# SVD Decoding Algorithm of Quasi-orthogonal STBC 

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#### Abstract

Antenna diversity is a practical and effective technique for combating multipath fading in a scattering environment. One attractive method for realizing transmit diversity is to employ quasi-orthogonal spacetime block code (QOSTBC). Based on analyzing the structure and character of the QOSTBC, a novel encoding method of QOSTBC is proposed for four antennas. It enriched the code word of QOSTBC. The BER performance is given by comparison of different decoding algorithms. It was proposed to use QR decomposition and SVD decoding algorithm for novel code. The SVD decoding algorithm is simplified to decoding of four independent single symbols. Further simulations show that the computation complexity of receiver is reduced. The SVD is a suboptimal decoding algorithm for QOSTBC.


Keywords: diversity; multiple input multiple output (mimo); space-time block code; quasi-orthogonal stbc; svd decoding algorithm; qrdecomposition

## 1 Introduction

Providing high-speed and reliable broadband data services are the focus of wireless communications. Without increasing the system bandwidth and transmit power of antennas, multi-antenna MIMO technology is applied to make full use of space resource. The channel fading of wireless communications can be decreased. The spectrum efficiency and channel capacity are improved. The spacetime signal processing is an effective method to acquire these benefits of MIMO system. It needs effective combination of channel coding, modulation, transmit and receive diversity technique. The transmission performance can be greatly improved.

The space-time block code (STBC) scheme designed by Alamouti [1] uses two transmit antennas, which provides full diversity and full transmission rate, and the maximum-likelihood decoding algorithm is applied at the receiver. The transmission matrices are designed with orthogonal columns. The system can get full diversity gain. Unfortunately, one cannot achieve both full rate and full diversity at the same time with STBC for more than two transmit antennas[2]. Therefore, the QOSTBC based on full transmission rate is proposed by Jafarkhani in [3]. The transmission matrix is quasi-orthogonal, and it can achieve maximum transmission rate 1. It also has an advantage of a simple decoder.

## 2 System Model

We consider a multiple-antenna wireless communication system with N transmission antennas and M receiver antennas. The channel is a quasi-static flat fading channel. The discrete time equivalent model of receiving signal can be defined by

$$
\begin{equation*}
\mathrm{y}(k)=\mathbf{H x}(k)+\mathrm{n}(k), \tag{1}
\end{equation*}
$$

where $y(k)=\left[y_{1}(k), \mathrm{y}_{2}(k), \cdots, y_{M}(k)\right]^{\mathrm{T}}$ is the received signal. $x(k)=\left[x_{1}(k), x_{2}(k), \cdots, x_{N}(k)\right]^{\mathrm{T}}$ is the transmitted signal. $n(k)=\left[n_{1}(k), n_{2}(k), \cdots, n_{M}(k)\right]^{\mathrm{T}}$ is the additive white Gaussian random noise. $\mathbf{H}$ is $M \times N$ channel matrix.

Assuming the transmission and receiver are independent, the channel parameters are known. So $\mathbf{H}$ is an independent identical distributed complex Gaussian random matrix with mean 0 and variance 1 . The fading coefficients are assumed constant in $T$ symbols block and change independently inter block. Assuming matrix of the receiver is $r_{i}{ }^{j},(t=1,2, \cdots, p)$ at time $t$, the decision metric can be computed as $\sum_{t=1}^{p} \sum_{j=1}^{M}\left|y_{t}^{j}-\sum_{i=1}^{N} h_{i j} x_{t}^{i}\right|^{2}$. The $h_{i j}$ of matrix H is gain of the channel from the ith transmit to the jth receive antenna. Over all possible of x , decide in favor of the code word that minimizes this sum.

## 3 Quasi-orthogonal Space-time Block Code

### 3.1 Jafarkhani Code and TBH Code

Based on the encode scheme of Alamouti, QOSTBC of four antennas schemes are given by Jafarkhani. An ex-
ample of Jafarkhani scheme with two antennas is defined by the following transmission matrix:

$$
\mathbf{A}_{12}=\left[\begin{array}{cc}
x_{1} & x_{2}  \tag{3}\\
-x_{2}^{*} & x_{1}^{*}
\end{array}\right] .
$$

