

# Back Analysis of Probability Integration Parameters Based on BP Neural Network

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Abstract: In order to obtain probability integration method parameters of surface movement after coal mining, based on analysis of mining and geological conditions, BP neural network model was built to back analysis the parameters with mining and geological conditions. Typical surface movement observation data in China were used as training and testing samples. Mean square error and mean absolute percentage error were used to evaluate the accuracy of the model. The calculated results show that model accuracy of fitting is goodness. Probability integration method parameters of 4 test samples were calculated by the inversion model, all mean square error of the results tested were less than 3 times of mean square error, and can meet the requirement of mining subsidence prediction, also show that the method to calculate probability integration method based on neural network inversion model is feasible. Various factors can be considered overall comprehensively with the BP neural network and nonlinear relationship between probability integration method parameters and mining and geological factors was established. The study provide basis to calculate mining subsidence prediction parameters for mining subsidence prediction data.

Keywords: mining subsidence, neural network, probability integration method, parameter back analysis

# 1. Introduction

To predict surface movement and deformation accurately is one of the core content of the study mining under building, water body and railways. Probability integration method based on stochastic medium theory is the most widely used and most successful method in China<sup>[1]</sup>, and also one of the methods in the "Regulation of Mining and Pillar Leaving under Building, Water-body, Railway and Main Underground engineer" <sup>[2]</sup>. Therefore, to determine the probability integration method parameters accurately have great significance on mining subsidence.

Filed survey is the main method to get the probability integration method parameters traditionally, and calculate the surface movement parameters with measured data <sup>[3-4]</sup>. In the long-term course of mining practice in China, all the major mining areas have established a large number of surface movement observation stations, which has played a significant role in promoting the mining subsidence study in China. In the aspect of theoretical study, it is reported that similar material simulation, and mechanical theory, mathematical and statistical theory and as well as robust regression, fuzzy and support vector machine methods are mainly used <sup>[5-10]</sup>. But the mining subsidence is a mechanical process contain highly nonlinear, due to the complexity of the geological and mining conditions, it is difficult for traditional mechanical mathematical

formulas to convey the highly nonlinear relationship between probability integration method parameters and geological and mining conditions. In addition, to establish observation stations needs large quantities of consumption in manpower and material resources and a long time. It cannot meet the needs of mine production.

Neural network is a nonlinear science which developed rapidly in recent years<sup>[11]</sup>. Due to its highly nonlinear, parallelism and powerful fault tolerance, neural network was used widely in artificial intelligence, automatic, classification and pattern recognition. Currently, BP neural network or its variations is the most widely used forms of neural network. It has proven in theory that a three-layer BP neural network can fit all continuous functions with arbitrary precision. This provides the theoretical basis for BP neural network used in function approximation and data fitting. In recent years, it raised a very good solution in highly nonlinear, non reversible and other complex issues. Also, in mining engineering, gratifying results have achieved in parameters inversion [<sup>12</sup>], mining subsidence prediction <sup>[13]</sup>, and et.al.

On the basis of comprehensive analysis of mining and geological factors which influence probability integration method parameters, data from typical surface observation station of main mining areas in China was used as training samples, probability integration method parameters inversion model was built based on BP neural network. Research results have great practical significance to calculate surface movement parameters especially for mining areas lack of actual observation data.

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# 2 Analysis of mining and geological conditions which influencing probability integration method parameters

A large number of research results and actual measurement data indicate that the probability integration method parameters have close relation with geological and mining conditions.

# 2.1 The subsidence factor q

Subsidence factor q is ratio of maximum vertical subsidence to mining thickness in condition of supercritical mining the horizontal or approximate horizontal coal seam. Studies have shown that subsidence factor have correlation with mechanical properties of the cover rock, ratio of mining depth to thickness, thickness of loose layer, mining degree, number of mining times, coal-mining method, and roof control method<sup>[14-15]</sup>. Empirical formula often seen in the reference is shown as follow. Equation (1) shows the relationship between mechanical properties of cover rock and subsidence factor <sup>[2]</sup>. Equation (2) approves the effect of mechanical properties, mining depth, and mining thickness <sup>[5]</sup>. Equation (3) shows the influence of repeat mining  $[1\sim 2]$ . (1)

q = 0.5(0.9 + P)

Where *P* is evaluation factor of cover rock.

