Combined Fuzzy Temporal Reasoning System of Qualitative and Quantitative on Points or Intervals

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Abstract: The durations of events in real world are difficult to measure precisely. The starting points and ending points of events are fuzzy. It makes it difficult to describe the fuzzy time intervals of events in Allen's interval algebra. To deal with this problem, makes use of Heisenberg's uncertainty principle to compress Meiri's five pairs of relations between points and intervals into three ones, the thirteen relations of interval algebra into six ones to get a kind of fuzzy interval algebra. It simplifies the composition operations between intervals and intervals, points and intervals, intervals and points, points and points. It decreases the complexity of temporal reasoning, improves the efficiency of computations. We provide the theoretical research and experiments. The result of the experiments shows that the efficiency of the fuzzy interval algebra is better than the efficiency of Allen's interval algebra.

Keywords: fuzzy system; interval algebra; temporal reasoning; uncertainty principle

1. Introduction

Allen proposed the interval algebra[1] in 1983. The theory has gotten applications in natural language understanding, agents, plan recognition, combat cooperation and so on[2-5]. Interval algebra defined thirteen basic temporal relations. It can be represented by I= {b,bi,m,mi,o,oi, s,si,d,di,f,fi,eq }.

Vilain, and Kautz proposed the point algebra[6] in 1986. There are three cases for any two time points P1 and P2: P1<P2; P1=P2; P1>P2. It can be represented by PP={<,=,>}.

There are five pairs of relations between points and intervals[7]: They are before(b), starts(s), during(d), finishes(f), after(a). Similarly, there are five pairs of relations between intervals and points: before by(bi), starts by(si), during by(di), finishes by(fi), after by(ai). Use P to represent time point, I to represent time intervals. The uncertainty relations between points and intervals can be represented by P{b, s, d, f, a}I. The uncertainty relations between intervals and points can be represented by I{bi, si, di, fi, ai}P. There are 2^5-1=31 (The empty set has no meaning) temporal relations between points and intervals. There are also 2^5-1=31 temporal relations between intervals and points.

2. Fuzzy Interval Algebra

First, according to Heisenberg's uncertainty principle we know that the endpoints of time intervals of events in real world can not be given precisely. Secondly, given two real numbers a and b randomly. Suppose they are the left end points of two intervals. Then the possibility of a=b is 1/K[R]=0, where K[R] is the cardinality of the real set.

For the two reasons above we say that the seven basic temporal relations {m, mi, s, si, f, fi, eq} are trivial, they can not happen in real word, and write Trivial={ m, mi, s, si, f, fi, eq}. We delete Trivial={ m, mi, s, si, f, fi, eq}from Allen's thirteen basic temporal relations to get a fuzzy interval algebra FI={b, bi, d, di, o, oi}. The expressing ability of FI is nearly as strong as Allen's interval algebra, but FI is much simpler than I.

We can convert the trivial temporal relations to relations in FI. For example, Suppose that [a1, b1] is the time interval of one event measured by someone, and [a2, b2] is the interval of another event measured by the same person. If [a1, b1] s [a2, b2], or , a1=a2, b1<b2. Then according to Heisenberg's uncertainty principle, it means that a1 and a2 are very close. We can express it in mathematics as follows: There exists a positive number ε which is small enough such that a1 $-\varepsilon \leqslant a2 \leqslant a1 + \varepsilon$. We get [a1, b1]o[a2, b2] or [a1, b1]d [a2, b2]. Hence, in this fuzzy temporal reasoning system, we convert s to {o, d }. It can be represented by s →{o,d }. Similarly, we have m→{b,o}; mi→{bi,oi}; si →{oi,di}; f→{d,oi}; fi→{di,o}; eq→{o,oi,d,di}. We denote this fuzzy interval algebra with FIA, and write FI={b, bi, d, di, o, oi }.

For the same principle, we can delete the trivial relations {s, f} from the five basic temporal relations {b, s, d, f, a} between points and intervals, and get the fuzzy system {b, d, a}. We can delete the trivial relations {si, fi} from the five basic temporal relations {bi, si, di, fi, ai} between intervals and points, and get the fuzzy system {bi, di, ai}. These trivial relations can be converted as follows: s→{b,d};si→{bi,di};f→{d,a};fi→{di,ai}. Finally, we get the fuzzy system between points and points {<, >}.