Here the subscript 12 represent the indeterminate $x_{1}$ and $x_{2}$ in the transmission matrix. The encoding matrix of Jafarkhani code can be written as

$$
\mathbf{C}_{J}=\left[\begin{array}{rr}
\mathbf{A}_{12} & \mathbf{A}_{34}  \tag{4}\\
-\mathbf{A}_{34}^{*} & \mathbf{A}_{12}^{*}
\end{array}\right]=\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
-x_{2}^{*} & x_{1}^{*} & -x_{4}^{*} & x_{3}^{*} \\
-x_{3}^{*} & -x_{4}^{*} & x_{1}^{*} & x_{2}^{*} \\
x_{4} & -x_{3} & -x_{2} & x_{1}
\end{array}\right] .
$$

Another code different from Jafarkhani scheme is proposed by O. Tirkkonen, A. Boariu and A. Hottinen in [4]. The TBH case has

$$
\mathbf{C}_{\text {TBH }}=\left[\begin{array}{ll}
\mathbf{A}_{12} & \mathbf{A}_{34}  \tag{5}\\
\mathbf{A}_{34} & \mathbf{A}_{12}
\end{array}\right]=\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
-x_{2}^{*} & x_{1}^{*} & -x_{4}^{*} & x_{3}^{*} \\
x_{3} & x_{4} & x_{1} & x_{2} \\
-x_{4}^{*} & x_{3}^{*} & -x_{2}^{*} & x_{1}^{*}
\end{array}\right] .
$$

### 3.2 Novel Quasi-orthogonal Space-time Block Code

To consider a wireless system with four transmit antennas and one receiving antenna, a full rate QOSTBC was introduced. A two encoding unit scheme is applied instead of Alamouti method. The encoding units of the novel scheme are as follows:

$$
\mathbf{A}_{12}^{\prime}=\left[\begin{array}{ll}
x_{1} & x_{2} \\
x_{2} & x_{1}
\end{array}\right], \quad \mathbf{A}_{12}^{\prime \prime \prime}=\left[\begin{array}{cc}
x_{1} & x_{2} \\
-x_{2} & -x_{1}
\end{array}\right] .
$$

The encoding matrix can be written as:

$$
\mathrm{C}_{\mathrm{W} 1}=\left[\begin{array}{cc}
\mathrm{A}_{12}^{\prime} & \mathrm{A}_{34}^{\prime}  \tag{6}\\
\left(\mathrm{A}_{34}^{\prime \prime}\right)^{*} & -\left(\mathrm{A}_{12}^{\prime \prime}\right)^{*}
\end{array}\right]=\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{2} & x_{1} & x_{4} & x_{3} \\
x_{3}^{*} & x_{4}^{*} & -x_{1}^{*} & -x_{2}^{*} \\
-x_{4}^{*} & -x_{3}^{*} & x_{2}^{*} & x_{1}^{*}
\end{array}\right] .
$$

It has the character matrix as:

$$
\mathbf{C}_{w 1}^{H} \mathbf{C}_{w 1}=\left[\begin{array}{cccc}
a & b_{w 1} & 0 & 0  \tag{7}\\
b_{w 1} & a & 0 & 0 \\
0 & 0 & a & -b_{w 1} \\
0 & 0 & -b_{w 1} & a
\end{array}\right],
$$

Where $a=\sum_{t=1}^{4}\left|x_{i}\right|^{2}, b_{W 1}=x_{1} x_{2}{ }^{*}+x_{2} x_{1}{ }^{*}+x_{3} x_{4}{ }^{*}+x_{4} x_{3}{ }^{*}$ is real.

## 4 Decoding Algorithm of the Novel QOSTBC

### 4.1 Encoding and Transmission Model

Take the novel scheme of QOSTBC for example, after constellation mapping of the signal, take every 4 bits signals input to QOSTBC system. For c = [c1 c2 c3 c4]T , we can get

$$
\left[\begin{array}{l}
c_{1}  \tag{8}\\
c_{2} \\
c_{3} \\
c_{4}
\end{array}\right]=\left[\begin{array}{cccc}
c_{1} & c_{2} & c_{3} & c_{4} \\
c_{2} & c_{1} & c_{4} & c_{3} \\
c_{3}^{*} & c_{4}^{*} & -c_{1}^{*} & -c_{2}^{*} \\
-c_{4}^{*} & -c_{3}^{*} & c_{2}^{*} & c_{1}^{*}
\end{array}\right] .
$$

