Modulation Instability in Decreasing Dispersion Fibers

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Abstract: Starting from the nonlinear Schrödinger equation governing the propagation of optical pulses in optical fibers, modulation instability in decreasing dispersion fiber is analytically investigated. The synthetic effects of dispersion parameter, slowly decreasing dispersion parameter, incident power, propagation distance, loss coefficient, nonlinear coefficient, etc. on the gain spectrum of modulation instability are analyzed. Furthermore, gain spectra of modulation instability in the ideal fiber, loss fiber and decreasing dispersion fiber are calculated and compared. The gain spectrum of decreasing dispersion fiber is much broader than that of ordinary optical fibers .The broad gain bandwidth can be obtained by selecting the dispersion parameter of decreasing dispersion fiber. Accordingly, the decreasing dispersion fiber in anomalous dispersion regime is good kind of fiber to produce modulation instability. This research result shows that a new method can be obtained to generate the broadband spectrum of modulation instability in the fibers with slowly decreasing dispersion which is promising to produce the ultra short pulses and so on.

Keywords: nonlinear optics; decreasing dispersion fiber; modulation instability; gain spectrum

1 Introduction

Modulation instability is very important and universal in physical phenomenon. It can be observed in Hydrodynamics, Plasma Physics, Condensed Physics, Optics, and so on . Modulation instability in optics result from the interaction between dispersion and nonlinear effects. It include time domain, airspace and the coupling between time domain and airspace. The modulation instability in time domain is usually in optical fiber. It registers as the secondary lobe of spectral line in frequency domain and a series of burst pulses in time domain [1]. To solve the fiber loss, people study out the decreasing dispersion fiber whosesecond-order dispersion is not uniform in transmission direction,, It can compensate fiber nonlinear effect by same rate of decrease in fiber loss. modulation instability in decreasing dispersion fiber is analytically investigated in this paper, characteristics of the gain spectrum are compared in deal fiber, loss of optical fiber and dispersion-decreasing fiber [2].

2 Theory Foundation of Optical Pulse in Optical Fiber Transmission

We can get the transmission equation from Maxwell's equations. Concrete steps are as follows [3]:

Make the vorticity for both sides of

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.1}$$

then eliminate $\vec{B}, \vec{D}$ with $\vec{E}, \vec{P}$ in

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{2.2}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad \text{and} \tag{2.3}$$

$$\vec{B} = \mu_0 \vec{P} + \vec{M} \tag{2.4}$$

from above, we can get the equation :

$$\nabla \times \nabla \times \vec{E} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \tag{2.5}$$

Where $\mu_0 \varepsilon_0 = 1/c^2$, c is vacuum speed of light.

Optical pulse shape and spectrum can be affected by dispersion and nonlinear effect in optical pulse transmission [4], Transfer equation can be expressed into the following :

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \vec{P}_{NL}}{\partial t^2} \tag{2.6}$$

Where $\vec{P}_L$ is the linear part of polarization, $\vec{P}_{NL}$ is Non-linear part of polarization.

Solution of equation (2.6) obtained under the formula:

$i\frac{\partial A}{\partial z}+\frac{i\alpha}{2}A-\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}+\gamma|A|^2A=0$ (2.7) This equation is called nonlinear Schrödinger equation (NLS). Where β_2 is Optical second-order dispersion coefficient, α is fiber loss coefficient , γ is fiber nonlinear coefficient.

3 The Analysis of Modulation Instability

Transmission satisfied for the above nonlinear equation Light waves in optical fiber s (2.7) type described in , the steady-state solution for equation (2.7) is

$$\overline{A}=\sqrt{P_0}\exp(i\gamma P_0 z) \quad (3.1)$$

Where P_0 is initial light intensity.

(3.1)-type shows that the waveform remains unchanged in optical fibers transmission with the increase of its phase. We will consider the perturbation characteristics of steady-state solution. MakA is $(\sqrt{P_0}+a)\exp(i\gamma P_0 z)$. where a is perturbation parameter, and $|\alpha|<<\sqrt{P_0}$ [5]. We substitute (3.1)-type in (2.7)-type, only consider a one-item, equations are simplified as follows:

$$\frac{\partial a}{\partial z}=\frac{1}{2}\beta_2\exp(\alpha z)\frac{\partial^2 a}{\partial T^2}-\gamma P_0(a+a^*) \quad (3.2)$$