3. Combined Fuz zy Qualita tive Te mporal Reasoning System on Points or Intervals

As above, we can simplify Allen's 213-1=8191 temporal relations to fuzzy II's (Interval-Interval) 26-1=63 temporal relations. Simplify PI (Point-Interval) to 23-1=7 temporal relations, IP(Interval-Point) to 23-1=7 temporal relations, PP(Point-Point) to 22-1=3 temporal relations.

Now there are four kinds of constraints: II, PI, IP, and PP. Suppose that R' and R'' are two constraints. We define the compositional operation as follows[7]:

R' $\otimes$ R''={r' $\otimes$ r''| r'∈R', r'' ∈R''}, r', r'' are basic temporal relations.

For example, {b} $\otimes$ {d, di}={b, o, d}; {<} $\otimes$ {d}={b, d}.

The compositional operation of basic temporal relations be given at its operation table. There are six operation tables: T1, T2, T3, T4, TPA, TIA. The compositional operation of different kinds of constraints can be found at different operation tables[7] as in Table1.

Table 1. Combined operation table of points and intervals

	PP	PI	IP	II
PP	[TPA]	[T1]	[Φ]	[Φ]
PI	[Φ]	[Φ]	[T2]	[T4]
IP	$[T1]^t$	[T3]	[Φ]	[Φ]
II	[Φ]	[Φ]	$[T4]^t$	[TIA]

The Φ means no meaning, or, the compositional operation can not be found here. We will give the six operation tables later.[TPA]: P1P2 $\otimes$ P2P3 = P1P3. The operation table is as follows:

Table 2. Operation table of PP and PP

TPA	<	>
<	<	?
>	?	>

The ? means uncertain completely, or, {<, >}.

[TIA]: I1I2 $\otimes$ I2I3=I1I3. The operation table is as follows:

Table 3. Operation table of II and II

TIA	b	bi	d	di	o	oi
b	b	?	b,o,d	b	b	b,o,d
bi	?	bi	bi,oi,d	bi	bi,oi,d	bi
d	b	bi	d	?	b,o,d	bi,oi,d
di	b,o,di	bi,oi di	o,oi, d,di	di	di, o	di,oi
o	b	bi,oi, di	o,d	b,o, di	b,o	o,oi, d.di
oi	b,o, di,	bi	oi,d	bi,oi, di	o,oi, d,di	bi,oi

The ? means uncertain completely, or, {b,bi,d,di, o,oi }.

[T1]: P1P2 $\otimes$ P2I=P1I ; $[T1]^t$: IP1 $\otimes$ P1P2=IP2, $[T1]^t$ means the transpose of [T1]. The operation table of [T1] is as follows:

Table 4. Operation table of PP and PI

T1	b	d	a
<	b	b,d	?
>	?	d,a	a

The ? means uncertain completely, or, { b,d,a }

[T2]: P1I $\otimes$ IP2=P1P2. The operation table is as follows:

Table 5. Operation table of PI and IP

T2	ai	di	bi
b	<	<	?
d	<	?	>
a	?	>	>

The ? means uncertain completely, or, {<, > }.

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[T3]: I1P ⊗ PI2=I1I2. The operation table is as follows:

Table 6. Operation table of IP and PI

T3	b	d	a
ai	b	b,o,d	?
di	b,o,di	o,oi,d,di	bi,oi,di
bi	?	bi,oi,d	bi

The ? means uncertain completely, or, { b,bi,d,di,o,oi }.