Columns of transmission matrix correspond to signal of different transmit antenna, rows of transmission matrix correspond to the different time slot. Assuming r1, r2, r3, r 4 are the signal of receiving antenna, and $\mathrm{h} 1, \mathrm{~h} 2, \mathrm{~h} 3, \mathrm{~h} 4$ are the path loss, n1, n2, n3, n4 are noise of system, then we can get

$$
\left[\begin{array}{l}
r_{1}  \tag{9}\\
r_{2} \\
r_{3} \\
r_{4}
\end{array}\right]=\left[\begin{array}{cccc}
c_{1} & c_{2} & c_{3} & c_{4} \\
c_{2} & c_{1} & c_{4} & c_{3} \\
c_{3}^{*} & c_{4}^{*} & -c_{1}^{*} & -c_{2}^{*} \\
-c_{4}^{*} & -c_{3}^{*} & c_{2}^{*} & c_{1}^{*}
\end{array}\right]\left[\begin{array}{l}
h_{1} \\
h_{2} \\
h_{3} \\
h_{4}
\end{array}\right]+\left[\begin{array}{l}
n_{1} \\
n_{2} \\
n_{3} \\
n_{4}
\end{array}\right] .
$$

From (9) we can get

$$
\left[\begin{array}{c}
r_{1}  \tag{10}\\
r_{2} \\
r_{3}^{*} \\
r_{4}^{*}
\end{array}\right]=\left[\begin{array}{cccc}
h_{1} & h_{2} & h_{3} & h_{4} \\
h_{2} & h_{1} & h_{4} & h_{3} \\
-h_{3}^{*} & -h_{4}^{*} & h_{1}^{*} & h_{2}^{*} \\
h_{4}^{*} & h_{3}^{*} & -h_{2}^{*} & -h_{1}^{*}
\end{array}\right]\left[\begin{array}{c}
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{array}\right]+\left[\begin{array}{l}
n_{1} \\
n_{2} \\
n_{3}^{*} \\
n_{4}^{*}
\end{array}\right] .
$$

The channel matrix H can be written as

$$
\mathbf{H}=\left[\mathbf{h}_{1} \mathbf{h}_{2} \mathbf{h}_{3} \mathbf{h}_{4}\right]=\left[\begin{array}{cccc}
h_{1} & h_{2} & h_{3} & h_{4}  \tag{11}\\
h_{2} & h_{1} & h_{4} & h_{3} \\
-h_{3}^{*} & -h_{4}^{*} & h_{1}^{*} & h_{2}^{*} \\
h_{4}^{*} & h_{3}^{*} & -h_{2}^{*} & -h_{1}^{*}
\end{array}\right] .
$$

It has the character matrix as

$$
\boldsymbol{\Delta}_{4}=\boldsymbol{\Delta}_{4}^{\mathrm{H}}=\mathbf{H}^{\mathrm{H}} \mathbf{H}=\left[\begin{array}{cccc}
a_{H} & b_{H W 1} & 0 & 0  \tag{12}\\
b_{H W 1} & a_{H} & 0 & 0 \\
0 & 0 & a_{H} & -b_{H W 1} \\
0 & 0 & -b_{H W 1} & a_{H}
\end{array}\right] .
$$

Where $a_{H}, b_{H W 1}$ are real, and $a_{H}=\left\|\mathrm{h}_{1}\right\|^{2}=\left\|\mathrm{h}_{2}\right\|^{2}$ $=\left\|\mathrm{h}_{3}\right\|^{2}=\left\|\mathrm{h}_{4}\right\|^{2}=\sum_{i=1}^{4}\left|h_{i}\right|^{2}, b_{H W 1}=\mathrm{h}_{1}{ }^{\mathrm{H}} \mathrm{h}_{2}=\mathrm{h}_{2}{ }^{\mathrm{H}} \mathrm{h}_{1}=\mathrm{h}_{3}{ }^{\mathrm{H}} \mathrm{h}_{4}$ $=\mathrm{h}_{4}{ }^{\mathrm{H}} \mathrm{h}_{3}=h_{1} h_{2}{ }^{*}+h_{2} h_{1}{ }^{*}+h_{3} h_{4}{ }^{*}+h_{4} h_{3}{ }^{*}$. From (10) we can get

$$
\begin{equation*}
\mathbf{r}=\mathbf{H c}=\mathbf{n} . \tag{13}
\end{equation*}
$$