 $q=0.991-0.238E/E_m-0.224\rho H^2/100E_m M$ (2)

Where *E* is comprehensive deformation modulus of rock mass;  $E_{\rm m}$  is deformation modulus of hard rock mass and can see as 3600Mpa; pis average density of rock mass; M is mining thickness and H is mining depth.

 $q = (1+\alpha) q_f$ 

Where  $q_r$  is factor of repeat mining,  $q_f$  is the first mining subsidence factor, and  $\alpha$  is rock activated factor.

(3)

(4)

# 2.2 The displacement factor b

Displacement factor b is ratio of maximum horizontal displacement to maximum mining vertical subsidence in condition of supercritical mining the horizontal or approximate horizontal coal seam. There is little change of displacement when mining the horizontal coal seam. Displacement factor can be calculated by dip angle of coal seam with empirical equation  $(4)^{[2]}$ 

$$b_c = b(1+0.0086\alpha)$$

Where  $b_c$  is displacement of declining coal seam, and  $\alpha$ is dip angle of coal seam.

Studies have shown that displacement factor vary with mining thickness obliviously<sup>[8]</sup>, and can be described by statistical equation (5).

b=0.0063M+0.27±0.12 (5)

Without regard for dip angle of mining horizontal coal seam, displacement factor has close relation with ratio of mining width to depth<sup>[16]</sup>.

Also, thickness of loose laver has strong effect on character of ground movement. Loose layer move with bedrock and fill subsidence space of bedrock with flows form at Scientific

the same time because of the rheological behaviour. That makes displacement increase with incremental of loose layer thickness. The empirical equation (6) gives relationship between displacement factor, loose layer thickness and mining depth in HeBi mining field<sup>[17]</sup>.

*b*=0.1379+0.1237*h*/*H*±0.04221 (6)

Where b is displacement factor; h is thickness of loose layer and H is mining depth.

In addition, lose of water in loose layer also can cause extent and degree of horizontal displacement increase.

# 2.3 Mining effect transference angle $\theta_0$

On tendency major section of subsidence basin, virtual boundary of working face calculated by deviation of inflection point wire splice with inflection point of subsidence curve, separation angle of the line with horizontal line named mining effect transference angle. Mining effect transference angle is coursed by dip angle of coal seam, and then they have the relationship as shown in equation (7).

 $\theta_0 = 90^\circ - k\alpha$ (7)Where k is parameter related to mechanical properties of the cover rock. k is bigger when the rock is harder. Generally, k is taken  $0.7 \sim 0.8$  when the rock is hard; and 0.6~0.7 when mid-hard, and 0.5~0.6 of soft rock<sup>[1]</sup>.

Mining effect spread vertically in loose layer, so the mining effect transference angle also affected by thickness of loose layer, and it is larger with thicker loose layer<sup>[17]</sup>.

# 2.4 Tangent of main effect angle tan β

On strike major section of subsidence basin, ratio of strike mining depth to main influence radius is the tangent of main effect angle. When mining depth is the same, tan  $\beta$  is related to mechanical of the cover rock, and is smaller if the rock is hard. When mechanical is the same,  $tan\beta$  will larger with increase of the mining depth. Reference [17] gives an equation to calculate  $tan \beta$  with cover rock mechanical, mining depth, and angle of coal seam, as shown of equation (8).

 $tan\beta = (1-0.0038\alpha)(D+0.0032H)$ 

Where D is a constant number changed with mechanical of the rock; H is mining depth, and  $\alpha$  is coal seam angle. Moreover,  $tan\beta$  also under impact of repeat mining,  $tan\beta$ of repeat mining is greater than first mining subsidence.