[T4] : P1I1 ⊗ I1I2=P1I2 , $[T4]^t$: I1I2 ⊗ I2P1=I1P1. $[T4]^t$ means the transpose of [T4]. The operation table of [T4] is as follows:

Table 7. Operation table of PI and II

T4	b	bi	d	di	o	oi
b	b	?	b,d	b	b	b,d
d	b	a	d	?	b,d	d,a
a	?	a	d,a	a	d,a	a

The ? means uncertain completely, or, { b,d,a }

4. Quantitative constraints on points

For the quantitative constraints[7] on points we have:

Unary quantitative constraints on points: To constraint a point Pi to given intervals {I1 , I2 , …, Ik} may be expressed:

(Pi∈ I1) ∨ (Pi ∈ I2) ∨------- ∨ (Pi ∈ Ik), or, Pi ∈ {I1 ,I2 ,…Ik}.

Binary quantitative constraints on points: To constraint the distance of two points Pi and Pj may be expressed:

(Pj-Pi∈ I1) ∨ (Pj-Pi∈ I2) ∨------- ∨ (Pj-Pi ∈ Ik), or, Pj-Pi ∈ {I1 ,I2 ,…Ik}.

The initial time point is called P0. It is the minimal time point.

Example: Helen left home between 7:05 or 7:10. Given P0=7:00. Unary constraint: If denote the time Helen left home as P1. Then P1∈{(5,10)}. Binary constraint: P1-P0∈{(5,10)}.

Scenario: It needs Helen thirty minutes or more from home to the office on foot. It only spends her ten minutes or less if by car.

Suppose that the time when Helen get to office is P2. The binary constraint is: P2-P1∈{(0,10),(30,+∞)}.

Define operations on the quantitative constraints as follows: C’, C’’ are quantitative constraints, the interval sets that they belong to are I’, I’’ respectively. The operations on the interval sets may be defined as follows[7]:

Intersection: C’∩ C’’={x | x∈I’, x∈I’’}

Composition: C’ ⊗ C’’=’’={z | x∈I’,y∈I’’,x+y=z}

For example, If C1={(1,4),(6,8)} and C2={(3,5),(6,7)}. Then C1∩C2={(3,4),(6,7)}. Another example: If C3={(1,2),(6,8)} and C4={(0,3),(12,15)}. Then C3 ⊗ C4={(1,5),(6,11),(13,17),(18,23)}.

5. The relatio ns bet ween qualitativ e constraints and quantitative constraints

Consider a pair of points Pi and Pj. If a quantitative constraints, C, between Pi and Pj is given (by an interval set {I1,…,Ik}), then the implied qualitative constraint, QUAL(C), is defined as follows[7]:

If there exists a value v>0 such that v∈{I1 ,I2 ,…Ik}, then “<” ∈QUAL(C);

If there exists a value v<0 such that v∈{I1 ,I2 ,…Ik}, then “>” ∈QUAL(C);

Similarly, if a qualitative constraint, C, between Pi and Pj is given (by a relation set R), then the implied quantitative constraint, QUAN(C), is defined as follows[7].

If “<”∈R, then (0,+∞) ∈QUAN(C);

If “>”∈R, then (-∞,0) ∈QUAN(C).

They can be expressed in table 8.

Table 8. Relational table of qualitative and quantitative

QUAL(C)	QUAN(C)
<	(0,+∞)
>	(-∞,0)
?	(-∞,+∞)

The intersection and composition operations can be extended to cases where the operands are constraints of different types[7]. If C’ is a quantitative constraint and C’’ is qualitative, then intersection is defined as quantitative intersection:

C’∩C’’=C’∩QUAN(C’’).

Composition, on the other hand, depends on the type of C''.

If C'' is a PP relation, then composition (and consequently the resulting constraint) is quantitative:

$$C' \otimes C'' = C' \otimes QUAN(C'');$$

If C'' is a PI relation, then composition is qualitative:

$$C' \otimes C'' = QUAL(C') \otimes C''.$$

Illustration Let C1={(0,3}} be a quantitative constraint, C2={<} be a PP relation, and C3={b, d} be a PI relation. Then, $C1 \otimes C2=\{(0,3)\} \otimes \{(0,+\infty)\} =\{(0,+\infty)\}$. According to table 4, we have $C1 \otimes C3=\{<\} \otimes \{b,d\}=(\{<\} \otimes \{b\})\cup(\{<\} \otimes \{d\})=\{b\}\cup\{b,d\} =\{b,d\}$.