### 4.2 Decoding

### 4.2.1 Maximum-likelihood Decoding Algorithm

The transmission matrix of QOSTBC is not completely orthogonal, so the signal needs joint detection at the receiver during maximum-likelihood (ML) decoding. Therefore, the complexity of decoding is higher than orthogonal STBC. From ML algorithm, the decoding can be calculated as follows:

$$
\begin{aligned}
\hat{c} & =\arg \min \left\|\mathbf{r}-\mathbf{C}_{w_{\mathbf{w}}} \mathbf{h}\right\|^{2} \\
& =\arg \min \left(\|\mathbf{r}\|^{2}-\mathbf{r}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}} \mathbf{h}-\mathbf{h}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}}^{\mathrm{H}} \mathbf{r}+\mathbf{h}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}} \mathbf{h}\right) \\
& =\arg \min \left(-2 \operatorname{Re}\left(\mathbf{r}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}} \mathbf{h}\right)+\mathbf{h}^{\mathrm{H}} \mathbf{C}_{\mathbf{w}_{1}}^{\mathrm{H}} \mathbf{C}_{w_{1}} \mathbf{h}\right)
\end{aligned}
$$

Where $\hat{c}$ is the symbol after mapping. Assuming $\mathbf{V}_{i}(i=1$, $2,3,4$ ) is the ith column of transmission matrix, we can get, $\left\langle\mathrm{v}_{1}, \mathrm{v}_{3}\right\rangle=\left\langle\mathrm{v}_{1}, \mathrm{v}_{4}\right\rangle=\left\langle\mathrm{v}_{2}, \mathrm{v}_{3}\right\rangle=\left\langle\mathrm{v}_{2}, \mathrm{v}_{4}\right\rangle=0$. Where $\left\langle\mathrm{v}_{i}, \mathrm{v}_{j}\right\rangle=\sum_{t=1}^{4}\left(\mathrm{v}_{i}\right)_{l}\left(\mathrm{v}_{j}\right)_{l}^{*}$ is the inner product of $\mathbf{v}_{i}$ and $\mathbf{v}_{j}$.

ML detection value at the receiver can be simplified to the sum of $f_{12}$ and $f_{34}$.

$$
\begin{aligned}
f_{12}\left(c_{1}, c_{2}\right)= & \arg \min \left(\sum_{i=1}^{4}\left|h_{i}\right|^{2}\left(\left|c_{1}\right|^{2}+\left|c_{2}\right|^{2}\right)\right. \\
& -2 \operatorname{Re}\left\{\left(h_{1} r_{1}^{*}+h_{2} r_{2}^{*}\right) c_{1}+\left(-h_{3} r_{3}^{*}+h_{4} r_{4}^{*}\right) c_{1}^{*}\right. \\
& +\left(h_{1} r_{2}^{*}+h_{2} r_{1}^{*}\right) c_{2}+\left(h_{3} r_{4}^{*}-h_{4} r_{3}^{*}\right) c_{2}^{*} \\
& \left.\left.+\left(h_{1} h_{2}^{*}+h_{2} h_{1}^{*}-h_{3} h_{4}^{*}-h_{4} h_{3}^{*}\right)\left(c_{1} c_{2}^{*}+c_{2} c_{1}^{*}\right)\right\}\right) \\
f_{34}\left(c_{3}, c_{4}\right)= & \arg \min \left(\sum_{i=1}^{4}\left|h_{i}\right|^{2}\left(\left|c_{3}\right|^{2}+\left|c_{4}\right|^{2}\right)\right. \\
& -2 \operatorname{Re}\left\{\left(h_{3} r_{1}^{*}+h_{4} r_{2}^{*}\right) c_{3}+\left(h_{1} r_{3}^{*}-h_{2} r_{4}^{*}\right) c_{3}^{*}\right. \\
& +\left(h_{3} r_{2}^{*}+h_{4} r_{1}^{*}\right) c_{4}+\left(-h_{1} r_{4}^{*}+h_{2} r_{3}^{*}\right) c_{4}^{*} \\
& \left.\left.+\left(h_{1} h_{2}^{*}+h_{2} h_{1}^{*}-h_{3} h_{4}^{*}-h_{4} h_{3}^{*}\right)\left(c_{3} c_{4}^{*}+c_{4} c_{3}^{*}\right)\right\}\right)
\end{aligned}
$$

Where $f_{12}\left(c_{1}, c_{2}\right)$ is determined by $c_{1}$ and $c_{2}$, and $f_{34}\left(c_{3}\right.$, $c_{4}$ ) is determined by $c_{3}$ and $c_{4}$.It needs the joint detection of $c_{1}, c_{2}$ and $c_{3}, c_{4}$ at the receiver.