To sum up, geological mining conditions affect the probability integration method are mechanical of cover rock, dip angle of coal seam  $\alpha$ , mining depth *H*, mining thickness M, mining degree, thickness of loose layer h, repeat mining, and roof control method etc. In this paper, average consistence coefficient f is used to indicator the influence of cover rock mechanical, f can be calculated by equation (9).

$$f = \sum_{1}^{n} m_{i} Q_{i} / 10 \sum_{1}^{n} m_{i}$$
(9)

(8)



Where  $m_i$  is normal thickness of *i*th Rock layer (m) and  $Q_i$  is one-way compression strength of the rock (Mpa).

# **3** Theory of **BP** neural network and parameters inversion method

#### 3.1 Theory of BP neural network

BP neural network trained the network using back propagation method, and then network weights can be got. The main idea is modify the weight or threshold to make the error function decline along the negative gradient direction <sup>[18]</sup>. Signal transmission and error backpropagation are two process procedures of BP neural network learning. In the forward propagation, the signal inputted from input layer, and propagated to output layer after processed by hidden layer. Each neurons only could affect the neurons state of the next layer. The error back propagated if desired output could not get. Output error which is the basis to modify the neurons weight back propagate to input layer through hidden layer, repeat until output error permitted<sup>[19]</sup>. Assume that there are n neurons in the input layer of a three feedforward BP neural network which have one hidden layer, which are  $X \in \mathbb{R}^n$ ,  $X = (x_1, x_2, ..., x_n)^T$ , there are  $n_1$  neurons in the second layer, which are  $X' \in \mathbb{R}^{n_1}, X' = (x'_1, x'_2, ..., x'_{n_1})^T$ , and *m* neurons in the output layer, which are  $Y \in \mathbb{R}^m, Y = (y_1, y_2, \dots, y_m)^T$ . Weight between the input layer and hidden layer is  $w_{ii}$ , and threshold is  $\theta_i$ . Weight between the hidden layer and output layer is  $w_{ij}$ , and threshold is  $\theta_i$ , in which  $(i=1,2,...,n; i=1,2,...,n_1; k=1,2,...,m)$ , and then all output of the neurons should satisfy the formulas (10) and (11).

$$y_{k} = f(\sum_{j=1}^{n_{1}} w_{jk} x_{j} - \theta_{k})$$
(10)

$$x'_{j} = f(\sum_{i=1}^{n} w_{ij} x_{i} - \theta_{j})$$
(11)

The purpose is to get connection weights  $(w_{ij}, wjk)$  and threshold  $(\theta_j, \theta_k)$  between neurons by training of P samples like  $(x^1, yI)$ ,  $(x^2, y2)$ ...  $(x^p, y^p)$ , and make it mapping successful. If the training samples  $x^1, x^2, ..., x^p$  known as input data, and  $t^1, t^2, ..., t^p$  known as expected corresponding output. The learn algorithms is modify their weights and thresholds with error of actual output  $y^1, y2... y^p$  and expected output  $t^1, t^2, ..., t^p$ , and make it closely as possible. If the  $p_l$  sample input into the network and  $y_l$  obtained as output, in which l=1,2,...,m. The error is sum error of all output unit, can be calculated by formula (12). And if training with all P samples, total error can be calculated by formula (13).

$$E_{P_l} = \frac{1}{2} \sum_{l=1}^{m} \left( t_l^{P_l} - y_l^{P_l} \right)^2$$
(12)

$$E_{\underline{\otimes}} = \frac{1}{2} \sum_{P_l}^{P} \sum_{l=1}^{m} \left( t_l^{P_l} - y_l^{P_l} \right)^2$$
(13)