Where α^* is Complex conjugation for a. Based equation (3.3) solution for

$$a(z,T)=a_1\cos(kz-\Omega T)+a_2\sin(kz-\Omega T) \quad (3.3)$$

Where k is perturbation wave number and Ω is perturbation wave frequency, a_1 and a_2 are amplitude parameter of perturbation wave][6]. We substitute (3.2)-type in (2.7)-type, the relation between k and Ω are simplified as follows:

$$k=\pm\frac{\Omega}{2}|\beta_2|\exp(\alpha z)[\Omega^2+\operatorname{sgn}\beta_2\frac{4\gamma P_0}{|\beta_2|\exp(\alpha z)}]^{1/2} \quad (3.4)$$

(3.4)-type shows the dispersion relation. For some perturbance , when k becomes plural, then the modulation instability generated. It shows that modulation instability can only happen at the negative dispersion fiber region. For conventional optical fiber, fiber dispersion is a constant, so we can get that $\beta_2(z)=\beta_2(0)$ [7],(3.4)-type can become

$$k=\pm\frac{1}{2}|\beta_2|\exp(\alpha z)\Omega(\Omega^2-\Omega_c^2)^{1/2} \quad (3.5)$$

where $\Omega_c^2=\frac{4\gamma P_0}{|\beta_2|\exp(\alpha z)}$.

For dispersion-decreasing fiber, make dispersion to meet the conditions of $\beta_2=\beta_2(0)\exp(-\mu z)$,where μ is longitudinal changes parameters of dispersion, from above ,(3.6)-type can be getted:

$$k=\pm\frac{\Omega}{2}|\beta_2(0)|\exp(\alpha z)\exp(-\mu z)(\Omega^2-\Omega_c^2)^{1/2} \quad (3.6)$$

where $\Omega_c^2=\frac{4\gamma P_0}{|\beta_2(0)|\exp(\alpha z)}\exp(\mu z)$.

It can be seen that the relation between k and Ω is not only with the fiber loss, but also with the fiber-optic vertical dispersion parameter. For the purposes of the general fiber, fiber loss is a constant, so (3.6)-type can be simplified as

$$k=\pm\frac{\Omega}{2}|\beta_2(0)|\exp(\alpha-\mu)z(\Omega^2-\Omega_c^2)^{1/2} \quad (3.7)$$

where $\Omega_c^2=\frac{4\gamma P_0}{|\beta_2(0)|}\exp(\mu-\alpha)z$.

To do further study of optical properties of modulation instability, we usually adhibit gain $g(\Omega)$,it can be defined as

$$g(\Omega)=2\operatorname{Im}k=\Omega|\beta_2(0)|\exp(\alpha-\mu)z(\Omega_c^2-\Omega^2)^{1/2} \quad (3.8)$$

(3.8) –type shows that the gain spectrum is not only in the frequency range, but also with the parameter μ and the transmission distance z , if

$$\Omega_{\max}=\pm\frac{\Omega}{\sqrt{2}}=\pm\sqrt{\frac{2\gamma P_0}{|\beta_2(0)}}\exp(\frac{\mu-\alpha}{2})z \quad (3.9)$$

the gain have maxima, its value is

$$g_{\max}=g(\Omega_{\max})=\frac{1}{2}|\beta_2|\exp(\alpha z)\Omega_c^2=2\gamma P_0 \quad (3.10)$$

From (3.10)-type,we can know that peak of gain spectrum has nothing to do with the fiber dispersion parameter, but has a linear proportional relationship with the power of light waves .

4 Simulation Analysis

From the theoretical analysis ,we can get that modulation instability only arises from the anomalous dispersion fib-

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er area. Gain spectra of modulation instability in the ideal fiber, loss fiber and decreasing dispersion fiber are calculated and compared.

4.1 Gain Spectra of Modulation Instability in the Ideal Fiber

Figure 4.1 shows the change curve between gain spectra of modulation instability in the ideal fiber and optical power. Figure 4.2 shows change curve between cut-off frequency and optical power. From the Figure 4.1,we can get that gain spectrum are symmetrical and it is zero at $\Omega = 0$, and with the increase of fiber power, the peak of gain spectrum increases and the gain spectrum will also be widened, And only frequency instability less than the cut-off frequency, it will produce modulation instability. Figure 4.2 shows the relation between Ω_c and P_0. With the increase of optical power P_0, the cut-off frequency almost linear increase. From the two Figure, we can get that the more big fiber power is, the more easy to produce modulation instability.