### 4.2.2 QR Decomposition Decoding Algorithm

Channel H is decomposed by QR algorithm, we can get

$$
\begin{equation*}
\mathrm{H}=\mathrm{QR} . \tag{14}
\end{equation*}
$$

Where $\mathbf{R}=\left[\begin{array}{cccc}R_{11} & R_{12} & 0 & 0 \\ 0 & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & 0 & R_{44}\end{array}\right]$ is a $4 \times 4$ upper triangular matrix, and $\mathbf{R}_{11}=\mathbf{R}_{12}, \mathbf{R}_{33}=\mathbf{R}_{44}, \mathbf{R}_{12}=\mathbf{R}_{34}$. Where Q is a $4 \times 4$ orthogonal matrix, $\mathrm{Q}^{\mathrm{H}} \mathrm{Q}=\mathrm{I}_{4}, \mathrm{I}_{4}$, is a 4 $\times 4$ unit matrix. By multiplying QH to (14), we can get

$$
\begin{equation*}
\mathrm{r}^{\prime}=\mathrm{Rc}+\mathrm{n}^{\prime} . \tag{15}
\end{equation*}
$$

Where $\mathbf{r}^{\prime}=\left[r_{1}^{\prime}, r_{2}^{\prime}, r_{3}^{\prime}, r_{4}^{\prime}\right]^{\mathrm{T}}=\mathbf{Q}^{\mathrm{H}} \mathbf{r}$ and $\mathrm{n}^{\prime}=\left[n_{1}^{\prime}, n_{2}^{\prime}, n_{3}^{\prime}, n_{4}^{\prime}\right]^{\mathrm{T}}$ $=\mathbf{Q}^{\mathrm{H}} \mathbf{n}$ are signal vector and noise vector of QR decomposition. $\mathbf{Q}$ is an orthogonal matrix. So $n$ is also a complex white Gaussian noise vector. According to ML algorithm, we can get

$$
\begin{equation*}
\hat{c}=\underset{c \in C}{\arg \min }\left\|\mathbf{r}^{\prime}-\mathbf{R c}\right\|^{2} . \tag{16}
\end{equation*}
$$

When $d_{1}=\left\|r_{1}^{\prime}-R_{11} c_{1}-R_{12} c_{2}\right\|^{2}, d_{2}=\left\|r_{2}^{\prime}-R_{22} C_{2}\right\|^{2}$, $d_{3}=\left\|r_{3}^{\prime}-R_{33} C_{3}-R_{34} C_{4}\right\|^{2}, d_{4}=\left\|r_{4}^{\prime}-R_{44} C_{4}\right\|^{2}$, (16) is equivalent with

$$
\begin{equation*}
\hat{c}=\underset{c \in C}{\arg \min } \sum_{k=1}^{4} d_{k} \tag{17}
\end{equation*}
$$

From the expression of $\boldsymbol{d}_{1}$ and $\boldsymbol{d}_{3}$, we can use joint decoding of ( $c_{1}, c_{2}$ ) and ( $c_{3}, c_{4}$ ) in order to achieve best performance of ML decoding algorithm. From (17), the joint decoding decision expression can be written as

$$
\begin{align*}
& \left(\hat{c}_{1}, \hat{c}_{2}\right)=\underset{\left(c_{1}, c_{2}\right) \in C}{\arg \min }\left(d_{1}+d_{2}\right), \\
& \left(\hat{c}_{3}, \hat{c}_{4}\right)=\underset{\left(c_{1}, c_{2}\right) \in C}{\arg \min }\left(d_{3}+d_{4}\right) . \tag{18}
\end{align*}
$$

Assuming the decision $\hat{c}_{2}$ and $\hat{c}_{4}$ are accurate, by taking $\hat{r}_{1}=r_{1}^{\prime}-R_{12} \hat{c}_{2}, \hat{r}_{3}=r_{3}^{\prime}-R_{34} \hat{c}_{4}$ into (18), we can get the decision vector of ML decoding.

