Cost Optimization of Gompertz Distribution Accelerated Burn-in

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Abstract: Many products have a high failure rate in their early operating lives. Burn-in has been widely accepted as a method of screening out defects before a product is shipped to the customer. In the literature it is often assumed that the failure pattern follows a specific distribution and the burn-in process is operated under approximately the same environment as that of the early operating life of the product. In this paper we require the product life distribution to have some specified properties. The burn-in process is operated under severe (stress) conditions involving high temperature, voltage, etc. and the product's residual life depends on the burn-in stress level and the length of burn-in period. Accelerated burn-in before shipment will reject poor-quality products and improve product reliability within a warranty period. Accelerated burn-in saves time but may cost more. We found the appropriate testing parameters to minimize the total of testing, manufacturing, quality and reliability costs. The upper and lower bounds for the optimal burn-in time are derived.

Keywords: gompertz distribution; accelerated burn-in; cost optimization

1. Introduction

Many products have a high failure rate in their early operating lives. Burn-in has been widely accepted as a method of screening out defects before a product is shipped to the customer. A common practice is to test the product until it reaches the change-point where the product failure rate decreases in the infant mortality stage to a constant level in the normal stage [1-3]. [4-7] study the effect of burn-in on the mean residual life of the product. [17-21] study failure rate model. [8-11] study Economic designs of burn-in procedures. [12-14] study General discussions about burn-in.

In the literature it is often assumed that the failure pattern follows a specific distribution and the burn-in process is operated under approximately the same environment as that of the early operating life of the product. In this paper we require the product life distribution to have some specified properties. The burn-in process is operated under severe (stress) conditions involving high temperature, voltage, etc. and the product's residual life depends on the burn-in stress level and the length of burn-in period. Accelerated burn-in before shipment will reject poor-quality products and improve product reliability within a warranty period. Accelerated burn-in saves time but may cost more. Our goal is to find the appropriate testing parameters to minimize the total of testing, manufacturing, quality and reliability costs. The upper and lower bounds for the optimal burn-in time are derived.

2. Notations and assumptions

We assume that every finished product is subject to an accelerated burn-in test and the product is non-repairable. There is a known relationship between stress conditions and the product life distribution [15, 16]. Let λ_0 and λ_1 be stress level of normal operation and the stress level of burn-in test respectively. There levels may be a function of several stress parameters of operating conditions. Without loss of generality, we assume $\lambda_0 \le \lambda_1$. The burn-in process may affect the residual life of the product so that the life distribution of the product is not the same as that of no burn-in under normal stress level λ_0. We assume that the product's residual life is equivalent to that under a more severe stress level than normal level and that this stress level λ is a function of λ_0 and λ_1. A typical function form is $\lambda_r = \left(\lambda_1 / \lambda_0\right)^k \lambda_0$, where $0 \le k \le 1$.

Let t_1 be the burn-in time. The life times of the product under stress levels λ_0, λ_1 and λ_r are denoted by X_0, X_1 and X_r respectively. The product is scrapped and has a life time X_1 if the product fails during the burn-in period. The product is shipped to the customer and has a residual life time X_r if the product passes the burn-in test. Products with no burn-in have life time X_0.

Notations

These include:

c_s: burn-in set-up cost;

$c_0(\lambda_1)$: burn-in cost per unit time which is an increasing function of the stress level;

c_1: burn-in failure cost per unit;

$c_2(t)$: loss of goodwill cost when a failure occurs at time t burn-in cost per unit time which is an increasing function of the stress level;

$c_3(t)$: operating failure cost when a failure occurs at time t with the customer;

$h_1(t)$: hazard rate function under stress level λ_i, i=0, 1 and r;

$F_i(t)$: distribution function associated with $F_i(t)$.

Assumptions

These include:

(1) $h_1(t)=\lambda_i g(\lambda_i t)$, i=0, 1 and r, where $g(t)$ is a hazard rate function satisfying;

$g(t)\to\infty$ as $t\to 0$;

$g(t)$ is decreasing for $t\geq 0$;

$g(t+s)/g(t)$ is an increasing function of t for $s\geq 0$

Some often used distributions satisfying assumption (1) are: Gompertz distribution with

$F(y)=1-\exp\left\{-\frac{\theta_1}{\beta}\left(e^{\beta y}-1\right)\right\}$, $\theta_1>0$ tampered failure rate model with $\lambda_1(y)=\theta_1 e^{\beta\cdot y}$, $\beta>0$ hazard rate function with $g(t)=\delta\cdot t^{\delta-1}, 0<\delta<1$.

