Application of Nonlinear and Eigenvalue Buckling Analysis in Packaging Test

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Abstract: Both physical and mathematical models of eigenvalue and nonlinear buckling analysis were established based on the mechanics of materials. The buckling strength expressions of nonlinear analysis for simple structure were also derived. Thus a new method combined with nonlinear and eigenvalue buckling analysis was developed after analyzing the advantages and disadvantages of the two buckling analysis methods. Lastly finite element analysis software ANSYS was adopted to simulate the buckling strength of honeycomb paperboard with the former two measures. The results show that: 1) compared with experiments, nonlinear method is superior to eigenvalue one, but the preloaded load in nonlinear analysis is difficult to determine and therefore the combinations of the two make a better result in efficiency and accuracy. 2) ANSYS is an effective tool to carry out nonlinear and eigenvalue buckling analysis for packaging related parts as well as their combination, and the antecedently obtained buckling analysis results can offer valuable references for packaging design, testing and prediction of mechanical properties of packaging related parts.

Keywords: nonlinear buckling analysis; eigenvalue buckling analysis; packaging test; finite element

1. Introduction[*]

Buckling is one the of the most important failure modes of materials and structures, especially in transport packaging. Buckling refers to materials or structure losing their stabilities or balance in nature. And honeycomb board is widely used as a substitute for wood and metal in pallet and packaging chest as a result of its excellent mechanical performance and friendly environmental. Thus, many buckling related studies [1-3] on honeycomb paperboard have been done home and aboard, most of which is experimental based [1,2]. There have also been a number of theoretical explorations including physical and mathematical models on the buckling mechanism and mechanical performance established to explore the buckling mechanism and engineering properties. However, there are numbers of influencing factors which contribute to the destabilizations of this material [4], which brings great difficulties to determine the mechanism and establish reliable mathematical models to predict the mechanical performance.

2. Theoretical Analysis

2.1 Eigenvalue buckling analysis

Eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. For instance, an eigenvalue buckling analysis of a column will match the classical Euler solution. However,

imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength. Thus, eigenvalue buckling analysis often yields un-conservative results, and should generally not be feasible in actual day-to-day engineering analyses. But this method always goes before nonlinear analysis and contributes to the determination of the preload in nonlinear analysis, for it can calculate out the approximate critical load, a little higher than real one. For eigenvalue buckling analysis the following Equation is applied.

$$\left([K] - \lambda_i [P]\right)\{\phi_i\} = 0 \qquad (1)$$

where $[K]$ is the structural rigidity matrix, $[P]$ is the preset load matrix, and $\{\phi_i\}$ represents the buckling mode the structure performs. By solving above equation we can obtain the minimum eigenvalue λ_{min}. Then the critical load of the structure can be written as

$$\{P\}_{cr} = \lambda_{min}\{P\} \qquad (2)$$

2.2 Nonlinear buckling analysis

Nonlinearity is composed of material nonlinearity and geometric nonlinearity as depicted in Fig.1. Material nonlinearity herein implies the nonlinear mechanical properties, like nonlinear elastic modulus (Fig.1 (A)) for instance; while geometric nonlinearity refers to the nonlinear change of unit structure as illustrated in Fig.1 (B). The nonlinear mechanical properties is difficult to applied in practical calculations, as a result, finite element method is frequently adopted by discretizing the nonlinear mechanical properties and applying the load as a stepped or ramped increment form on the object. For the

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simple geometric nonlinearities, analytical solution can be obtained sometimes, and it must introduce other tools to deal with the complex situation.
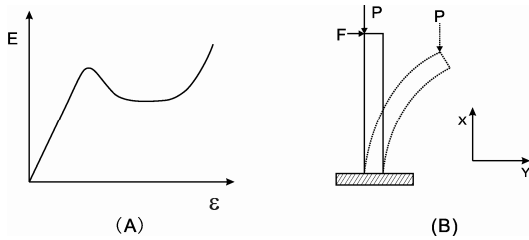