### 4.2.3 MMSE decoding algorithm

The signal expression can be transformed as

$$
\begin{equation*}
\tilde{\mathbf{r}}=\mathbf{H}^{\mathrm{H}} \mathbf{r}=\mathbf{H}^{\mathrm{H}} \mathbf{H c}+\mathbf{H}^{\mathrm{H}} \mathbf{n}=\Delta_{4} \mathbf{c}+\tilde{\mathbf{n}} . \tag{19}
\end{equation*}
$$

Let $\tilde{\mathbf{n}}=\mathbf{H}^{\mathrm{H}} \mathbf{n}=\left[\begin{array}{llll}\tilde{\mathbf{n}}_{1} & \tilde{\mathbf{n}}_{2} & \tilde{\mathbf{n}}_{3}^{*} & \tilde{\mathbf{n}}_{4}^{*}\end{array}\right]^{\mathrm{T}}, \tilde{\mathbf{r}}=\left[\begin{array}{llll}\tilde{\mathbf{r}}_{1} & \tilde{\mathbf{r}}_{2} & \tilde{\mathbf{r}}_{3}^{*} & \tilde{\mathbf{r}}_{4}^{*}\end{array}\right]^{\mathrm{T}}$, we can get

$$
\left[\begin{array}{c}
\tilde{r}_{1} \\
\tilde{r}_{2} \\
\tilde{r}_{3}^{*} \\
\tilde{r}_{4}^{*}
\end{array}\right]=\left[\begin{array}{cccc}
a_{H} & b_{H W 1} & 0 & 0 \\
b_{H W 1} & a_{H} & 0 & 0 \\
0 & 0 & a_{H} & -b_{H W 1} \\
0 & 0 & -b_{H W 1} & a_{H}
\end{array}\right]\left[\begin{array}{c}
c_{1} \\
c_{2} \\
c_{3} \\
c_{4}
\end{array}\right]+\left[\begin{array}{c}
\tilde{n}_{1} \\
\tilde{n}_{2} \\
\tilde{n}_{3}^{*} \\
\tilde{n}_{4}^{*}
\end{array}\right] .
$$

According to MMSE (Minimum Mean Square Error) decoding algorithm, it needs to find matrix $\mathbf{W}_{4}^{\mathrm{H}}$ to match

$$
\begin{equation*}
\hat{c}=\arg \min \left\|\mathbf{W}_{4}^{\mathrm{H}} \tilde{\mathbf{r}}-\hat{\mathbf{c}}\right\|^{2}, \tag{20}
\end{equation*}
$$

so we can get $\mathbf{W}_{4}^{\mathrm{H}}$ by minimizing (20).

$$
\begin{equation*}
\mathbf{W}_{4}^{\mathrm{H}}=\left(\Delta_{4}^{\mathrm{H}} \Delta_{4}+\sigma_{n}^{2} \mathbf{I}_{4}\right)^{-1} \Delta_{4}^{\mathrm{H}}=\left(\Delta_{4} \Delta_{4}+\sigma_{n}^{2} \mathbf{I}_{4}\right)^{-1} \Delta_{4} \tag{21}
\end{equation*}
$$

### 4.2.4 Singular Value Decomposition decoding algorithm

The channel matrix $\mathbf{H}$ can be described as (11). $\mathbf{H}$ is a normal matrix, which can be discussed by using the singular value decomposition (SVD).

$$
\mathbf{H}=\mathbf{U} \boldsymbol{\Lambda} \mathbf{U}^{\mathrm{H}}
$$

where $\boldsymbol{\Lambda}=\operatorname{diag}\left[\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}\right], \lambda_{n}$ is the character value of H . By multiplying $\mathbf{U}^{\mathrm{H}}$ to (11), we can get

$$
\begin{equation*}
\mathbf{U}^{\mathrm{H}} \mathbf{r}=\boldsymbol{\Lambda} \mathbf{U}^{\mathrm{H}} \mathbf{c}+\mathbf{U}^{\mathrm{H}} \mathbf{n} \tag{22}
\end{equation*}
$$

Equation (22)can be expressed in short:

$$
\begin{equation*}
\mathbf{r}^{\prime}=\boldsymbol{\Lambda} \mathbf{c}^{\prime}+\mathbf{n}^{\prime} \tag{23}
\end{equation*}
$$

where, $\quad \mathbf{r}^{\prime}=\left[r_{1}^{\prime}, r_{2}^{\prime}, r_{3}^{\prime}, r_{4}^{\prime}\right]^{\mathrm{T}}=\mathbf{U}^{\mathrm{H}} \mathbf{r}, \quad \mathbf{n}^{\prime}=\left[n_{1}^{\prime}, n_{2}^{\prime}, n_{3}^{\prime}, n_{4}^{\prime}\right]^{\mathrm{T}}$ $=\mathbf{U}^{\mathrm{H}} \mathbf{n}$ are signal vector and noise vector after diagonalization. $\mathrm{U}^{\mathrm{H}}$ is a unitary matrix, so $\mathbf{n}^{\prime}$ is also a complex white Gaussian noise vector. The four independent symbols can be decoded respectively.