6. Applications of fuzzy interval algebra

Consider a fuzzy temporal reasoning problem. We are given the following information.

Example Bill and John work for the same company. It takes Bill less than twenty minutes to get to work by taxi, or at least sixty minutes on foot. While John needs 15-20 minutes to get to work. Today, Bill left home between 7:05-7:10 a.m., and John arrived at work between 7:50-7:55 a.m. We also know that Bill and John met at a traffic light on their way to work. Now we have two questions: Is the information in this story consistent? Who was the first to arrive at work?

Solution: Let B=(P1, P2) denote the event: Bill get to work from his home, where P1 is the time when Bill leave his home and P2 is the time when Bill get to work. Similarly, let J=(p3, p4) denote the event: John get to work from his home, where P3 is the time when John leave his home and P4 is the time when John get to work. Let P0 is the time at 7:00 a.m. that day.

We know that Bill and John met at a traffic light on their way to work. So B overlaps J, or, B{o,d,oi,di}J. It is easy to see that P1{b}B, P3{b}J, P2{a}B, P4{a}J.

There are quantitative constraints between P1 and P2, P3 and P4: P1{(0,20),(60,+∞)}P2, P3{(15,20)}P4.

Consider these constraints on points and intervals all together and then have the path-consistency calculation. Finally we get the table as follows:

Table 9. Combined operation table

	P0	P1	P2	P3	P4	B	J
P0		(5,10)	(65,+∞)	(30,40)	(50,55)	b	b
P1	(-10,-5)		(60,+∞)	(20,35)	(40,50)	b	b
P2	(-∞, -65)	(-∞, 60)		(-∞, -25)	(-∞, -10)	a	a
P3	(-40, -30)	(-35, -20)	(25, +∞)		(15,20)	d	b
P4	(-55, -50)	(-50, -40)	(10, +∞)	(-20, -15)		d	a
B	bi	bi	ai	di	di		di
J	bi	bi	ai	bi	ai	d	

The temporal relations in each cell of table 9 are basic, and no contradiction. So the information in this story is consistent. Because P2{(-∞,-10)}P4, So P4<P2, or, John arrived at work first.

Experiments

The configurations of the experiments are: Windows XP, 512M main memory, Intel's Celeron 2.0G CPU, Use Visual C++ to program. The constraints of interval algebra network are generated by Beek's function s(n,p)[8], where n is the number of intervals, p is a positive number that is less than 1.

Step 1. Generate the underlying constraints graph by indicating which of the possible n(n-1)/2 edges is present. Let each edge be present with the probability p, independently of the presence or absence of other edges.

Step 2. If an edge occurs in the underlying constraint graph, randomly chose a nonempty subset of I for the edge. If an edge does not occur, label the edge with I, the set of all thirteen basic relations.

Step 3. Generate a "solution" of size n as follows. Generate n real intervals by randomly generating values for the end points of the intervals. Determine the consistent scenario by determining the basic relations which are satisfied by the intervals. Finally, add the solution to the IA network generated in Steps 1-2.

From this IA network, we can get a fuzzy IA network: Convert the trivial temporal relations to relations in FI as in section 2. So, for the same problem, we label the edges in different ways, and get an IA network and a fuzzy IA network respectively. The differences exist only in the

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labels of edges. After the experiments of path consistency on IA and fuzzy IA, we get the data as follows.

Table 10. Experiments of path consistency on IA and FIA

	20	30	50
IA	0.010	0.030	0.136
FIA	0.005	0.010	0.033

Notice: The cell "0.136" means that the time required for the path consistency of the IA network with fifty intervals is 0.136 seconds.

It is easy to see that the time required in FIA is much less than in IA.

7. Summary

Simplify the Allen's interval algebra IA to get a fuzzy interval algebra FIA, and show that the expressing ability of FIA is nearly as strong as IA. Define the combined qualitative and quantitative temporal operation table. Have the fuzzy temporal reasoning on points and points, points and intervals, intervals and points, intervals and intervals. The experiments show that the efficiency of FIA is much better than IA.

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