(2) The loss of goodwill cost $c_2(t)$ and the operating failure cost $c_3(t)$ satisfies: Cost optimization $c_1(t)$ is decreasing for $0\leq t\leq T_i$ and $c_i(t)=0$, for $t>T_i$, i = 2 and 3.

$c_i(t)$ is differentiable for $0\leq t\leq T_i$ and $|\partial c_i(t)/\partial t|\leq M$,where $M>0$.

Some typical function of $c_2(t)$ are constant cost: $c_2=\begin{cases}c_2, & 0\leq t\leq T_2\\ 0, & t>T_2\end{cases}$; Linear decreasing cost: $c_2=\begin{cases}c_2(1-t/T_2), & 0\leq t\leq T_2\\ 0, & t>T_2\end{cases}$; and exponentially decreasing cost $c_2(t)=c_2 e^{-\beta\cdot t^{\alpha}}, t\geq 0$.

If the product passes the burn-in test, then testing for time t_1 under stress level λ_1 is equivalent to using the product for time t_r under stress level λ_r, where

$$F_r(t_r)=F_1(t_1) \tag{1}$$

Here, the residual life is affected by the length of the burn-in period as well as the stress level λ_1.

3. COST MODEL

If the product fails during the burn-in period, the cost per unit consists of the burn-in failure cost and the testing cost. The product is shipped to the customer if it passes the burn-in test. The cost per unit consists of three parts. They are the burn-in cost in the burn-in period; the loss of goodwill cost when the product fails in the warranty period; and the operating failure cost which occurs when the product fails during the servicing period. The total cost per unit subject to a burn-in time t_1 is

$$\begin{aligned}TC(t_1)&=c_s+c_0(\lambda_1)X_1 I(X_1\leq t_1)\\&+c_0(\lambda_1)t_1 I(X_1>t_1)+c_1 I(X_1\leq t_1)\ c_2(X_r-t_r)I(X_r>t_r)\\&+c_3(X_r-t_r)I(X_r>t_r), t_i>0\end{aligned} \tag{2}$$

where $I(\cdot)$ is the indicator function. Taking expectation, we have

$$\begin{aligned}m(t_1)&=E[TC(t_1)]\\&=c_c+c_0(\lambda_1)\int_0^{t_1} s\,dF_1(s)+c_0\lambda_1 t_1(1-F_1(t_1))+c_1F_1(t_1)\\&+\int_{t_r}^{t_r+T_2}c_2(s-t_r)dF_r(s)+\int_{t_r}^{t_r+T_3}c_3(s-t_r)dF_r(s)=M(t_r)\end{aligned} \tag{3}$$

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Where $t_r = \lambda_1 t_1 / \lambda_r$ (4)

from assumption (1) and equation (1).

The total cost for the product without burn-in is

$$TC(0) = c_2(X_0) + c_3(X_0) \quad (5)$$

and $m(0) = \int_0^{T_2} c_2(s)dF_0(s) + \int_0^{T_3} c_3(s)dF_0(s)$ (6)

4. Optimal Burn-in Time

The optimal burn-in time is determined by minimizing the expected total cost $m(t_1)$ of operating an item in the burn-in period and the servicing period. Taking a derivative of the expected total cost with respect to t_r,

$$\partial M(t_r)/\partial t_r = (1 - F_r(t_r))$$
$$\{c_0(\lambda_1)\lambda_r/\lambda_1 - (c_2(0) + c_3(0) - c_1)h_r(t_r) + A(t_r)h_r(t_r)\} \quad (7)$$

where

$$A(t_r) = \int_0^{T_2} (-\partial c_2(s)/\partial s)(f_r(t_r + s)/f_r(t_r))ds$$
$$+ \int_0^{T_3} (-\partial c_3(s)/\partial s)(f_r(t_r + s)/f_r(t_r))ds$$
$$+ c_2(T_r) f_r(t_r + T_2)/f_r(t_r) + c_3(T_3) f_r(t_r + T_3)/f_r(t_r) \quad (8)$$

and $f_r(t)$ is the density function of $F_r(t)$.