$$
\begin{equation*}
\hat{c}^{\prime}=\underset{c^{\prime} \in C}{\arg \min }\left\|\mathbf{r}^{\prime}-\boldsymbol{\Lambda} \mathbf{c}^{\prime}\right\|^{2} \tag{24}
\end{equation*}
$$

## 5 Simulation and Analysis

The measurement of error probability can be obtained by calculating the diversity product [6]. We show it as

$$
\begin{equation*}
\lambda=\min _{\{\mathbf{C} \neq \bar{C}\}}\left|\operatorname{det}\left[(\mathbf{C}-\tilde{\mathbf{C}})^{\mathrm{H}}(\mathbf{C}-\tilde{\mathbf{C}})\right]\right|^{1 / 2 N} \tag{25}
\end{equation*}
$$

where N is the number of transmit antennas, $\tilde{\mathbf{C}}$ is the erroneous estimate of $\mathbf{C}$. Assuming $\mathbf{B}=(\mathbf{C}-\tilde{\mathbf{C}})$, and $\tilde{x}_{i}=\left(x_{i}-\tilde{x}_{i}\right), \mathrm{i} \in\{1,2,3,4\}$ as elements of error matrix $\mathbf{B}$, the diversity products from the first novel code is as follows:

$$
\begin{align*}
& \left.\lambda_{W 1}=\min _{\{C \neq C\}} \mid \operatorname{det}\left[\mathbf{B}_{W 1}^{\mathrm{H}} \mathbf{B}_{W 1}\right]\right]^{1 / 2 N} \\
& \quad=\min _{\{C \neq C\}}\left|\operatorname{det}\left[\begin{array}{cccc}
a & b_{W 1} & 0 & 0 \\
b_{W 1} & a & 0 & 0 \\
0 & 0 & a & -b_{W 1} \\
0 & 0 & -b_{W 1} & a
\end{array}\right]\right|^{1 / 2 N} \tag{26}
\end{align*}
$$

where $a=\sum_{i=1}^{4}\left|x_{i}\right|^{2}, b_{W 1}=x_{1} x_{2}{ }^{*}+x_{2} x_{1}{ }^{*}+x_{3} x_{4}{ }^{*}+x_{4} x_{3}{ }^{*}$, $\operatorname{det} \mathbf{B}_{w 1}^{\mathrm{H}} \mathbf{B}_{w 1}=\left(a^{2}-b_{w 1}^{2}\right)^{2}$.

Figure 1 provides the simulation result of the novel QOSTBC under different decoding methods. The simulation conditions of Figure 1 are quasi-static flat fading channel. The transmit power is equivalent, and receiving
noise is independent complex Gaussian random variable. It uses the QPSK modulation, and the signal power is 1.


Figure 1. The BER performance comparison of decoding algorithms

As shown in Fig.1, the BER of ML algorithm is the best among the given algorithm. MMSE algorithm is a suboptimal method. And the performance of QR decomposition is the worst. It is because that the weight vectors of MMSE algorithm are generated by the principle of minimum mean square error. The channel response matrix and the influence of noise in receiving signal are both taken into account. Therefore, the BER of the system is improved. There is a lack of eliminating noise process of channel in QR decomposition algorithm and SVD algorithm. However, SVD algorithm for novel schemes can decode for four independent single symbols respectively. The complexity of decoding is lower. SVD is a suboptimal decoding algorithm for QOSTBC.

## 6 Conclusions and Discussions

The existing ML, MMSE and QR decomposition decoding algorithm for QOSTBC are analyzed in this paper. A SVD decoding algorithm was developed for the novel QOSTBC. The performance of different encoding algorithm and decoding algorithm are compared by simulation. It can be concluded that SVD decoding algorithm is not the optimal. However, it has the advantage of low complexity due to decoding independent symbols respectively. It is a suboptimal decoding algorithm for QOSTBC.

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