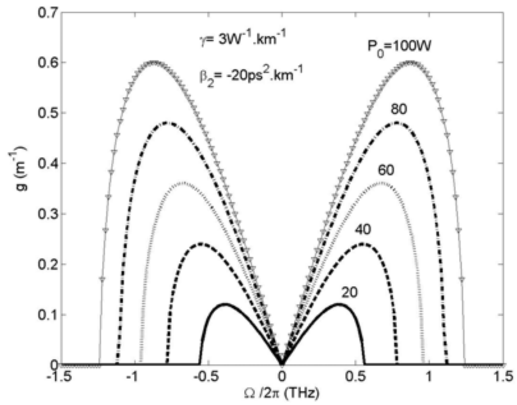

Figure 4.1. the change curve between gain spectra of modulation instability in the ideal fiber and optical power.

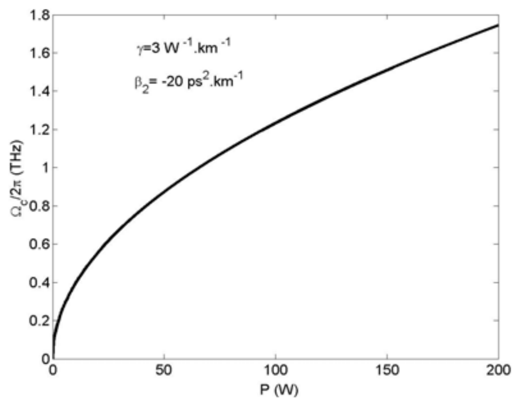

Figure 4.2. change curve between cut-off frequency and optical power.

4.2 Gain Spectra of Modulation Instability in the Loss Fiber

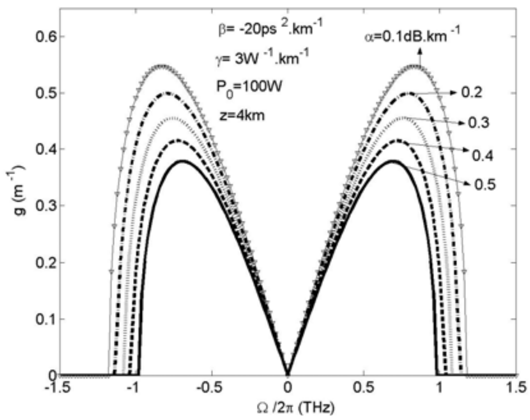

Figure 4.3. the change curve between gain spectra of modulation instability in the loss fiber and loss coefficient

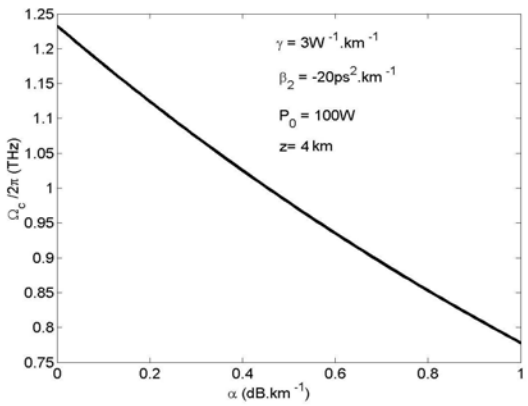

Figure 4.4. the change curve between cut-off frequency and loss coefficient

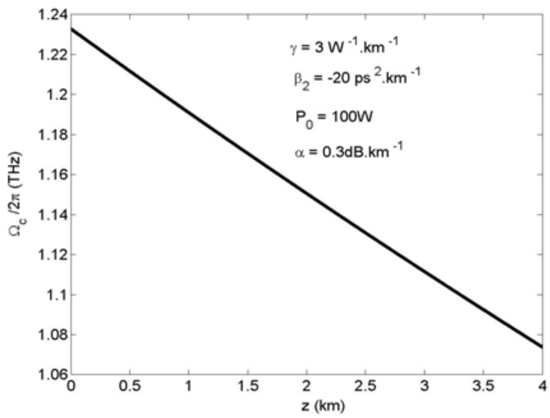

Figure 4.5. the change curve between cut-off frequency and transmission distance.

Figure 4.3 shows the change curve between gain spectra of modulation instability in the loss fiber and loss coefficient. Fig.4.4 shows the change curve between cut-off frequency and loss coefficient .Fig.4.5 shows the change curve between cut-off frequency and transmission distance. From Figue 4.3,we can get that the peak of gain spectra gradually reduce and the wide of gain spectrum is gradually narrowed with the loss coefficient increased gradually. From Figure 4.4, we can get that the peak of gain spectra almost linearly decreases with the loss coefficient increases when the transmission distance fixed. It can also be getted from (3.7)-type, mainly because of the fiber loss reduces the optical power, the cut-off frequency exponentially decay with the fiber loss. From Figure 4.5 ,we can get that the peak of gain linearly decrease with the transmission distance increases when the loss coefficient keep a constant. By the above Fig,it can be seen that the more big iber loss is, the more difficult to produce greater modulation instability because of the existence of fiber loss .