Theorem 1

(a) If $c_2(0) + c_3(0) \le c_1$, the optimal burn-in time is 0:

$$m(0) = \min_{t_1 \ge 0}(t_1).$$

(b) If $c_2(0) + c_3(0) > c_1$, there exists t_1^* such that

$$m(t_1^*) = \min_{t_1 > 0} m(t_1)$$

and t_1^* is finite if and only if

$$c_0(\lambda_1)\lambda_1/\lambda_1 - (c_2(0) + c_3(0) - c_1 - A_0)\lambda_r d > 0$$

where

$$d = \lim_{t\to\infty} g(t)$$

$$A_0 = \lim_{t_r\to\infty} A(t_r) = \int_0^{T_2}\left(-\frac{\partial c_2(s)}{\partial s}\right)g_r(s)ds +$$

$$\int_0^{T_3}\left(-\frac{\partial c_3(s)}{\partial s}\right)g_r(s)ds + c_2(T_2)g_r(T_2) + c_3(T_3)g_r(T_3)$$

and $g_r(s) = \lim_{t\to\infty} f_r(t+s)/f_r(t)$.

The optimal burn-in time is greater than 0 when $m(0) - m(t_1^*) > 0$.

Proof: Note that $f_r(t+s)/f_r(t)$ is increasing with respect to t for $s \ge 0$. Assumption (2) gives that A(t) is increasing for $t \ge 0$ and $\lim_{t\to 0} A(t) = 0$.

Let $\partial M(t_r)/\partial t_r = f_r(t_r)R(t_r)$

where

$$R(t_r) = (\lambda_r/\lambda_1)c_0(\lambda_1)/h_r(t_r) - (c_2(0) + c_3(0) - c_1) + A(t_r)$$

and the sign $\partial M(t_r)/\partial t_r$ depends on $R(t_r)$.

From the properties of $A(t)$ and assumption (1), we have

$$\lim_{t_r\to 0} R(t_r) = -(c_2(0) + c_3(0) - c_1)$$

$$\lim_{t_r\to\infty}(t_r) = c_0(\lambda_1)/(\lambda_1 d) - (c_2(0) + c_3(0) - c_1) + A_0$$

and $R(t_r)$ is increasing.

In case (a), $R(t_r) \ge 0$ for all $t_r > 0$ gives $\partial M(t_r)/\partial t_r \ge 0$. Thus, $M(t_r)$ is an increasing function of t_r and $M(t_1)$ is an increasing function of t_1. The optimal plan is no burn-in.

In case (b), $\lim_{t_r\to\infty} R(t_r)$ is positive if and only if $c_0(\lambda_r)/\lambda_1 - (c_2(0) + c_3(0) - c_1 - A_0) - \lambda_1 d > 0$.

If $\lim_{t_r\to\infty} R(t_r)$ is positive, there exists a unique t_r^* such that $R(t_r^*) = 0$. Also t_r^* is infinite when $\lim_{t_r\to\infty} R(t_r)$ is negative.

Theorem2

If the optimal burn-in time is finite and positive, then

$$\frac{\lambda_r}{\lambda_1} f_r^{-1}\left[\frac{\frac{\lambda_r}{\lambda_1}c_0(\lambda_1) + c_2(0)f_r(T_2) + c_3(0)f_r(T_3)}{c_2(0) + c_3(0) - c_1}\right] \le t_1^*$$

$$\le \frac{\lambda_r}{\lambda_1} h_r^{-1}\left(\frac{\lambda_r}{\lambda_1}\frac{c_0(\lambda_1)}{c_2(0) + c_3(0) - c_1}\right)$$

Proof: Observe that $A(t) \ge 0$ and $R(t_r^*) = 0$. It follows that $(\lambda_r/\lambda_1)c_0(\lambda_1)/h_r(t_r^*) \le c_2(0) + c_3(0) - c_1$

Hence

$$t_r^* \le h_r^{-1}\left[\frac{\lambda_r}{\lambda_1}\frac{c_0(\lambda_1)}{c_2(0) + c_3(0) - c_1}\right]$$

Note that $t_1^* = t_r^* \lambda_r / \lambda_1$ .We obtain the right-hand side of the inequality. The proof of the left-hand side of the inequality follows from the fact that

$A(t) \le c_2(0) f_r(T_2)/ f_r(t) + c_3(0) f_r(T_3)/ f_r(t)$

and $f_r(t) \le h_r(t)$.

The results in Theorems 1 and 2 identify the appropriateness of implementing the burn-in process before shipment. Theorem 2 gives the upper and lower bounds of the optimal burn-in time for a burn-in stress level λ_1. The result is useful in estimating the optimal burn-in time numerically.

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