According to the gradient algorithm, general principle to modify the weights of connections is to make the total error always decrease until it meets the conditions. Through derivation calculation of error, all layer weights can be calculated by formula (14) and (15).

$$w'_{jk}(n+1) = w'_{jk}(n) + \eta \sum_{P_l=1}^{r} \delta_{jk}^{P_l} x'_j^{P_l}$$
(14)

Where 
$$\delta_{jk}^{P_l} = (t_l^{P_l} - y_l^{P_l})y_l^{P_l}(1 - y_l^{P_l})$$
, and  $\eta$  is step length

$$w_{ij}(n+1) = w_{ij}(n) + \eta \sum_{P_i=1}^{i} \delta_{ij}^{P_i} x_i^{P_i}$$
(15)

Where 
$$\delta_{ij}^{P_i} = \sum_{k=1}^m \delta_{jk}^{P_i} w'_{jk} x'_j^{P_i} (1 - x'_j^{P_i}).$$

Above is the basis formula to modify the weight for a three-layer BP neural network. The entire network learning process contains two phases; the first stage is calculated from input layer to output layer. Output of all neurons can be calculated by training samples by initial structure and weight; the second stage is to modify the weights and threshold, and it start from output layer to input layer, and weight of neurons connect to output can be modified according to error of output, and also hidden layer weight can be modified too. The two stages are iterative process, repeat until convergence. Essentially, BP neural network is a nonlinear optimization problem issues for a set of input and output samples. It can be seen as a mapping from n-dimensional to m-dimensional. BP neural network structure is shown in Fig.1.



Fig.1 Structure of neural network

#### **3.2 Sample Data Selection**

To inverse the probability integration method parameters by neural network, enough samples include training samples and testing samples are needed to select first. Each sample should include the multidimensional input parameters related to probability integration parameters and an output parameter (probability integration parameter). Mass of data were obtained by actual measured during many years of mining subsidence study in China. Reference [2] has given data from 208 typical observatories; part of those data includes geological and



mining conditions. By analyzing and testing, 43 working faces data were selected as training and testing samples,

shown in Table 1.Part of the samples are omitted for simpleness.

Table 1 Training and Testing Samples											
Sequence Number	geological and mining conditions						Parameters of probability integration method				
	f	M(m)	α(°)	H0(m)	D1/H0	D3/H0	H(m)	q	b	tanβ	θ₀(°)
1	3.5	1	9	67.5	1.42	3.63	8.1	0.66	0.27	1.41	83
2	3.5	1.6	7	47	3.40	6.38	8.1	0.62	0.29	1.68	84
3	3.1	1.6	10	318.5	0.79	0.88	2.7	0.67	0.15	3.67	80
:	:	:	:	:	:	:	:	:	:	÷	:
40	3.5	2	16	98.5	0.83	2.08	64	0.92	0.370	1.700	83.0
41	1.82	7	9	95	0.84	2.87	48	1.16	0.27	2.10	86
42	3.1	1.5	8	81	1.01	2.59	3.2	0.62	0.24	2.50	83
43	2.45	0.92	6.5	305	4.26	1.44	55	0.8	0.33	1.80	89

# **3.3 BP neural network inversion model and accuracy evaluation**

It need to build A mapped function model with 7 mining and geological conditions parameters as input and 4 probability integration method parameters as output if parameters inversed with data in table 1. In order to expres the nonlinear relationship between input and output, S-type function was selected as transfer function of hidden layer and output neurons transfer function is linear function. In order to make sure that all neurons weights modified at the maximum changes of the S-type activation function, 7 input mining and geological conditions were initialized to the interval (-1,1). Single hidden layer neural network and 15 neurons in hidden layer were selected to ensure the system stability and to prevent over learning. 0.05 was selected as learning rate. No. 1~39 samples were chosen to train the BP neural network as training samples. And then, nonlinear relationship between mining and geological conditions was built. Calculation accuracy must be evaluated before the model used.