4.3 Gain Spectra of Modulation Instability in the Decreasing Dispersion Fiber

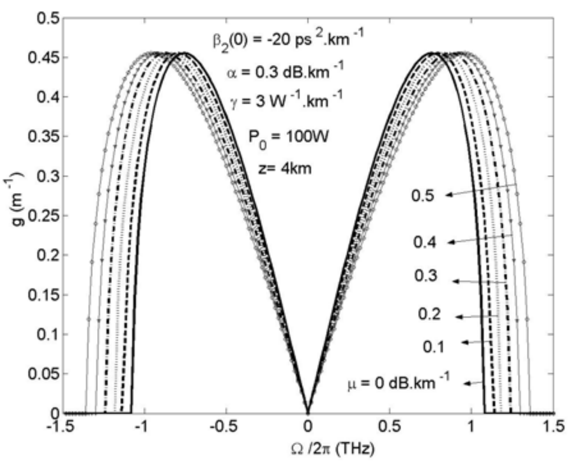

Figure 4.6. the change gain spectra curve of modulation instability in the decreasing dispersion fiber

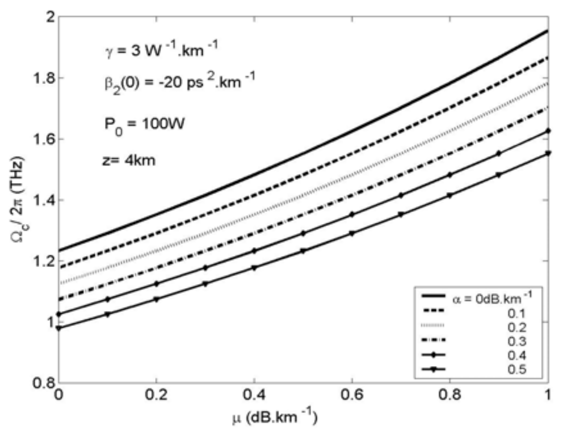

Figure 4.7. the change curve between cut-off frequency and vertical dispersion parameter

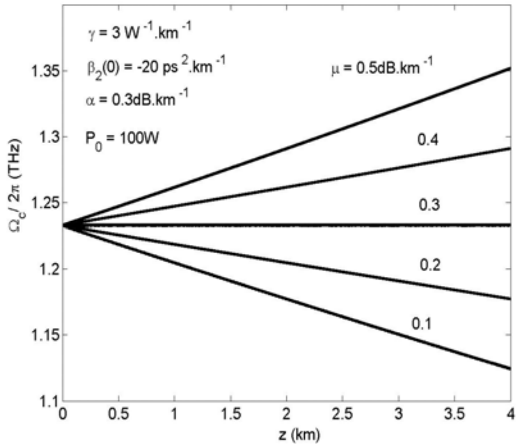

Figure 4.8. the change curve be tween cut-off frequency and t ransmission distance in the decreasing dispersion fiber.

Figure 4.6 shows the change gain spectra curve of modulation instability in the decreasing dispersion fiber. Figure 4.7 shows the change curve between cut-off frequency and vertical dispersion parameter. Figure 4.8 shows the change curve between cut-off frequency and transmission distance in the decreasing dispersion fiber. Figure 4.6 shows that the peak of gain has nothing to do with the GVD parameter, it can also be getted from (3.10)-type, the peak of gain remain unchanged with vertical dispersion parameter changes and the wide of gain spectrum will be widen with the vertical dispersion parameter increases. From Figure 4.7, we can get that the smaller Loss coefficient is, the greater the cut-off frequency is. From Figure 4.8, we can get that the cut-off frequency will be extended in the decreasing dispersion fiber. It shows that decreasing dispersion fiber can provide the wide range which can make modulation instability effect.

5 Conclusion

Starting from the nonlinear Schrödinger equation governing the propagation of optical pulses in optical fibers, modulation instability in decreasing dispersion fiber is analytically investigated. gain spectra of modulation instability in the ideal fiber, loss fiber and decreasing dispersion fiber are calculated and compared. The synthetic effects of dispersion parameter, slowly decreasing dispersion parameter, incident power, propagation distance, loss coefficient, nonlinear coefficient, etc. The results show that self-phase modulation instability generate in anomalous dispersion and anomalous dispersion region of decreasing dispersion fiber is better dispersion me-

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dium chich can generate modulation instability effect.

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