to establish the Simulink simulation models.

$$\begin{cases} \ddot{x} = -\dfrac{C + C_s}{m}\dot{x} + \dfrac{C_s}{m}\dot{x}_s - \dfrac{k + k_s}{m}x + \dfrac{k_s}{m}x_s - \dfrac{r}{m}x^3 \\ \ddot{x}_s = -\dfrac{C_s}{m_s}\dot{x}_s + \dfrac{C_s}{m_s}\dot{x} - \dfrac{k_s}{m_s}x_s + \dfrac{k_s}{m_s}x \end{cases} \tag{7}$$

The simulation models of the mathematics differential equation are established with the principle of Matlab/Simulink, which could get the shock properties, as shown in Fig. 4.

The model parameter values are given as the following: $m = 80$ kg, $C = 165$ N·s/m, $k = 20000$ N/m, $r = 560000$ N/m^3, $m_s = 2$ kg, $C_s = 20$ N·s/m,

$k_s = 3000$ N/m. And the height $h = 0.8$ m.

Though the mathematics equation of double-degree-of-freedom cubic cushioning packaging systems in the process of dropping has an additional module rx^3 than that of linear system, the simulation model of cubic systems is much more complex than that of linear systems. The acceleration response curves of products and vulnerable parts are plotted together in Fig. 5 to compare their shock properties easily.

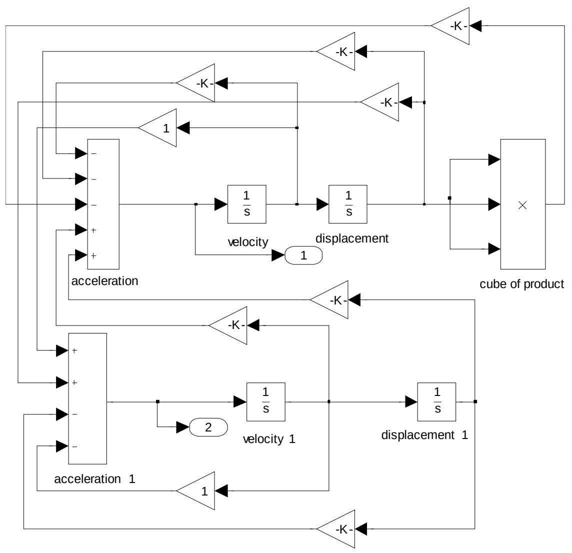

Figure 4. The simulation diagram of shock properties of nonlinear system.

It can be seen from the simulation result that the system get stabilized as the time goes on. The conclusion could be made that the maximum of the vulnerable parts acceleration response is bigger than that of the product, and it coincides with the linear system and the fact, so it is necessary to protect the vulnerable parts with cushioning material. It can be seen from Fig. 5 that it takes 0.1844s for the acceleration of product to reach the maximum, while it takes 0.8236s for the acceleration of vulnerable parts to reach the maximum. That is to say, the vulnerable parts reach the acceleration maximum later than product in the nonlinear systems for the function of the nonlinear chrematistics, and it is the difference between the nonlinear systems and the linear systems.

The acceleration response curves of linear systems and nonlinear systems with the same parameters are compared in Fig. 6 and 7, and the simulation time is 10s.

Two conclusions can be made from the response curves. The first conclusion is the cycle of the nonlinear systems is shorter than that of the linear systems; the second conclusion is that the maximum of the nonlinear

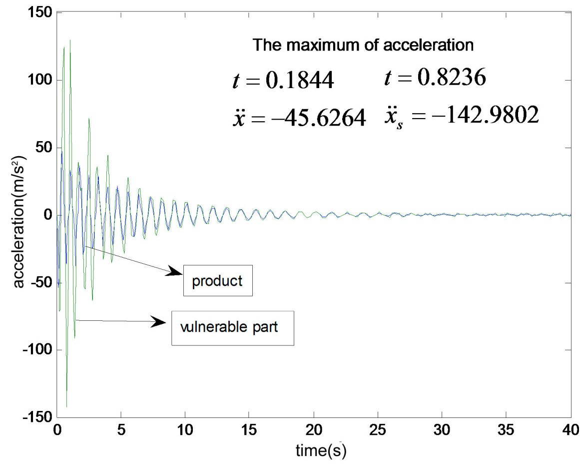

Figure 5. The acceleration response curves of product and vulnerable part（nonlinear system).

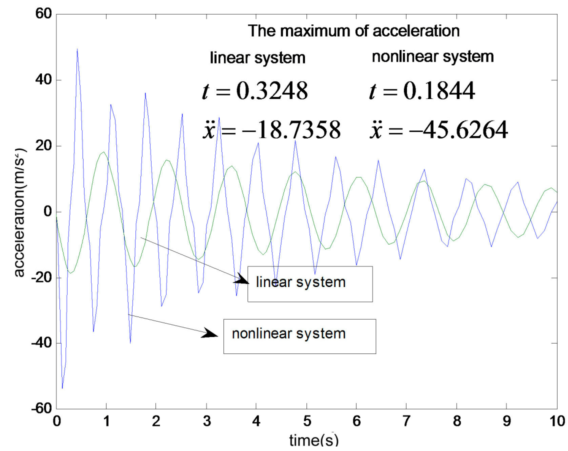

Figure 6. The acceleration response curves of product between linear and nonlinear system.

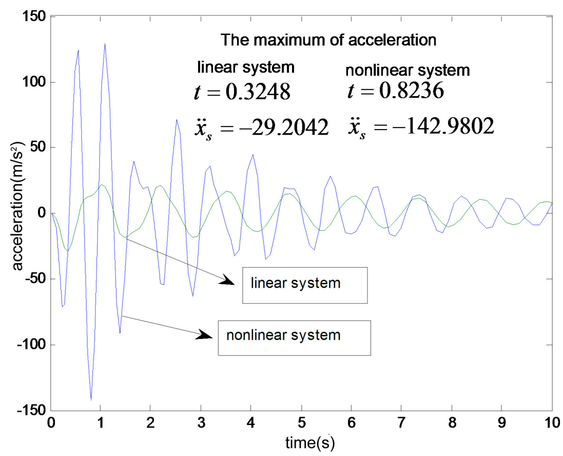

Figure7. The acceleration response curves of vulnerable part between linear and nonlinear system.

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system acceleration response is bigger than that of linear systems. When the nonlinear system is simplified as a linear system, the acceleration response will decrease. If the result is used for designing cushioning packaging systems, it will cause big mistakes, and it could lead to inadequate packaging, and then the intactness rate of products will decrease seriously.

5. Conclusion

This paper does a simulink simulation research on the shock properties of double-degree-of-freedom linear and nonlinear transport packaging systems, and the conclusion can be made that the nonlinear system could not be simplified to linear system in the process of designing cushioning packaging systems, or it will cause inadequate packaging. The shock properties of the multi-degree-of-freedom cushioning packaging systems could also be solved with the same method of double-degree-of-freedom cushioning packaging systems. Though the model is more complex, the basic theories are the same.

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