Since the observations are independent of each other, the mean square error (MSE) and mean absolute percentage error (MAPE) of probability integration method parameters can be used to evaluate the accuracy of the model. MSE denote the regression error, and MAPE denote the relative error of the models. MSE and MAPE of the model can be calculated by equations 16 and 17.

$$m_{b} = \pm \sqrt{\sum_{i=1}^{39} (p_{i} - \hat{p}_{i})^{2} / 39}$$
(16)  
$$MAPE = \left(\sum_{i=1}^{39} \left| p_{i} - \hat{p}_{i} \right| / p_{i}\right) / 39 \times 100\%$$
(17)

Where  $m_b$  is mean square error;  $p_i$  is parameters of probability integration method by actual measured and

 $\hat{p}_i$  is parameters calculated by the model; n is total amount of samples and n=39; and MAPE is mean absolute percentage error.

The model was evaluated by equations 16, 17 . Results are listed in table (2).  $R^2$  is correlation coefficient.

**Table 2 Models calculation results** 

Probability integration parameters	correlation coef- ficient	MSE	MAPE (%)	
q	0.96	0.04	3.3	
b	0.91	0.02	4.6	
tanβ	0.93	0.15	5.7	
00	0.98	1	0.8	

As shown in table 2, correlation coefficient indicates the correlation of model output and actual measured parameters, the closer of squared correlation coefficient to 1, the closer model output to target output. The minimum squared correlation coefficient of all groups is 0.91, and the maximum one is 0.98, it show that the established models has a high similarity to actual measured value, and the model has a great fitting performance.

MSE and MAPE value in table(2) show that the calculation results of the model has a small mean square error and the maximum mean absolute percentage error is 5.7%, also Computing model has a high precision. MSE in table (2) can serves as evaluating indicator to control model accuracy.

# 4 Model test and discussions

NO. 40~43 samples were chosen as test samples. Test results compared with measured values are shown in table 3.

Table3 Comparison of calculation results and actual measured results

Parameters	Samples



		40	41	42	43
q	Measured Predicted	0.92 0.95	1.16 1.20	0.62 0.63	0.8 0.83
	Absolute error	0.03	0.04	0.01	0.03
	Relative error (%)	3.2	3.4	1.6	3.8
b	Measured Predicted	0.37 0.37	0.27 0.28	0.24 0.25	0.33 0.31
	Absolute error	0	0.01	0.01	0.02
	Relative error (%)	0	3.7	4.2	6.7
tanβ	Measured Predicted	1.7 1.60	2.1 2.02	2.5 2.25	1.8 1.93
	Absolute error	0.1	0.08	0.25	0.13
	Relative error (%)	5.9	3.8	10	7.2
θ₀	Measured Predicted	83 85	86 88	83 85	89 84
	Absolute error	2	2	2	5
	Relative error (%)	2.4	2.3	2.4	5.6

Results of table 3 shows that all parameter predicted are less than three-times of MSE and all relative error are less than 10%. Therefore, results calculated by neural network inversion model are accurate and reliable, and to calculate the probability integration parameters are feasible and effective. Effect factors can be considered synthetically with neural network model and the result is accurate and reliable, also can meet the requirements of engineering site.

#### **5** Conclusions

(1) Mining and geological conditions which impact probability integration method parameters were analyzed comprehensively; method to build the probability integration method parameters inversion model was detailed in the paper.

(2) Data from some typical observatory were chosen as training and testing samples. Results show that parameters inversion model has a good performance on prediction. Testing results show that value predicted by BP neural network can fit the actual perfectly, all values predicted are within the permission error, and relative error is small. The accuracy of the inversion model can meet the requirements of engineering site. Test results prove that it is accurate and reliable to calculate the probability integration parameters by neural network.

(3) Factors that affect the probability integration parameters can be considered synthetically with neural network model; and built the nonlinear relationship between mining and geological conditions and mining subsidence parameters; model can be established easily and also the results are accurate and reliable.

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