Fig.1 Presentations of (A) material nonlinearity and (B) geometric nonlinearity

And the nonlinear buckling analysis can boil down to solving (3) in the perspective of mathematics.

$$[K]\{D\} = [P] \qquad (3)$$

where $\{D\}$ is the total deformation. In order to solve above equation, the critical load is divided into certain steps of load increments, indicating below.

$$\{P\}_{cr} = \sum_{i=1}^{n} \{\Delta P_i\} \qquad (4)$$

For each load application of $\{\Delta P\}$ the load-deflection curve is linear. Then the total nonlinear process can be superimposed by above mentioned linear ones. That is, the load is applied in a little by little manner, and it can be hold as the critical load when the slope of the load-displacement curve reaches zero, and further load increment will lead to minus slope.

3. Finite Element Analysis of Honeycomb Paper

3.1 Physical model

Honeycomb paperboard is a kind of interlaminar paperboard composed of facial, inner and core layer parts. The core part is the main part to bear the total load, especially on the flat compression condition. Then the buckling behaviors of this structure dominantly focus on the core. For simplification, the unit honeycomb structure was adopted as the subject investigated with the assumption that there exists good adhesion between the three parts. Let the edge length of the core a=0.006 m, and the height h=0.005 m. Then we have the equivalent area per unit core.

$$S_0 = 6 \times \frac{\sqrt{3}a^2}{4} = 0.000054 m^2 \qquad (5)$$

We again propose the thinness of the paper used in facial, inner and core parts are identical with the some thickness of b=0.000269 m, and the mechanical parameters cited from [5] are listed as follows in Table.1.

Table. 1 Mechanical parameters of honeycomb paperboard material

Elastic modulus (MPa)			Poisson's ratio			Shear modulus (MPa)		
Ex	Ey	Ez	Vxy	Vxz	Vyz	Gxy	Gxz	Gyz
7600	4020	38	0.34	0.01	0.01	2140	20	70

3.2 Finite element model

In order to investigate the numerical solution, we discrete the physical model above with shell63 element, producing 2280 nodes. In the finite element model each edge of the unit core was desecreted into 20 equivalent parts. Fig.2 gives a clear illustration of the finite element model of the core.
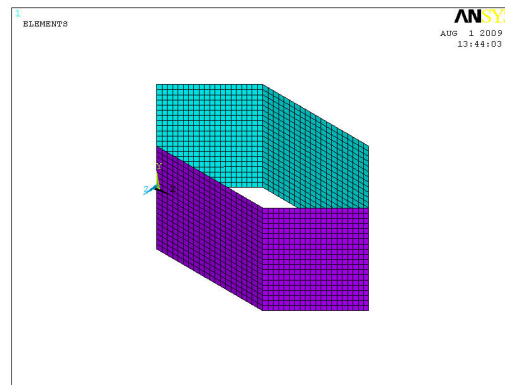

Fig. 2 Finite Element model of honeycomb structure

Before the nonlinear analysis was preceded, eigenvalue analysis was carried out in advance, which can yield an approximate buckling strength value. This value offered a reference for determination of the initial preset pressure value we set for the nonlinear analysis. A load around former obtained value was applied along the height of the core with a small offset force perpendicular to both the vertical load and the edge of the core to induce a perturbation. Switch the 'large deformation' and 'auto time step' in the software option on.

4. Results and Discussions

4.1 Eigenvalue buckling analysis

The unit force applied on the top edge nodes was f_0=1 MN and we propose all the nodes had a good adhesion with facial and inner paperboard. The first order eigenvalue was adopted as the first buckling eigenvalue, for the second or more order stabilization state is difficult to achieve in engineering applications. Fig.3 displays the contours plot of von-mises stress, from which we obtain the first order eigenvalue as F=2.35E-5. Therewith we have the critical load value of each node

$$f_{cr} = f_0 \cdot F = 2.35 \times 10^{-5} MN \qquad (6)$$

Then the bearing force of unit core structure can be

written as:

$$F = 3 \times 20 f_{cr} = 1.41 \times 10^{-3} MN \qquad (7)$$

Combining (5) and (7), the flat compression strength is obtained.
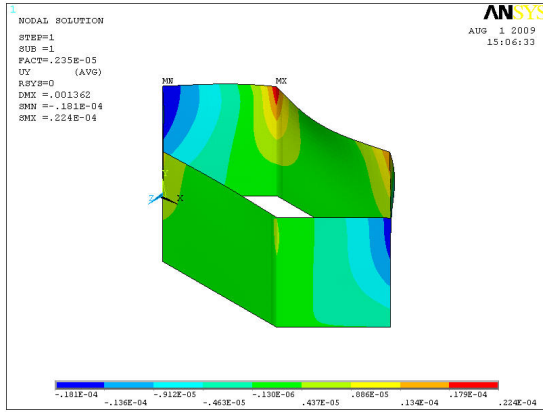
$$P_{cr} = \frac{F}{S_0} = 2.61 MPa \qquad (8)$$

Fig. 3 Contour plot of displacement in Y direction of eigenvalue buckling

4.2 Nonlinear buckling analysis

The critical load per unit node of the FE model here was set to 2.5E-5 MN according to the eigenvalue analysis results. In addition, a smaller load 1/50 of above applied value was also applied on the center of the top edge of core, perpendicular to both the edge and the vertical load. Fig.4 displays the nonlinear buckling analytical results. At the initial stage, offset displacement is small and the deformation of the core is in a linear manner, which contributes to the linear mechanical properties and structure. The linearity of vertical behavior was destroyed with the offset displacement increasing. Thus a sudden change of the vertical movement of the loaded nodes took place as illustrated in Fig.4. The turning point can be hold for the buckling point of the core structure, with the value of 2.03E-5 MN, as indicating in the x-coordinate. In other words, the critical load per unit node of the honeycomb structure equals f_{non}=2.03E-5 MN.

Then the bearing force of the single core can be obtained.

$$F_{non} = 3 \times 20 f_{non} = 1.22 \times 10^{-3} MN \qquad (9)$$

Next, we can calculate out the flat compression strength of the honeycomb structure as

$$P_{crn} = \frac{F_{non}}{S_0} = 2.26 MPa \qquad (10)$$

4.3 Comparison with experimental

The effect of structure dimension on the mechanical performance of the honeycomb paper board was explored by Zeng [6]. The flat compression strength of the honeycomb paperboard with different edge length was measured and the results were quoted in Table.2 for comparison with our calculated ones.
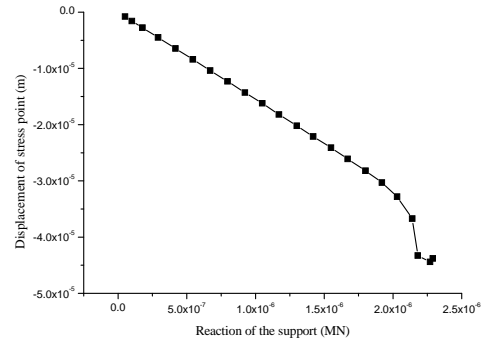

Fig.4 Relations of displacement in Y direction at loaded node versus reaction force at related fixed node

Table. 2 Flat compression buckling strength of honeycomb paperboard with different edge length

Edge length/m	0.005	0.00625	0.007	0.0075	0.0085
compression strength/MPa	3.46	2.31	2.26	1.865	2.183

The falter compression strength of the honeycomb paperboard with the edge length of 0.006 m can be obtained by cubic spline interpolation with the value of 2.305 MPa. Compared with the experimental results, the calculated one has a good agreement with it and very approximately reflects the real situation.

5. Conclusion Remarks

The differences of eigenvalue and nonlinear buckling analysis were summarized. By comparison between two, we found that the combination of the two makes a better results in theoretical analysis. At last Zeng's experiments were introduced to validate the feasibility of this analysis method, and it turned out that the combined analysis can give a very accurate prediction of buckling strength of complex structures. At the same time, it offers a good way to investigate the mechanism of buckling behavior of honeycomb board.

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Scientific Research

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